E 2.1

For a single error correcting (7,4) cyclic code with a generator polynomial $G(p)=p^3+p^2+0+1$, find generator matrix G and construct the code.

E 2.2

For a systematic cyclic code, we defne the message bit, check bit and codeword polynomial as M(p), C(p) and X(P), respectively.

These three polynomials are related to each other as $X(P)=p^qM(p)+C(P)$, where $q=n-k$. Each codeword corresponds to the polynomial product $X(P)=Q_M(p)G(p)$, in which $Q_M(p)$ represents a block of k message bits. X and M are code and message vectors which correspond to X(p) amd M(p), respectively.

The two equations above for X(p) require that

$$\frac{P^q M(p)}{G(p)} = Q_M(P) + \frac{C(p)}{G(p)}$$

In the receiver side, Every valid received code word R*(p)* must be a multiple of *G(p),* otherwise an error has occurred. Therefore dividing the *R(p)/G(p)* and considering the remainder as a syndrome can reveal if the error has happened. The syndrome of (*n,k,l)* degree is therefore

$$S(P) = rem\left[\frac{R(p)}{G(p)}\right]$$

Consider a systematic (7,3) cyclic code generated by $G(p)=p^4+p^3+p^2+0+1$. Find $Q_M(p)$, C(p), and X when M=(1 1 1). Then take received vector, Y=X' and confirm that S(p)=0.

E 2.3

Diagram the encoders for

a. A systematic (3,2,3) convolutional code
b. A systematic (4,3,1) convolutional code

Label the input and output rates and the current input state and state at arbitrary time.

E 2.4

A (3,1,2) achieves maximum free distance when

$$x_j^{'} = m_{j-2} \oplus m_j \qquad\qquad x_j^{''} = x_j^{'''} = m_{j-2} \oplus x_j^{'}$$

(a) Construct the code trellis and state diagram
(b) Find the state and output sequence produced by the input sequence 1011001111.

Homework return box is located at Otakaari 5, 2nd floor, near the E-wing. You can also return the answers to the assistant just before the class.

Error detection/correction over a communications channel can be done in numerous ways. One of the most popular means to accomplish less-erroneous transmission is to use *convolutional codes*. Convolutional codes generally outperform linear block codes.

a) In this homework, you are asked to explain in your own words how a four-state convolutional block-code operates.

Hints:
- Use a state diagram to begin with.
- Use a set of trellis diagrams to further explain convolution coding.
- Demonstrate the operation of an imaginary (invent one of your own!) 4-state convolutional code.
- Show how it works to receive a minimum of 4 decoded bits.

b) What are catastrophic codes?

c) What is the free distance of a convolutional code? Why free distance is an important quantity?