19. The exercise deals with the properties of a Rayleigh-distributed desired signal and interference signal with and without diversity.

a) Derive the probability density function of the instantaneous carrier-to-interference ratio (cir) when both are independently Rayleigh-distributed. Use the following formula from probability theory:

$$z = x/y \rightarrow p_z(z) = \int_{-\infty}^{\infty} |y| p_x(zy) p_y(y) dy$$

b) Derive the probability density function of the instantaneous carrier-to-interference ratio (cir) when 2-branch diversity (equally strong branches) with selection combining is used and both diversity paths and interference are independently Rayleigh-distributed. Use the same formula as above and assume that the interference is still Rayleigh-distributed after combining.

20. Application of the results of problem 19:

a) Use the results of problem 19 to calculate the probability that the instantaneous *cir* goes below a given value in a normal receiver in a diversity receiver with selection combining where both branches have the same cir_m.

b) Calculate the needed cir_m when the requirement $P\{\text{cir} < 10\} = 0.01$ is fulfilled in both cases.

21. A rapidly fading narrow-band Rayleigh-channel is defined as a channel where all fading states are realized in the time interval used for a BER-measurement. The measured BER-value gives then an estimate of the average bit error probability P_b of a narrow-band digital system which is $P_b = \int_{\gamma} p(\gamma) P_b(\gamma) d\gamma$, where γ is the instantaneous signal to noise ratio and $p(\gamma)$ is its p.d.f. (However, the channel should be quasi-invariant over the symbol duration).
The diversity gain is determined as the difference in average SNR:s required for a given P_b-value without diversity and with diversity.
In a slowly fading narrow-band Rayleigh-channel the channel remains fairly constant during a BER-measurement. Instead of the average P_b a better performance measure is the time percentage during which a given P_b-value is exceeded. This equals the time (probability) the instantaneous γ-value goes below the value required to obtain the given P_b-value.

Diversity improvement is defined as the ratio between the times instantaneous BER is exceeding a given value without and with diversity.

a) Derive the expression for average bit error probability of DPSK in the rapidly fading Rayleigh-channel without diversity and when two-fold diversity with selection combining is applied. The bit error probability of DPSK in the AWGN-channel is $P_b(\gamma) = \frac{1}{2}e^{-\gamma}$.
b) Calculate the diversity gain in dB for the average bit error probability values 10^{-2} and 10^{-4}.
c) Calculate the diversity improvement on the bit error probabilities 10^{-2} and 10^{-4} when the average SNR in each branch is 20 dB
d) If the bit error probability 10^{-2} and 10^{-4} may be exceeded during 1% and 0,01 % of the time so what is then the diversity gain based on the required average SNR:s to obtain these outage performances?

22. In a radio communication system space diversity is used where the second branch average power level is lower than in the first branch. The branches are independently Rayleigh-fading.

a) Derive the SNR p.d.f. when selection combining is used.
b) Derive the SNR p.d.f. when maximum ratio combining is used
Narrow-band DPSK is used and the average power level difference of the branches is 6 dB. Based on average bit error probability 10^{-3}, what is the diversity gain penalty in dB compared to a system with equally strong branches,
c) when selection combinig is used,
d) when maximum ratio combining is used?

HOMEWORK 5. Return time 1 month, at latest before the May 2005 exam

The received signal in a mobile communication system is slowly Rayleigh-fading and the stationary noise is AWGN-type. It is requried that the signal to noise ratio should not be below 6 dB more than 0.01 % of the time.

a) Determine the required average signal to noise ratio in dB
b) Space diversity with two independent antennas with equal average power is used. Determine the required branch signal to noise ratio in dB with selection combining.