S-72.232 RADIO COMMUNICATION SYSTEMS
EXERCISE 2/2005, 11.2.2005

7. A radio channel can be modelled as a two-path channel with a fixed delay τ, $h(\lambda,t) = h_1(t)\delta(\lambda) + h_2(t)\delta(\lambda - \tau)$. The time varying complex gains of each path are independent, stationary, zero-mean normal processes.

a) Derive the autocorrelation function $R_h(\lambda, \Delta t)$ of the impulse response of this channel in terms of the autocorrelation functions of the path gains.

b) Derive the expression of standard deviation of the delay spread of the channel.

c) Calculate the numerical value of the delay spread std. as a function of the path gain ratio in dB when $\tau = 1$ μs.

8. The scattering function of a fading multipath channel is

$$S(\lambda, \nu) = c \cdot \text{tria}\left(\frac{\lambda - \lambda_o}{\lambda_o}\right) \cdot \text{tria}\left(\frac{\nu}{B}\right),$$

where λ_O=10 μs ja B=100 Hz. Estimate

a) the multipath spread,
b) the Doppler spread,
c) the coherence time,
d) the coherence bandwidth,
e) the spread factor

of this channel.

9. A multipath channel comprises 6 independently Rayleigh-fading components having delays and average power levels relative to the strongest component according to the table below.

i	1	2	3	4	5	6
τ_i/μs	0	0.3	1.0	1.6	5.0	6.6
P_{im}/dB	–2.5	0	–3.0	–5.0	–2.0	–4.0

a) Present the equivalent low-pass transfer function of the quasi-static channel in symbolic form.

b) What is the transfer function value (again in symbolic form) on the carrier frequency?
c) How many dB larger is the signal to noise ratio of a narrow-band output signal compared to the signal to noise ratio of the strongest component? Hint! The narrow-band signal is still Rayleigh-fading.
d) What is the probability that the instantaneous output signal to noise ratio goes below the average signal to noise ratio of the strongest component?

10. The received radio signal amplitude is Rayleigh-distributed. The amplitude of an interference signal is also Rayleigh-distributed and independent of the first signal.
    a) Derive an expression of the probability that the signal to interference ratio (SIR) is below the value k, when the average SIR is c.
    b) Calculate the probability that SIR is below i) 20 dB, ii) 10 dB, iii) 0 dB, when the average SIR is 20 dB.
    c) How large should the average SIR in dB be, if the instantaneous SIR would be below 20 dB no more than i) 10% of the time, ii) 1% of the time iii) 0,1% of the time.

HOMEWORK 2, return time 1 month, at latest before the May 2005 exam

The GSM Hilly Terrain multipath channel model comprises 6 independently Rayleigh-fading components having delays and average power levels relative to the strongest component according to the table below.

i	1	2	3	4	5	6
$\tau_i/\mu s$	0	0.1	0.3	0.5	15.0	17.2
P_{im}/dB	0	−1.5	−4.5	−7.5	−8.0	−17.7

a) Derive the autocorrelation function $R_h(\lambda, \Delta t)$ of the impulse response of this channel in terms of the mean power of each path gain.
b) Derive the expressions and the numerical values of the mean delay, r.m.s. delay, and delay standard deviation (a measure of half delay spread) of the of this channel.