S-72.227 Digital Communication Systems

Spring 2002 Home work#1 Solution

Homework-1

The original (7,4) Hamming code is a *non-systematic code* with

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

a) Construct the lookup table and explain the rationale for this non-systematic form

b) Write the equations for s_1,s_2, and s_3 taking Y=X=(x_1 x_2 … x_7). Then determine which are the message bits and check bits in X; finally obtain the check-bit equations.

Answer:

a)

	S_1 S_2 S_3	E
	0 0 0	0 0 0 0 0 0 0
H^T	0 0 1	1 0 0 0 0 0 0
	0 1 0	0 1 0 0 0 0 0
	0 1 1	0 0 1 0 0 0 0
	1 0 0	0 0 0 1 0 0 0
	1 0 1	0 0 0 0 1 0 0
	1 1 0	0 0 0 0 0 1 0
	1 1 1	0 0 0 0 0 0 1

The binary number S_1 S_2 S_3 equals the error location, i.e.

0 0 0 ⇒ no error
0 0 1 ⇒ 1st bit
0 1 0 ⇒ 2nd bit
etc.

b)

$S_1 = X_4 \oplus X_5 \oplus X_6 \oplus X_7 = 0$
$S_2 = X_2 \oplus X_3 \oplus X_6 \oplus X_7 = 0$
$S_3 = X_1 \oplus X_3 \oplus X_5 \oplus X_7 = 0$

Since x_1, x_2 and x_4 appear only once, they must be the check bits.

Thus, X= (c_1 c_2 m_1 c_3 m_2 m_3 m_4)

Here,

$C_1 = m_1 \oplus m_2 \oplus m_4$
$C_2 = m_1 \oplus m_3 \oplus m_4$
$C_3 = m_2 \oplus m_3 \oplus m_4$