E 4.1

Consider a **PRK** signal with pilot carrier added for synchronisation purposes, results in

$$s_1(t) = [A_c \cos w_c t + aA_c \cos(w_c t + q)]P_{T_b}(t)$$

$$s_0(t) = [-A_c \cos w_c t + aA_c \cos(w_c t + q)]P_{T_b}(t)$$

Take $q = 0$ and show an optimum coherent receiver with AWGN yields $P_e = Q[\sqrt{2g_b/(1+a^2)}]$

ANSWER:

Given $s_1(t)$ and $s_0(t)$ and θ=0; we can proceed as follows:

$$E_1 = \int_0^{T_b} s_1^2(t)dt = \int_0^{T_b} A_c^2(1+a)^2 \cos^2(w_c t)dt = \frac{(1+a)^2 A_c^2 T_b}{2}; \quad here, \; w_c T_b = 2pN_c$$

$$E_0 = \int_0^{T_b} s_0^2(t)dt = \int_0^{T_b} A_c^2(1-a)^2 \cos^2(w_c t)dt = \frac{(1-a)^2 A_c^2 T_b}{2};$$

$$E_{10} = \int_0^{T_b} s_1(t)s_0(t)dt = \int_0^{T_b} -A_c^2(1+a)(1-a)\cos^2(w_c t)dt = \frac{-(1-a^2)A_c^2 T_b}{2};$$

$$E_b = \frac{E_1 + E_0}{2} = \frac{1}{2}\left[\frac{(1+a)^2 A_c^2 T_b}{2} + \frac{(1-a)^2 A_c^2 T_b}{2}\right] = \frac{(1+a)^2 + (1-a)^2}{2}\frac{A_c^2 T_b}{2} = (1+a^2)\frac{A_c^2 T_b}{2}$$

$$P_e = Q\left[\sqrt{\frac{E_b - E_{10}}{h}}\right] = Q\left[\sqrt{\frac{\left[(1+a^2)\frac{A_c^2 T_b}{2}\right] - \left[\frac{-(1-a^2)A_c^2 T_b}{2}\right]}{h}}\right]$$

$$= Q\left[\sqrt{\frac{2A_c^2 T_b}{2h}}\right] = Q\left[\sqrt{\frac{2}{h} * \frac{A_c^2 T_b}{2}(1+a^2) * \frac{1}{(1+a^2)}}\right]$$

$$= Q\left[\sqrt{\frac{2E_b}{h} * \frac{1}{(1+a^2)}}\right] = Q\left[\sqrt{\frac{2g_b}{(1+a^2)}}\right]$$

E 4.2

It is understood that when the noise in a coherent binary system is Gaussian but has a *non-white* power spectrum $G_n(f)$, the noise can be "whitened" by inserting at the front end of the receiver a filter with transfer function $H_w(f)$ such that $|H_w(f)|^2 G_n(f) = h/2$. The rest of the receiver must then be matched to the distorted signalling waveforms $\tilde{S}_1(t)$ and $\tilde{S}_0(t)$ at the output of the whitening filter. Furthermore, the duration of unfiltered waveforms $S_1(t)$ and $S_0(t)$ must be reduced to ensure that the whitening filter does not introduce ISI. Apply these conditions to show from

$$\left(\frac{z_1 - z_0}{2s}\right)^2_{\max} = \frac{E_1 + E_0 - 2E_{10}}{2h} = \frac{E_b - E_{10}}{h}$$

that

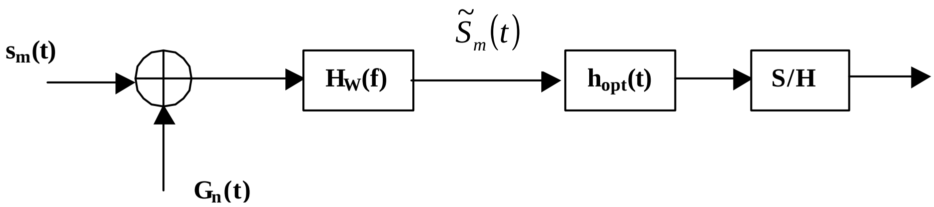
$$\left(\frac{z_1 - z_0}{2s}\right)^2_{\max} = \int_{-\infty}^{\infty} \frac{[S_1(f) - S_0(f)]^2}{4G_n(f)} df$$

where $S_1(f) = \Im[S_1(t)]$, etc.

Hint: Recall that if *v*(t) and *w*(t) are real, then,

$$\int_{-\infty}^{\infty} v(t)w(t)dt = \int_{-\infty}^{\infty} V(f)W^*(f)df = \int_{-\infty}^{\infty} V^*(f)W(f)df$$

ANSWER:



$$|H_w(f)|^2 G_n(f) = \frac{h}{2}$$

$$\tilde{S}_m(f) = \Im\{\tilde{s}_m(t)\} = S_m(f)H_w(f) \qquad here,\ S_m(f) = \Im\{s_m(t)\} \qquad for\ \ m = 0,1$$

$$If\ \ h_{opt}(t) = K[\tilde{s}_1(T_b - t) - \tilde{s}_0(T_b - t)],\ then\ \ given\ \left(\frac{z_1 - z_0}{2s}\right)^2_{\max} = \frac{E_1 + E_0 - 2E_{10}}{2h} = \frac{E_b - E_{10}}{h}$$

$$Here,\ E_m = \int_0^{T_b} \tilde{s}_m^2(t)dt = \int_{-\infty}^{\infty} |\tilde{S}_m(f)|^2 df = \int_{-\infty}^{\infty} |S_m(f)|^2 |H_w(f)|^2 df$$

$$\text{and} \qquad E_{10} = \int_0^{T_b} \tilde{s}_1(t)\tilde{s}_0(t)dt = \int_{-\infty}^{\infty} \tilde{S}_1(f)\tilde{S}_0^*(f)df = \int_{-\infty}^{\infty} S_1(f)S_0^*(f)|H_w(f)|^2 df$$

$$or \quad = \int_{-\infty}^{\infty} \tilde{S}_0(f)\tilde{S}_1^*(f)df = \int_{-\infty}^{\infty} S_0(f)S_1^*(f)|H_w(f)|^2 df$$

$$Thus, \ we \ can \ write, \quad 2E_{10} = \int_{-\infty}^{\infty} S_1(f)S_0^*(f)|H_w(f)|^2 df + \int_{-\infty}^{\infty} S_0(f)S_1^*(f)|H_w(f)|^2 df$$

$$So \quad E_1 + E_0 - 2E_{10} = \int_{-\infty}^{\infty}\left[|S_1(f)|^2 + |S_0(f)|^2 - S_1(f)S_0^*(f) - S_1^*(f)S_0(f)\right] |H_w(f)|^2 df$$

$$= \int_{-\infty}^{\infty} |S_1(f) - S_0(f)|^2 \frac{h}{2G_n(f)} df$$

$$Hence, \quad \left(\frac{z_1 - z_0}{2s}\right)^2_{\max} = \int_{-\infty}^{\infty} \frac{|S_1(f) - S_0(f)|^2}{4G_n(f)} df$$

E 4.3

Suppose the OOK receiver has a simple BPF with $H(f) = [1 + j2(f - f_c)/B]$ for f>0, where $2r_b \le B << f_c$. Assuming that $f_c >> r_b$ and $g_b >> 1$, show that the signal energy must be increased by at least 5 dB get the same error probability as incoherent receiver with a matched filter.

ANSWER:

$$s^2 = \frac{h}{2} \times 2\int_0^{\infty} |H(f)|^2 df = h \int_0^{\infty} \frac{df}{1 + \frac{4(f - f_c)^2}{B}}$$

$$= \frac{hB}{2}\left[\frac{p}{2} - \arctan\left(\frac{-2f_c}{B}\right)\right] = \frac{hB}{2}\left[\frac{p}{2} - \left(-\frac{p}{2}\right)\right] = \frac{hB}{2} * p$$

$$because \quad \frac{f_c}{B} >> 1, \quad so, \quad \arctan\left(\frac{-2f_c}{B}\right) \approx \arctan(-\infty) = -\frac{p}{2}$$

$$P_e = \frac{1}{2}P_{e_0} = \frac{1}{2}e^{-\frac{A_C^2}{8s^2}} \qquad where \quad \frac{A_C^2}{8s^2} = \frac{A_C^2}{4pBh} \qquad and \qquad A_C^2 = 4E_b r_b$$

$$Thus \quad \frac{A_C^2}{8s^2} = \frac{E_b r_b}{pBh} = \frac{r_b}{pB} * \frac{E_b}{h} = \frac{r_b}{pB} * g_b = \frac{2r_b}{pB} * \frac{g_b}{2}$$

$$So \ \ we \ \ should \ \ increase \ \ g_b \ \ by \ \ \frac{pB}{2r_b} \ge p \approx 5dB$$

E 4.4

A binary transmission system with S_T=200 mW, L=90 dB, and $h = 10^{-15}$ W/Hz to have $P_e \le 10^{-5}$.

Find the maximum allowable bit rate using:

(a) Non-coherent FSK;

(b) DPSK;

(c) coherent PRK.

ANSWER:

$$\frac{S_T}{L} = S_R = E_b r_b = h\gamma_b r_b \Rightarrow r_b = \frac{S_T}{Lh\gamma_b} = \frac{200\times10^{-3}}{10^9 * 10^{-15} * \gamma_b} = \frac{2\times10^5}{\gamma_b}$$

a) for non-coherent FSK,

$$\frac{1}{2}e^{-\frac{\gamma_b}{2}} \le 10^{-5} \Rightarrow \gamma_b \ge 2\ln\frac{1}{2\times10^{-5}} \approx 21.6 \Rightarrow r_b \le \frac{2\times10^5}{21.6} = 9.2kbps$$

b) for DPSK

$$\frac{1}{2}e^{-\gamma_b} \le 10^{-5} \Rightarrow \gamma_b \ge \ln\frac{1}{2\times10^{-5}} \approx 10.8 \Rightarrow r_b \le \frac{2\times10^5}{10.8} = 18.4kbps$$

c) for coherent PSK

$$Q\left(\sqrt{2\gamma_b}\right) \le 10^{-5} \Rightarrow \gamma_b \ge \frac{1}{2} * 4.27^2 \approx 9.1 \Rightarrow r_b \le \frac{2\times10^5}{9.1} = 21.9kbps$$

Homework-4 Deadline 25 February 2002 at 10.00

Homework return box is located at Otakaari 5, 2nd floor, near the E-wing. You can also return the answers to the assistant just before the class.

Binary data is to be transmitted at the rate with r_b = 500kbps on a radio channel having 400-kHz bandwidth.

a) Specify the modulation method that minimises signal energy, and calculate g_b in dB needed to get $P_e \leq 10^{-6}$.

b) Repeat part (a) with the additional constraint that coherent detection is not practical for channel in question.