## S-72.227 Digital Communication Systems (Spring 2005)

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### Cyclic Codes

**1.**

For a *systematic* cyclic code, we define the message bit, check bit and codeword polynomial as $M(p)$, $C(p)$ and $X(p)$, respectively. These three polynomials are related to each other as $X(p) = p^q M(p) + C(p)$, where $q = n - k$.

Each codeword corresponds to the polynomial product $X(p) = Q_M(p)G(p)$, in which $Q_M(p)$ represents a block of $k$ message bits. $X$ and $M$ are code and message vectors which correspond to $X(p)$ and $M(p)$, respectively. The two equations above for $X(p)$ require that

$$\frac{p^q M(p)}{G(p)} = Q_M(p) + \frac{C(p)}{G(p)}$$

In the receiver side, every valid received code word $R(p)$ must be a multiple of $G(p)$ otherwise an error has occurred. Therefore dividing the $R(p)/G(p)$ and considering the remainder as a syndrome can reveal if the error has happened. The syndrome is therefore

$$S(p) = rem\left[\frac{R(p)}{G(p)}\right]$$

Consider a systematic (7, 3) cyclic code generated by $G(p) = p^4 + p^3 + p^2 + 0 + 1$.

Find $Q_M(p)$, $C(p)$ and $X$ when $M = (1\ 0\ 1)$. Then take received vector, $Y = X'$ and confirm that $S(p) = 0$.

**2.**

Figure 1 shows is a shift-register circuit that divides an arbitrary *m*th-order polynomial $Z(p)$ by a fixed polynomial $G(p) = p^q + g_{q-1}p^{q-1} + \ldots + g_1 p + 1$. If the register has been cleared before $Z(p)$ is shifted in, then the output equals the quotient and the remainder appears in the register after $m$ shift cycles. Confirm the division operation by constructing a table containing the input message bits, register contents before shift, register content after shift, and the output. Take $Z(p) = p^3 M(p)$, with message bits 1100, and $G(p) = p^3 + 0 + p + 1$.

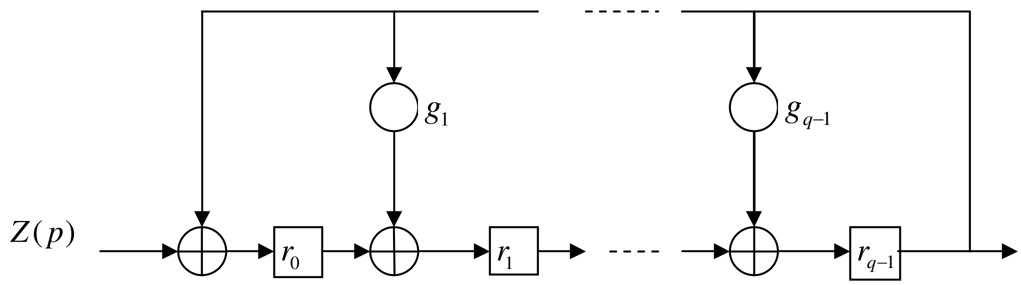


Fig. 1

**Convolutional Codes**

**3.**
Diagram the encoders for

a. A *systematic*[1] (3,2,3) convolutional code
b. A *systematic* (4,3,1) convolutional code

Label the input and output rates and the current input state and state at arbitrary time.

**4**.
A (3,1,2) encoder achieves maximum free distance when
$x'_j = m_{j-2} \oplus m_j \qquad x''_j = x'''_j = m_{j-2} \oplus x'_j$

a. Construct the code trellis and state diagram
b. Find the state and output sequence produced by the input sequence 1011001111.
c. Construct the modified state diagram (Splitting and labeling the state diagram), identify the minimum-weight path or paths and determine the values of the free distance $d_f$ .

**5. Convolutional Codes**
Repeat part c of last problem for a (2,1,3) encoder with
$x'_j = m_{j-3} \oplus m_{j-1} \oplus m_j \qquad x''_j = m_{j-2} \oplus x'_j$

**6. Viterbi algorithm**
Construct the code trellis diagram for a (2,1,2) code with
$$x'_j = m_{j-1} \oplus m_j \qquad x''_j = m_{j-2} \oplus m_{j-1}$$
Apply the Viterbi algorithm to find the transmitted sequence $Y + \hat{E}$ and the estimated message sequence $\hat{M}$ when *Y*=10 11 01 01 10 01 10 11. If two paths arriving at a given node have equal running metrics, arbitrarily keep the upper path.

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[1] A. B. Carlson: Communication Systems.