# S72-238 WCDMA systems

Tutorial 1

18.01.2002.

## Solutions

#### 1.

Cost 231 - Okomura-Hata propagation model gives for 1880 *MHz* average attenuation  $L_c = 157.3 - 13.82 \lg (h_{BS}) + (44.9 - 6.55 \lg (h_{BS})) \lg (d) - a (h_r)$ 

How much increases the coverage area (  $50\,\%\,$  at the cell boundary) when Base Station

antennae height is increased from 30 m to 200 m, and link budget allows 155 dB attenuation?

### 1.

Okumura published an empirical prediction method based on measurements in and around Tokyo. The Okumura method is probably the most widely quoted of the available models. In an attempt to make Okomura model easy to apply, Hata established empirical mathematical relationship to describe the graphical information given by Okumura. For the urban areas Okumura-Hata

formula is  $L_c = 157.3 - 13.82 \lg (h_{_{BS}}) + (44.9 - 6.55 \lg (h_{_{BS}})) \lg (d) - a (h_r)$ 

where

 $\begin{array}{ll} 150 < f_c < 1500 \quad f_c & \mbox{in } MHz \ , \\ 30 < h_t < 200 & \mbox{height of transmitting antenna.} \\ 1 < d < 20 & \mbox{distance in km,} \\ a\left(h_r\right) & \mbox{correction term for mobile antenna height.} \end{array}$ 

For a small city

$$a(h_r) = (1.1 \lg f_c - 0.7) h_r - (1.56 \lg f_c - 0.8).$$

Here we use the given formula

$$\begin{split} &L_c = 157.3 - 13.82 \lg \left(h_{\scriptscriptstyle BS}\right) + \left(44.9 - 6.55 \lg \left(h_{\scriptscriptstyle BS}\right)\right) \lg \left(d\right) \\ &h_{\scriptscriptstyle BS} = 30 \, [m] \\ &L_c = 157.3 - 13.82 \lg \left(h_{\scriptscriptstyle BS}\right) + \left(44.9 - 6.55 \lg \left(30\right)\right) \lg \left(d\right) \\ &= 157.3 - 20.4 + 35.22 \lg \left(d\right) = 155.0 \\ &\Rightarrow d_{\scriptscriptstyle 30} = 10^{(155 - 136.9)/35.22} = 3.3 \, [km] \\ &h_{\scriptscriptstyle BS} = 200 \, [m] \\ &L_c = 157.3 - 13.82 \log_{10} \left(200\right) + \left(44.9 - 6.55 \lg \left(200\right)\right) \lg \left(d\right) \\ &= 157.3 - 31.8 + 29.8 \lg \left(d\right) = 155.0 \\ &\Rightarrow d_{\scriptscriptstyle 30} = 10^{(155 - 125.5)/29.8} = 9.77 \, [km] \end{split}$$

Cell areas are calculated as  $\pi d^2$ 

From the cell area ratio of increase is:  $\left(\frac{d_{200}}{d_{30}}\right)^2 = \left(\frac{9.77}{3.3}\right)^2 = 8.64$ 

2.						
Channel characterization						
Power $[dB]$	-3	0	-2	-6	-8	-10
Delay $[\mu s]$	0.0	0.2	0.5	1.6	2.3	5.0

a) Determine the rms delay and mean excess time for the channel

b) Determine the maximum excess delay -10 dB.

c) Estimate the coherence bandwidth of the channel.

d) (Coherence time). Assume that mobile using UMTS system traveling with  $50 \frac{km}{h}$ 

receives the signal through this channel determine the time over which the channel appears stationary (or at least highly correlated).

2.

Reference. Rappaport T.S. Wireless Communications. Prentice Hall 1996. a) Mean excess delay is the first moment of the power delay profile and is defined as,

$$\overline{\tau} = \frac{\sum_{k} P\left(\tau_{k}\right) \tau_{k}}{\sum_{k} P\left(\tau_{k}\right)} = 0.67 \left[\mu s\right]$$

For our channel we first transfer the powers into absolute scale Power 0.5 1 0.63 0.25 0.15 0.1By evaluating for the  $\overline{\tau}$  it becomes  $\overline{\tau} = 0.67 [\mu s]$ 

The *rms* delay is the square root of the second central moment of the power delay profile and is defined to be

$$\sigma_{\tau}^2 = \overline{\left(\tau^2\right)} + \left(\overline{\tau}\right)^2$$

Where  $\overline{(\tau^2)}$  is defined as  $\overline{(\tau^2)} = \frac{\sum_k P(\tau_k) \tau_k^2}{\sum_k P(\tau_k)} = 1.5816 [\mu s]$  $\sigma_{\tau}^2 = \overline{(\tau^2)} + (\overline{\tau})^2 = 1.13.$ 

b) *Maximum excess delay* is defined to be delay during which multipath energy falls X dB below the maximum. In our case it would be -10 dB. For the given channel that is delay till the last multipath component:  $5 \mu s$ .

c) The *coherence bandwidth* can be assumed to be the frequency separation at which the channel is highly correlated. For example it can be estimated to be at the level where the normalized value of the correlation coefficient attains a value 0.7.

The *coherence bandwidth* can be calculated as  $\Delta f = \frac{1}{2\pi T}$  where T is maximum excess delay. For the given channel it is 31.8 kHz.

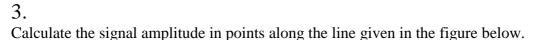
d) The Doppler speed and *coherence time* are inversely proportional to each other  $T_{C,1} \approx \frac{1}{f_m}$  where  $f_m$  is the maximum Doppler shift.

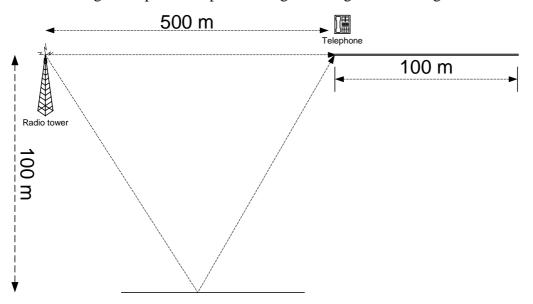
$$f_m = \frac{2 \times v \times f_{umts}}{c} = \frac{2 \times 50 \frac{1000}{3600} \times 2150}{3 \times 10^8} = 199.1 [Hz]$$

If the coherence time is defined as the time over which the time correlation function is above 0.5, then the coherence time is approximately  $T_{_{C,2}} \approx \frac{9}{16\pi f_m}$ .

The coherence time can be calculated as geometric mean of equations of these two definitions of  $T_{\!_C}$ 

$$T_{_C} \approx \sqrt{rac{9}{16\pi f_{_m}}} = rac{0.423}{f_{_m}} = 0.0021 \, [s]$$





The wall reflection coefficient is -1 and there is no phase change. The system operates on 2 GHz frequency where only the carrier is transmitted and transmission power is 1 W. The calculations are made with granularity of 5 m.

- a) Calculate the amplitude at the receiver when only the line of sight component is considered.
- b) Calculate the amplitude change at given locations on the figure. When both paths are summed together.
- c) Assume that the receiver at is capable to separate the paths and sum them together in phase. Calculate the resulting signal amplitude.
- d) Assume that the receiver is capable to separate paths that are apart at least by one symbol length. What is the symbol rate that system should have in order to separate these two paths.

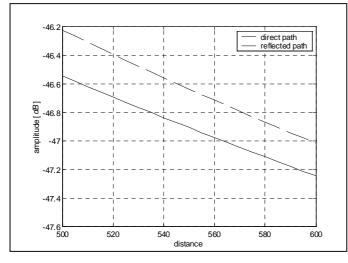
3.

a) We have to calculate the amplitude and phase of line of sight component.

We can describe the amplitude and phase of an arriving component  $a_i \times e^{j\varphi_i}$  where  $a_i$  is the signal amplitude along the *i*th path and  $\varphi_i$  is the signal phase along that path. The amplitude is subject of attenuation and depending on the distance it can be

calculated as  $a_i = \sqrt{P_i} = \frac{\sqrt{P_t}}{\frac{2\pi d}{\lambda}}$  where  $P_t$  is transmitted signal power.

For the given signal paths the amplitudes is calculated in the figure below.



b) In this calculation we have to consider also the fact that signals arrive at different time moments. The multipath component is described by the amplitude, phase change, and the delay in the channel:

 $\sum_{l} a_{l}(t) e^{-j2\pi f_{c}\tau_{l}(t)} \delta[\tau - \tau_{n}(t)], \text{ where } f_{c} \text{ is Doppler component in given path and}$ 

 $\tau_l(t)$  is delay along the path.

In our exercise there is no mention on Doppler change and in the following calculations it is assumed to be zero, in the similar way the amplitude and delay are not time dependent and can be assumed to be constant.

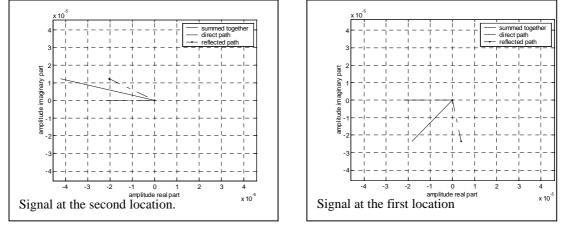
The component coming along direct path has shorter way to travel and when the other component arrives, the direct path component has value that the signal will have at  $\tau_{delta}$ . Where  $\tau_{delta}$  is time difference that signal has for travelling along these different paths.

The time  $\tau_i$  for electromagnetic wave for travelling is  $\tau_1 = \frac{l_1}{c}$  where  $l_1$  length of the line, c speed of light.

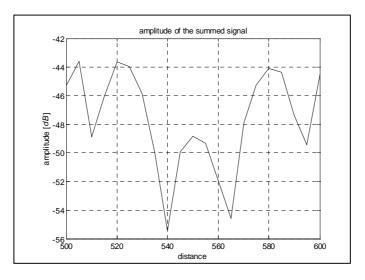
The time difference is accordingly  $\tau_{delta} = \tau_2 - \tau_1 = \frac{(l_2 - l_1)}{c}$ . During that time the phase of the first signal is changed  $\phi = \tau_{delta} \times \frac{c}{\lambda} \times 2\pi$  degrees. So we will have the baseband equivalence of the received signal  $a_1 e^{j\varphi_1} e^{j\phi} + a_2 e^{j\varphi_2}$ . The reflection factor -1 can be transformed to the phase change  $\varphi_2 = \pi$ . The signal from the first path does not have any phase change. (Similar analyze can be made when you write out the channel response of the signal and assume the delay along line of sight component to

be 0. The delay and corresponding phase of the reflected component in this case will be negative, the reflected component phase and component are behind of line of sight component. When delay of the line of sight component is zero the reflected component has still to arrive.)

The arriving signals and their sums for the first two locations are given on the figures below



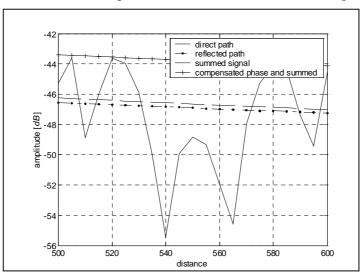
The total signal amplitude along the 100 m path with 5 m steps is given in the figure below.



c)

The compensation of the phase means multiplication with the coefficient  $e^{-j\phi_{tobe compensated}}$ . This multiplication is equivalent to delaying the signal. In our case the total phase to be compensated is  $\varphi_2 + \phi$ .

If we have compensated the phase the signal resulting summed signal is just sum of the absolute values of the amplitudes. That can be seen on the figure below.



d)

It was stated that separation of these signals at the receiver is only possible when their arrival difference is more than the symbol length. We can calculate the required symbol length by calculating what is the minimal arrival difference between the signals. This minimal difference occurs in the location where the distance for the two paths are minimum and that is at the location 500 + 100 m.

The time difference there is  $\frac{l_2 - l_1}{c} = \frac{632.45 - 600}{3 \times 10^8} = 1.08 \times 10^7$ . That corresponds to the symbol length. If to assume that a binary modulation is used this corresponds to transmission speed  $9.24 \times 10^6$  symbols per second.

#### 4.

What is minimal separation of channel taps that the receiver can recognize when user data rate is 30 kbit/s, the system uses coding rate 1/2 and each bit is spread with 64 bits? If two arriving signals can be separated what is the difference of path length they have traveled? The two paths can be separated if they are apart at least by one chip length.

4.

Bit rate 30 kbit/s with  $\frac{1}{2}$  coding symbol rate is 60 kbit/s. There are 60 chips for every symbol: chiprate: 60\*64=3864 kchip/s. Chip duration is  $T = \frac{1}{2} = \frac{1}{2}$ 

Chip duration is  $T_c = \frac{1}{R_c} = \frac{1}{3840000 [chips/s]} = 260 [ns]$ 

Path length difference is  $c \times T_c = 3 \times 10^8 \left[\frac{m}{s}\right] \times 260 \times 10^{-9} \left[s\right] = 78 \left[m\right]$