FOURIER-SARJA (FOURIER SERIES)

Jaksollisen signaalin $x(t)$ jaksonaika $T_0 = \frac{1}{f_0} = \frac{2\pi}{\omega_0}$, jossa f_0 on signaalin taajuus (ω_0 on kulmataajuus).

(The period of a periodical signal $x(t)$ is $T_0 = \frac{1}{f_0} = \frac{2\pi}{\omega_0}$ where f_0 is the frequency of a signal)

Jaksollinen signaali (A periodical signal) $x(t) = \sum_{n=-\infty}^{\infty} c_n \cdot e^{jn\cdot 2\pi f_0 t} = \sum_{n=-\infty}^{\infty} c_n \cdot \left[cos(n \cdot 2\pi f_0 t) + j \cdot sin(n \cdot 2\pi f_0 t) \right]$

F-sarjan kertoimet (Coefficients of the F-series)

$$c_n = \frac{1}{T_0} \int_{T_0} x(t) \cdot e^{-jn\cdot 2\pi f_0 t} dt = \frac{1}{T_0} \cdot \left[\int_{T_0} x(t) \cdot cos(n \cdot 2\pi f_0 t)\, dt - j \int_{T_0} x(t) \cdot sin(n \cdot 2\pi f_0 t)\, dt \right]$$

Kun $x(t)$ on reaalinen signaali, voidaan Fourier-sarja esittää trigonomisessa muodossa
(When $x(t)$ is real, its Fourier series may be expressed in the trigonometric form)

$x(t) = c_0 + \sum_{n=1}^{\infty} |2c_n| \cdot cos\left[n \cdot 2\pi f_0 t + arg(c_n) \right] = c_0 + \sum_{n=1}^{\infty} \left[a_n \cdot cos(n \cdot 2\pi f_0 t) + b_n \cdot sin(n \cdot 2\pi f_0 t) \right]$, jossa (where)

$a_n = 2 \cdot Re(c_n), \quad b_n = -2 \cdot Im(c_n)$ ja (and) $arg(c_n) = \frac{Im(c_n)}{Re(c_n)}$

FOURIER-MUUNNOS (FOURIER TRANSFORM)

Fourier-muunnos (Fourier transform)

$$X(f) = F\left[x(t)\right] = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} dt = \int_{-\infty}^{\infty} x(t) \cdot \cos(2\pi ft)\, dt - j \int_{-\infty}^{\infty} x(t) \cdot \sin(2\pi ft)\, dt$$

Fourier-käänteismuunnos (Inverse Fourier transform)

$$x(t) = F^{-1}\left[X(f)\right] = \int_{-\infty}^{\infty} X(f) \cdot e^{j2\pi ft} df = \int_{-\infty}^{\infty} X(f) \cdot \cos(2\pi ft)\, df + j \int_{-\infty}^{\infty} X(f) \cdot \sin(2\pi ft)\, df$$

Diskreetti Fourier-muunnos (Discrete Fourier transform, DFT)

$DFT\left[x(k \cdot T_S)\right] = X(l \cdot f_0) = \sum_{k=0}^{N-1} x(k \cdot T_S) \cdot e^{-j2\pi l f_0 k T_S}$,jossa (where) $f_0 = \frac{1}{N \cdot Ts} = \frac{f_S}{N}$.

f_S = näytetaajuus (sampling frequency), T_S = näyteväli (sampling interval) ja (and) N = näytemäärä (number of samples) .

LAPLACE- JA Z-MUUNNOKSET (LAPLACE AND Z-TRANSFORMS)

Laplace-muunnos (Laplace transform)

$X(s) = L\left[x(t)\right] = \int_{0}^{\infty} x(t) \cdot e^{-st} dt$, jossa (where) $s = \sigma + j\omega = \sigma + j2\pi f$

Z-muunnos (Z-transform)

$Z\left[x(n)\right] = X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$ 2-puoleinen muunnos (2-sided transform)

$Z\left[x(n)\right] = X(z) = \sum_{n=0}^{\infty} x(n) \cdot z^{-n}$ 1-puoleinen muunnos (1-sided transform)

FOURIER-MUUNNOKSEN TEOREEMAT (THEOREMS OF THE FOURIER TRANSFORM)

Operaatio (Operation)	*Funktio (Function)*	*Muunnos (Transform)*
Lineaarisuus (Linearity)	$a\cdot x(t)+b\cdot y(t)$	$a\cdot X(f)+b\cdot Y(f)$
Aikaviipe, -siirto (Time delay or time shift)	$x(t-t_d)$	$X(f)\cdot e^{-j2\pi f t_d}$
Asteikon vaihto (Scale change)	$x(a\cdot t)$	$\frac{1}{\lvert a\rvert}\cdot X\left(\frac{f}{a}\right)$
Konjugointi (Conjugation)	$x^*(t)$	$X^*(-f)$
Dualisuus (Duality)	$X(t)$	$x(-f)$
Taajuussiirto (Frequency shift)	$x(t)\cdot e^{j2\pi f_c t}$	$X(f-f_c)$
Lineaarinen modulaatio (Linear modulation)	$x(t)\cdot\cos(2\pi f_c t+\varphi)$	$\frac{1}{2}\left[e^{j\varphi}\cdot X(f-f_c)+e^{-j\varphi}\cdot X(f+f_c)\right]$
Derivointi (Differentiation)	$\frac{d^n x(t)}{dt^n}$	$(j2\pi f)^n\cdot X(f)$
Integrointi (Integration)	$\int_{-\infty}^{t} x(u)\,du$	$\frac{X(f)}{j2\pi f}$
Konvoluutio (Convolution)	$x(t)\otimes y(t)=\int_{-\infty}^{\infty} x(u)\cdot y(t-u)\,du$	$X(f)\cdot Y(f)$
Kertolasku (Multiplication)	$x(t)\cdot y(t)$	$X(f)\otimes Y(f)=\int_{-\infty}^{\infty} X(u)\cdot Y(f-u)\,du$
Kertolasku termillä t^n (Multiplication by t^n)	$t^n\cdot x(t)$	$\frac{1}{-j2\pi}\cdot\frac{d^n X(f)}{df^n}$

FOURIER-MUUNNOKSET (FOURIER TRANSFORMS)

Funktio (Function)	$x(t)$	$X(f)$
Suorakaide(pulssi) Rectangular (pulse)	$rect\left(\frac{t}{\tau}\right)=\Pi\left(\frac{t}{\tau}\right)$	$\tau\cdot sinc(f\tau)$
Kolmio(pulssi) Triangular (pulse)	$tria\left(\frac{t}{\tau}\right)=\Lambda\left(\frac{t}{\tau}\right)$	$\tau\cdot sinc^2(f\tau)$
Gaussin pulssi (Gaussian pulse)	$e^{-\pi\left(\frac{t}{\tau}\right)^2}$	$\tau\cdot e^{-\pi(f\tau)^2}$
Yksipuolinen (kausaalinen) eksponenttipulssi (One sided exponential pulse)	$e^{-\frac{t}{\tau}}\cdot u(t)$	$\frac{\tau}{1+j2\pi f\tau}$
Kaksipuolinen (ei-kausaalinen) eksponenttipulssi (Two sided exponential pulse)	$e^{-\frac{\lvert t\rvert}{\tau}}$	$\frac{2\tau}{1+(j2\pi f\tau)^2}$
Sinc-pulssi (Sinc pulse)	$sinc(2Wt)$	$\frac{1}{2W}\cdot rect\left(\frac{f}{2W}\right)$
Vakio (Constant)	A	$A\cdot\delta(f)$
Osoitin (Phasor)	$e^{j(2\pi f_c t+\varphi)}$	$e^{j\varphi}\cdot\delta(f-f_c)$
Kosiniaalto (Cosine wave)	$cos(2\pi f_c t+\varphi)$	$\frac{1}{2}\left[e^{j\varphi}\cdot\delta(f-f_c)+e^{-j\varphi}\cdot\delta(f+f_c)\right]$
Viivästetty impulssi (Delayed impulse)	$\delta(t-t_d)$	$e^{-j2\pi f t_d}$
Näytteistys (Sampling)	$\sum_{k=-\infty}^{\infty}\delta(t-k\cdot T_S)$	$f_S\cdot\sum_{n=-\infty}^{\infty}\delta(f-n\cdot f_S)$
Signum (Signum)	$sgn(t)$	$-\frac{j}{\pi f}$
Yksikköaskel (Step)	$u(t)$	$\frac{1}{2}\cdot\delta(f)+\frac{1}{j2\pi f}$

TRIGONOMETRISIA YHTÄLÖITÄ (TRIGONOMETRIC FORMULAS)

$$e^{\pm j\varphi} = \cos\varphi \pm j\cdot\sin\varphi$$

$$\cos\varphi = \frac{1}{2}\left(e^{j\varphi} + e^{-j\varphi}\right) = \sin\left(\varphi + 90^\circ\right)$$

$$\sin\varphi = \frac{1}{2j}\left(e^{j\varphi} - e^{-j\varphi}\right) = \cos\left(\varphi - 90^\circ\right)$$

$$\sin^2\varphi + \cos^2\varphi = 1$$

$$\cos^2\varphi - \sin^2\varphi = \cos(2\varphi)$$

$$\cos^2\varphi = \frac{1}{2}\left[1 + \cos(2\varphi)\right]$$

$$\sin^2\varphi = \frac{1}{2}\left[1 - \cos(2\varphi)\right]$$

$$\cos^3\varphi = \frac{1}{4}\left[3\cdot\cos\varphi + \cos(3\varphi)\right]$$

$$\sin^3\varphi = \frac{1}{4}\left[3\cdot\sin\varphi - \sin(3\varphi)\right]$$

$$\sin(\alpha\pm\beta) = \sin\alpha\cdot\cos\beta \pm \cos\alpha\cdot\sin\beta$$

$$\cos(\alpha\pm\beta) = \cos\alpha\cdot\cos\beta \mp \sin\alpha\cdot\sin\beta$$

$$\tan(\alpha\pm\beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\cdot\tan\beta}$$

$$\sin(3\varphi) = 3\sin(\varphi) - 4\sin^3(\varphi)$$

$$\cos(3\varphi) = 4\cos^3(\varphi) - 3\cos(\varphi)$$

$$\tan(3\varphi) = \frac{3\tan(\varphi) - \tan^3(\varphi)}{1 - 3\tan^2(\varphi)}$$

$$\sin\alpha\cdot\cos\beta = \frac{1}{2}\cdot\sin(\alpha-\beta) + \frac{1}{2}\cdot\sin(\alpha+\beta)$$

$$\sin\alpha\cdot\sin\beta = \frac{1}{2}\cdot\cos(\alpha-\beta) - \frac{1}{2}\cdot\cos(\alpha+\beta)$$

$$\cos\alpha\cdot\cos\beta = \frac{1}{2}\cdot\cos(\alpha-\beta) + \frac{1}{2}\cdot\cos(\alpha+\beta)$$

$$A\cdot\cos(\varphi+\alpha) + B\cdot\cos(\varphi+\beta) =$$

$$C\cdot\cos\varphi - S\cdot\sin\varphi = R\cdot\cos(\varphi+\gamma),$$

jossa (where)

$$C = A\cdot\cos\alpha + B\cdot\cos\beta, \qquad S = A\cdot\sin\alpha + B\cdot\sin\beta,$$

$$R = \sqrt{C^2 + S^2} = \sqrt{A^2 + B^2 + 2AB\cdot\cos(\alpha-\beta)}$$ ja (and)

$$\gamma = \arctan\left(\frac{S}{C}\right) = \arctan\left(\frac{A\cdot\sin\alpha + B\cdot\sin\beta}{A\cdot\cos\alpha + B\cdot\cos\beta}\right)$$

MÄÄRÄÄMÄTTÖMIÄ INTEGRAALEJA (INDEFINITE INTEGRALS)

$$\int x^n dx = \frac{1}{n+1}\cdot x^{n+1} \ ; x \neq -1$$

$$\int x^{-1} dx = \ln x$$

$$\int \ln(x)dx = x\ln(x) - x$$

$$\int a^x dx = \frac{1}{\ln a}\cdot a^x$$

$$\int e^{ax} dx = \frac{1}{a}\cdot e^{ax}$$

$$\int x \cdot e^{ax} dx = \frac{1}{a}\cdot e^{ax}\cdot\left(x - \frac{1}{a}\right)$$

$$\int x^2 \cdot e^{ax} dx = \frac{1}{a^3}\cdot e^{ax}\cdot\left(a^2x^2 - 2ax + 2\right)$$

$$\int sin\left(ax\right)dx = -\frac{1}{a}\cdot cos\left(ax\right)$$

$$\int cos\left(ax\right)dx = \frac{1}{a}\cdot sin\left(ax\right)$$

$$\int sin^2(x)dx = \frac{1}{2}x - \frac{1}{4}sin(2x)$$

$$\int cos^2(x)dx = \frac{1}{2}x + \frac{1}{4}sin(2x)$$

$$\int sin^2(ax)dx = \frac{1}{2a}(ax - cos(ax)sin(ax))$$

$$\int cos^2(ax)dx = \frac{1}{2a}(ax + sin(ax)cos(ax))$$

$$\int e^{ax} sin(bx)dx = \frac{e^{ax}}{a^2+b^2}(asin(bx) - bcos(bx))$$

$$\int e^{ax} cos(bx)dx = \frac{e^{ax}}{a^2+b^2}(acos(bx) + bsin(bx))$$

$$\int arcsin(ax)dx = xarcsin(ax) + \frac{1}{a}\sqrt{1-a^2x^2}$$

$$\int arccos(ax)dx = xarccos(ax) - \frac{1}{a}\sqrt{1-a^2x^2}$$

$$\int arctan(ax)dx = \frac{1}{2a}[2axarctan(ax) - \ln(1+a^2x^2)]$$

MÄÄRÄTTYJÄ INTEGRAALEJA (DEFINITE INTEGRALS)

$$\int_0^\infty \frac{1}{1+x^a}dx = \frac{\pi}{a\cdot sin\left(\frac{\pi}{a}\right)}, a > 1$$

$$\int_0^\infty \frac{1}{x^a(1+x)}dx = \frac{\pi}{sin(a\pi)}, 0 < a < 1$$

$$\int_0^\infty \frac{x^{a-1}}{1+x^b}dx = \frac{\pi}{b\cdot sin\left(\frac{a\pi}{b}\right)}, 0 < a < b$$

$$\int_0^\infty \frac{1}{a^2+x^a}dx = \frac{\pi}{2a}, a > 0$$

$$\int_0^\infty sin^2(x)dx = \int_0^\infty cos^2(x)dx = , a > 1$$

$$\int_0^\infty \frac{sin(ax)}{x}dx = \frac{\pi}{2}$$

$$\int_0^\infty \frac{sin^2(ax)}{x^2}dx = \frac{\pi}{2}$$

$$\int_0^\infty e^{-ax} sin(bx)dx = \frac{b}{a^2+b^2}, a > 0$$

$$\int_0^\infty e^{-ax} cos(bx)dx = \frac{a}{a^2+b^2}, a > 0$$

$$\int_0^\infty xe^{-ax} sin(bx)dx = \frac{2ab}{\left(a^2+b^2\right)^2}, a > 0$$

$$\int_0^\infty xe^{-ax} cos(bx)dx = \frac{a^2-b^2}{\left(a^2+b^2\right)^2}, a > 0$$

SUODATTIMET (FILTERS)

Siirtofunktio (Transfer function, where Y(f) is the output and X(f) is the input of the system in frequency domain)

$H(f) = \dfrac{Y(f)}{X(f)}$, missä $Y(f)$ on järjestelmän vaste taajuusalueessa ja $X(f)$ järjestelmän heräte taajuusalueessa.

Amplitudifunktio (Amplitude function) $A(f) = |H(f)|$

Vaihefunktio (Phase function) $\phi(f) = -\arg(H(j))$

Energiansiirtofunktio/Tehonsiirtofunktio (Energy transfer function/Power transfer function) $G(f) = A^2(f)$

SUODATINPERHEET (FILTER FUNCTIONS)

- Butterworth (where W is 3dB boundary frequency ja n is the degree of the filter)

$A(f) = \dfrac{1}{\sqrt{1+\left(\dfrac{f}{W}\right)^{2n}}}$, missä W on suodattimen -3dB (puolen tehon) rajataajuus ja n suodattimen asteluku.

- Tšebyshev (where ε is the eccentricity of the ellipse and Cn(x) is the Tŝebyshev polynomial of power n)

$A(f) = \sqrt{\dfrac{1+\varepsilon^2 C_n^2(0)}{1+\varepsilon^2 C_n^2(\frac{f}{W})}}$, missä ε on ellipsin eksentrisyys, ja $C_n(x)$ on n:nnen kertaluvun Tŝebyshevin polynomi.

$C_0(x) = 1,\qquad C_1(x) = x,\qquad C_n(x) = 2C_{n-1}(x) - C_{n-2}(x)$

- Bessel (where Bn(x) is the Bessel polynomial of power n)

$H(f) = \dfrac{B_n(0)}{B_n(\frac{f}{W})}$, missä $B_n(x)$ on Besselin n:nnen kertaluvun polynomi.

$B_0(x) = 1,\qquad B_1(x) = 1 + jx,\qquad B_n(x) = (2n-1)B_{n-1}(x) - x^2 B_{n-2}(x)$

SUODATTIMIIN LIITTYVIÄ TUNNUSLUKUJA (PARAMETERS RELATED TO FILTERS)

Selektiivisyys (Selectivity) $r_{sel} = \dfrac{B_{-60dB}}{B_{-3dB}}$

Oktaaviselektiivisyys (Octave selectivity) $OS = min\left\{20\,log\dfrac{A(f_0)}{A(0{,}5f_0)}, 20\,log\dfrac{A(f_0)}{A(2f_0)}\right\}$

Nousuaika (Rise time) $t_r = t_{90\%} - t_{10\%}$, missä $h(t_{10\%}) = 0{,}1h(\infty)$ ja $h(t_{90\%}) = 0{,}9h(\infty)$

EPÄLINEAARISET JÄRJESTELMÄT (NON LINEAR SYSTEMS)

Harmoninen särökerroin (Harmonic distortion factor)

$d_n = \dfrac{u_n}{u_1}$, jossa (where) u_n= n:nnen harmonisen amplitudi (amplitude on the nth harmonic) ja

u_1= perusaallon (1. harmonisen) amplitudi (amplitude of the fundamental frequency).

Kokonaissärökerroin (Total distortion factor or THD, Total Harmonic Distortion)

$d_{tot} = \sqrt{d_2^2 + d_3^2 + ...}$

Särövaimennus desibeleinä (Distortion attenuation in desibels)
$A_n = -20 \cdot lg\left(d_n\right)$

MODULAATIOT (MODULATIONS)

Käytetyt symbolit (used symbols):
$s(t)$ = moduloitu signaali (modulated signal), U_c = kantoaallon huippuarvo (peak value of carrier),
m = modulaatioindeksi (0...1) AM:ssä (modulation index in AM),
$x_m(t)$ = moduloiva signaali (1) (modulating signal (1)), $y_m(t)$ = moduloiva signaali 2 (modulating signal 2),
f_c = kantoaallon taajuus (carrier frequency), β = modulaatioindeksi PM:ssä (modulation index in PM),
Δf = deviaatio eli taajuuspoikkeama FM:ssä (deviation in FM).

Amplitudimodulaatio, AM (Amplitude modulation)

$$s(t) = U_c \cdot \left[1 + m \cdot x_m(t)\right] \cdot \cos\left(2\pi f_c t\right)$$

Kaksisivukaistamodulaatio, DSB (Double Side Band modulation)

$$s(t) = U_c \cdot x_m(t) \cdot \cos\left(2\pi f_c t\right)$$

Kvadratuurimodulaatio, QAM (Quadrature amplitude modulation)

$$s(t) = U_c \cdot \left[x_m(t) \cdot \cos\left(2\pi f_c t\right) - y_m(t) \cdot \sin\left(2\pi f_c t\right)\right]$$

Vaihemodulaatio, PM (Phase modulation)

$$s(t) = U_c \cdot \cos\left[2\pi f_c t + \beta \cdot x_m(t)\right]$$

Taajuusmodulaatio, FM (Frequency modulation)

$$s(t) = U_c \cdot \cos\left[2\pi f_c t + 2\pi \cdot \Delta f \cdot \int_{-\infty}^{t} x_m(t)\,dt\right]$$

TILASTOLLISET ODOTUSARVOT (STATISTICAL EXPECTATION VALUES)

Yksi muuttuja: (Case of one variable)

Keskiarvo (Mean) $\overline{X} = E\{x\} = \int_{-\infty}^{\infty} x p(x) dx$

Keskiteho (Mean power) $\overline{x^2} = E\{x^2\} = \int_{-\infty}^{\infty} x^2 p(x) dx$

Varianssi (Variance)

$$\sigma_x^2 = \overline{(x - \overline{x})^2}$$

Momentti (Moment)

$$\overline{x^n} = E\{x^n\} = \int_{-\infty}^{\infty} x^n p(x) dx$$

Funktion y(t) keskiarvo ja varianssi: (Mean and variance of function y(t))

Keskiarvo (Mean) $\overline{y(x)} = E\{y(x)\} = \int_{-\infty}^{\infty} y(x) p(x) dx$

Varianssi (Variance) $\overline{\sigma_y^2} = E\left\{(y(x) - \overline{y(x)})^2\right\} = \int_{-\infty}^{\infty} (y(x) - \overline{y(x)})^2 p(x) dx$

Funktion x(t) aikakeskiarvon määritelmä:
Mean value of x(t) versus time:

$$E\{x(t)\} = \langle x(t) \rangle = \lim_{T \to \infty} \left\{ \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) dt \right\}$$

Keskeismomentti (Central moment)

$$m_n = E\left\{\left(x - \overline{x}\right)^n\right\} = \int_{-\infty}^{\infty} \left(x - \overline{x}\right)^n p(x) dx$$

GAUSSIN JAKAUMA (GAUSSIAN DISTRIBUTION)

Tiheysfunktio (Density function, where m = mean and σ = variance)

$N(m,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{(x-m)^2}{2\sigma^2}\right)}$, missä m on keskiarvo ja σ on varianssi.

Q-funktio (Q-function)

$1 - F(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{x^2}{2\sigma^2}\right)dx = Q(\frac{x}{\sigma})$, missä (where)

$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{x^2}{2\sigma^2}\right)dx$

SEKALAISIA KAAVOJA (OTHER EQUATIONS)

Parsevalin tehoteoreema (Parsevals power theorem) $P = \sum_n |c_n|^2$

Rayleighin energiateoreema (Rayleigh energy theorem) $E = \int_{-\infty}^{\infty} |z(t)|^2 dt = \int_{-\infty}^{\infty} |Z(f)|^2 df$

Autokorrelaatio (Autocorrelation) $R_x(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} f(\tau) f(t+\tau) d\tau$

Wiener-Kinchine teoreema (Wiener-Kinchine theorem)

$S_x(f) = F\{R_x(\tau)\} = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau$, missä (where) $S_x(f)$ on tehospektri (is the power spectrum).

KAISTANPÄÄSTÖJÄRJESTELMÄT (BANDPASS SYSTEMS)

Analyyttinen signaali, esiverhokäyrä (Analytic signal, Pre-envelope)

$q(t) = x(t) + j\hat{x}(t) = 2x_+(t)$, missä $x_+(t)$ on signaalispektrin oikeanpuoleista osaa vastaava signaali. where $x_+(t)$ is the signal respecting the right part of signal spectrum.

Kompleksinen verhokäyrä (Complex envelope)
$$Z(f) = 2X_+(f+f_0) = Q(f+f_0) \rightarrow$$
$$z(t) = 2x_+(t)e^{-j2\pi f_0 t} = q(t)e^{-j2\pi f_0 t}$$

Ekvivalenttinen alipäästösignaali (Equivalent lowpass signal) $W(f) = \frac{1}{2} Z(f) = X_+(f+f_0)$

Kvadratuuriesitys (Quadrature presentation)

$$x(t) = Re\{z(t)\}cos(2\pi f_0 t) - Im\{z(t)\}sin(2\pi f_0 t) = x_c(t)cos(2\pi f_0 t) - x_s(t)sin(2\pi f_0 t)$$

Verhokäyrä-vaihe-esitys (Envelope and phase presentation)

$$x(t) = \sqrt{x_c^2(t) + x_s^2(t)}\, cos\left(2\pi f_0 t + arctan\frac{x_s(t)}{x_c(t)}\right) = a(t)cos(2\pi f_0 t + \phi(t))$$

a(t) on verhokäyrän lauseke ja φ(t) on vaiheen lauseke.
a(t) is envelope equation and φ(t) is phase equation.

OHMIN LAKI JA DESIBELIT (OHM´S LAW AND DECIBELS)

$U = I \cdot R = \sqrt{P \cdot R}$ $\quad R = \dfrac{U}{I}$ $\quad I = \dfrac{U}{R} = \sqrt{\dfrac{P}{R}}$

$P = U \cdot I = I^2 \cdot R = \dfrac{U^2}{R}$

Kondensaattorin reaktanssi (Reactance of capasitor) $X_C = \dfrac{1}{j2\pi fC}$.

Kelan reaktanssi (Reactance of coil) $X_L = j2\pi L$.

Jos järjestelmän tulo- ja lähtöimpedanssit ovat samansuuruiset, pätee (If the input and output impedances of the system are the same, we can define)

$P1 = \dfrac{U1^2}{Z}$ *ja* $P2 = \dfrac{U2^2}{Z}$, $\quad G(dB) = 10 \cdot \log \dfrac{P1}{P2} = 20 \cdot \log \dfrac{U1}{U2}$ $\quad [G = gain]$

Usein vertailukohdaksi (P2 tai U2) valitaan tietty teho tai jännite. Seuraavassa taulukossa on esitetty tavallisia vertailukohteita dB-merkintöineen. (Often is chosen known power (P2) or voltage (U2) as a reference. Usual references are presented in the following table).

Merkintä (Mark)	Vertailukohde (Reference)
dBW	1 W
dBm	1 mW
dBf	1 fW
dBV	1 V
dBμV	1 μV

Vahvistus (gain, amplification) $G(dB) = -L(dB)$, L = vaimennus (loss, attenuation).

Tavallisia teho- ja jännitesuhteita (Usual power and voltage ratios)
Huomaa, että desibelien yhteenlasku vastaa absoluuttiarvojen kertolaskua ja desibelien vähennyslasku jakolaskua.
Please notice that desibel sum corresponds multiplication with absolute values and desibel subtraction corresponds division with absolute values.

G(dB)	*P1/P2*	*U1/U2*
0	*1*	*1*
3	*2*	$\sqrt{2} \approx 1{,}4142$
6 (=3+3)	*4 (=2 x 2)*	*2*
9 (=3+3+3)	*8 (= 2 x 2 x 2)*	$2 \cdot \sqrt{2} \approx 2{,}8284$
10	*10*	$\sqrt{10} \approx 3{,}18$
20 (=10+10)	*100 (=10 x 10)*	*10*
30 (=10+10+10)	*1000 (=10 x 10 x 10)*	$10 \cdot \sqrt{10} \approx 31{,}8$
40	*10000*	*100*
-3 (=0-3)	$0{,}5 = \dfrac{1}{2}$	$\dfrac{1}{\sqrt{2}} \approx 0{,}707$
-6 (=-3-3)	$0{,}25 = \dfrac{1}{4}$	$0{,}5 = \dfrac{\left(1/\sqrt{2}\right)}{\sqrt{2}}$
-20	*0,01*	*0,1*

AD/DA-MUUNNOKSET JA PCM (AD/DA CONVERSIONS AND PCM)

Kvantisointivirheen ε_k varianssi (Variance of the quantization error ε_k) $\sigma_{\varepsilon_k}^2 = \dfrac{\Delta x_k^2}{12}$, jossa (where) Δx_k = kvantisointiväli (quantization space between levels).

Tasavälinen kvantisointi (Linear quantization): $\Delta x_k = \Delta x = \dfrac{2 \cdot x_{max}}{N}$, jossa (where) x_{max} = muunnoksen maksimarvo (maximum value of the conversion) ja (and) N = tasojen määrä (number of levels), jolloin (when) kvantisointivirheen ε_k varianssi (variance of the quantization error ε_k) $\sigma_{\varepsilon}^2 = \dfrac{x_{max}^2}{3N^2}$.

A-lain mukainen kompressio (Compression using A-law):

$$x_{comp}(t) = \frac{A}{1+\ln(A)} \cdot x(t), \quad kun \quad 0 \le |x(t)| \le \frac{1}{A} \quad \underline{\text{tai}} \quad x_{comp}(t) = sign\left[x(t)\right] \cdot \frac{1+\ln(A \cdot |x(t)|)}{1+\ln(A)}, \quad kun \quad \frac{1}{A} < |x(t)| \le 1$$

A-lain mukainen dekompressio (Decompression using A-law):

$$x_{decomp}(t) = \frac{1+\ln(A)}{A} \cdot x_{comp}(t), \; kun \; 0 \le \left|x_{comp}(t)\right| \le \frac{1}{1+\ln(A)} \qquad tai$$

$$x_{decomp}(t) = sign\left[x_{comp}(t)\right] \cdot \frac{1}{A} \cdot e^{\left\{\left[1+\ln(A)\right] \cdot \left|x_{comp}(t)\right| - 1\right\}}, \; kun \; \frac{1}{1+\ln(A)} < \left|x_{comp}(t)\right| \le 1$$