



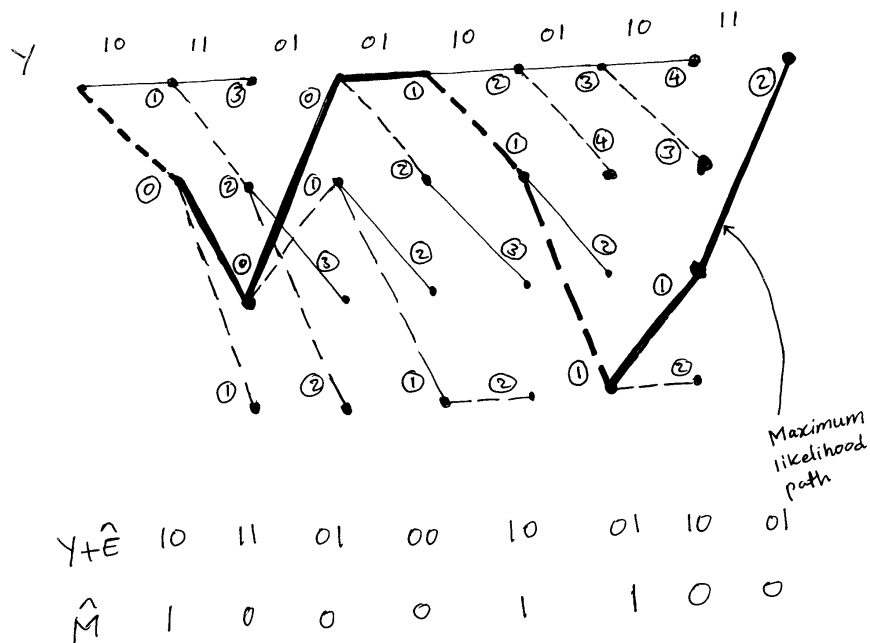
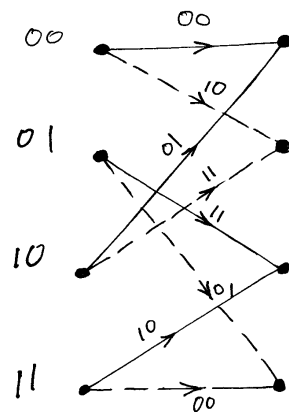
E 3.1 Construct the code trellis diagram for a (2,1,2) code with

$$x_j' = m_{j-1} \oplus m_j$$

$$x_j'' = m_{j-2} \oplus m_{j-1}$$

- a) Apply the Viterbi algorithm to find  $Y + \hat{E}$  and  $\hat{M}$  when  $Y = 10 \ 11 \ 01 \ 01 \ 10 \ 01 \ 10 \ 11$ . If two paths arriving at a given node have equal running metrics, arbitrarily keep the upper path.

ANS:



### E 3.2

From the following equation

$$G_{lp}(f) = G_i(f) = \frac{M^2 - 1}{12r} \text{sinc}^2 \frac{f}{r} + \frac{(M-1)^2}{4} \delta(f),$$

find the average power  $\overline{x_c^2}$  and the carrier-frequency power  $P_c$  of an  $M$ -ary ASK signal. Then form the ratio  $P_c / \overline{x_c^2}$  and simplify for  $M=2$  and  $M \gg 1$ .

ANS:

$$\begin{aligned} \overline{x_c^2} &= \int_{-\infty}^{+\infty} G_c(f) df = \frac{A_c^2}{4} \int_{-\infty}^{+\infty} [G_{lp}(f - f_c) + G_{lp}(f + f_c)] df \\ &= \frac{A_c^2}{4} * 2 * \int_{-\infty}^{+\infty} [G_{lp}(f)] df \quad [\text{Since } G_{lp}(f-f_c) \text{ and } G_{lp}(f+f_c) \text{ do not overlap if } f_c \gg r] \\ &= \frac{A_c^2}{2} \left[ \frac{M^2 - 1}{12r} \int_{-\infty}^{+\infty} \text{sinc}^2 \left( \frac{f}{r} \right) df + \frac{(M-1)^2}{4} \int_{-\infty}^{+\infty} \delta(f) df \right] \\ &= \frac{A_c^2}{2} \left[ \frac{M^2 - 1}{12r} * r + \frac{(M-1)^2}{4} \right] \\ &= \frac{A_c^2}{12} (M-1)(2M-1) \end{aligned}$$

$$P_c = 2 * \frac{A_c^2}{4} * \frac{(M-1)^2}{4}$$

$$\begin{aligned} \frac{P_c}{\overline{x_c^2}} &= \frac{3M-3}{4M-2} = \begin{cases} \frac{1}{2} & \text{when } M=2 \\ \frac{3}{4} & \text{when } M \gg 1 \end{cases} \end{aligned}$$

### E 3.3

The envelope and phase variations of a QAM signal are

$$A(t) = A_c [x_i^2(t) + x_q^2(t)]^{1/2} \text{ And } \phi(t) = \arctan[x_q(t)/x_i(t)] \text{ respectively.}$$

- By considering the time interval  $kD < t < (k+1)D$ , obtain expression for  $A(t)$  and  $\phi(t)$  with a rectangular pulse shape  $p_D(t)$ .
- Redo part a with an arbitrary pulse shape  $p(t)$  whose duration does not exceed  $D$ .

ANS:

a) for  $KD < t < (K+1)D$

$$x_i(t) = a_{2k}, \quad x_q(t) = a_{2k+1} \quad \text{with } a_k = \pm 1$$

Thus,  $x_i^2(t) + x_q^2(t) = 2$  for all  $t$  so that  $A(t) = \sqrt{2}A_c$

$$\text{Where } \Phi(t) = \sum_k \tan^{-1}(a_{2k+1} / a_{2k}) P_D(t - kD)$$

b) for  $kD < t < (k+1)D$

$$x_i(t) = a_{2k}P(t - kD) \quad \text{and} \quad x_q(t) = a_{2k+1}P(t - kD) \quad \text{with } a_k = \pm 1$$

Thus,  $x_i^2(t) + x_q^2(t) = 2 P^2(t - kD)$ ; so,  $A(t) = \sqrt{2} \sum_k |P(t - kD)|$

$$\text{And } \Phi(t) = \sum_k \tan^{-1}[x_q(t) / x_i(t)] P(t - kD) = \sum_k \tan^{-1}[a_{2k+1} / a_{2k}] P(t - kD)$$

### E 3.4

Suppose an OOK signal has raised-cosine pulse shaping so that

$$S_1(t) = A_c \sin^2(\pi t / T_b) P_{T_b}(t) \cos \omega_c t$$

Draw and label the block diagram of an optimum coherent receiver using:

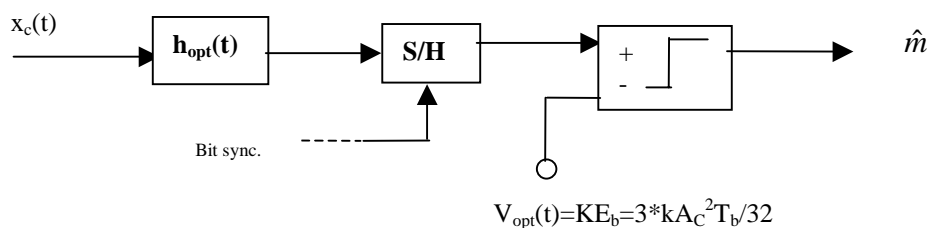
- (a) matched filter;
- (b) Correlation detection.

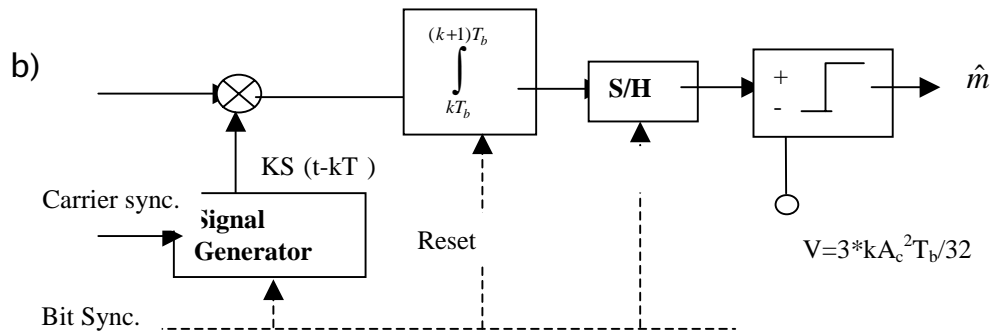
ANS:

a)

$$\begin{aligned} h_{opt}(t) &= kA_c \sin^2 \left[ \frac{\pi}{T_b} (T_b - t) \right] P_{T_b}(T_b - t) \cos(\omega_c (T_b - t)) \\ &= kA_c \sin^2 \left[ \pi - \frac{\pi t}{T_b} \right] P_{T_b}(t) \cos(\omega_c t - 2\pi N_c) \\ &= kA_c \sin^2 \left[ \frac{\pi t}{T_b} \right] \cos \omega_c t \quad 0 < t < T_b \end{aligned}$$

$$\begin{aligned} V_{opt}(t) &= \frac{k}{2} (E_1 - E_0) = \frac{k}{2} \int_0^{T_b} s_1^2(t) dt = \frac{k}{2} * A_c^2 \int_0^{T_b} \sin^4 \frac{\pi t}{T_b} \cos^2 \omega_c t dt \\ &= \frac{k}{2} * A_c^2 * \frac{3}{16} * T_b = \frac{3}{32} kA_c^2 T_b \end{aligned}$$





Homework-3 Deadline February 18,2002 at 10.00

Homework return box is located at Otakaari 5, 2nd floor, near the E-wing. You can also return the answers to the assistant just before the class.

Suppose a binary FSK signal with discontinuous phase is generated by switching between two oscillators with outputs  $A_c \cos(2\pi f_o t + \theta_o)$  and  $A_c \cos(2\pi f_1 t + \theta_1)$ .

Since the oscillators are unsynchronized, the FSK signal may be viewed as the interleaved sum of two independent binary ASK signals.

Use this approach to find, sketch, and label  $G_c(f)$  for  $f > 0$  when  $f_o = f_c - r_b/2$  and  $f_1 = f_c + r_b/2$  with  $f_c \gg r_b$ .