S72-238 WCDMA systems

Tutorial 2 25.01.2002.

Solutions

1.

Two spreading signals in vector form are $s_1 = [-111-1]$ and $s_2 = [1-11-1]$ with the chip length $\tau_1 = 1$.

a) Calculate the cross correlation function between the spreading sequence $s_1(t)$ and signal $s_{11}(t)$ that is bit sequence $[11]$ spread by $s_1(t)$.

b) Calculate the cross correlation function between the spreading sequence $s_1(t)$ and signal $s_{10}(t)$ that is bit sequence $[-11]$ spread by $s_1(t)$.

c) Calculate the cross correlation function between the spreading sequence $s_1(t)$ and signal $s_{21}(t) = [11]$ spread by $s_2(t)$.

d) Calculate the cross correlation function between the spreading sequence $s_1(t)$ and signal $s_{20}(t) = [-11]$ spread by $s_2(t)$.

1.

The spreading signal is described as sequence of values ± 1. Our spreading code has length four and we can express it as sum of four unit pulses

$s_1(t) = \sum_{n=0}^{3} s_1(n) p(t - n\tau_1)$, where $s_1(n)$ is the *n* element of the spreading signal vector, and $p(t - n\tau_1)$ is a unit pulse beginning at 0 and ending at τ_1.

A spread signal is generated by multiplying the bits of the s_{11} with the spreading sequence:

$s_{11}(t) = \sum_{n_b=0}^{1} a\left(n_b\right) p_b(t - n_b T_1) \left(\sum_{n_s=0}^{\tau_1} s_1\left(n_s\right) p_s\left(t - n_s \tau_1\right) \right)$, where n_b, n_s stand for the bit and chip numbers accordingly. $p_b(\ ), p_s(\ )$ are unit pulses with the bit and chip length accordingly and $a\left(n_b\right), s_1\left(n_s\right)$ stand for the bit and chip amplitudes.

The cross-correlation function $R_{xy}(\tau)$ between two signals $x(t)$ and $y(t)$ is defined as

$$R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t) y(t - \tau) dt.$$

Since we are dealing with a finite length signal the integration can be limited only to the values which are different from zero for out signal it would be interval $0 \ldots 7$. For our signal when τ is integer the correlation function can be written as sum of every chip of spread signal $s_{11}(t)$ and reference signal $s_1(t)$

$$R(\tau) = \frac{1}{4\tau} \int_{0+\tau}^{7+\tau} s_{11}(t + \tau) s_1(t) dt$$

Since the signals are compounded from triangular impulses we can calculate the correlation only for integer values of τ and between those $R(\tau)$ is given by a linear function connecting two neighboring values.
For integer τ_{int} values $R(\tau_{\text{int}})$ is calculated as:

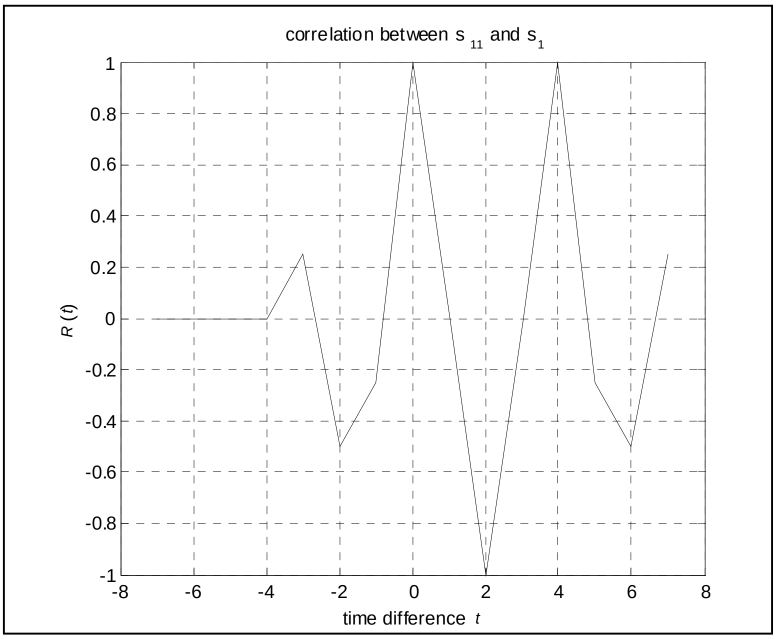
$$R(\tau_{\text{int}}) = \frac{1}{4\tau_1} \sum_{m=0+\tau}^{7+\tau} s_{11}(m+\tau_{\text{int}}) s_1(m)$$

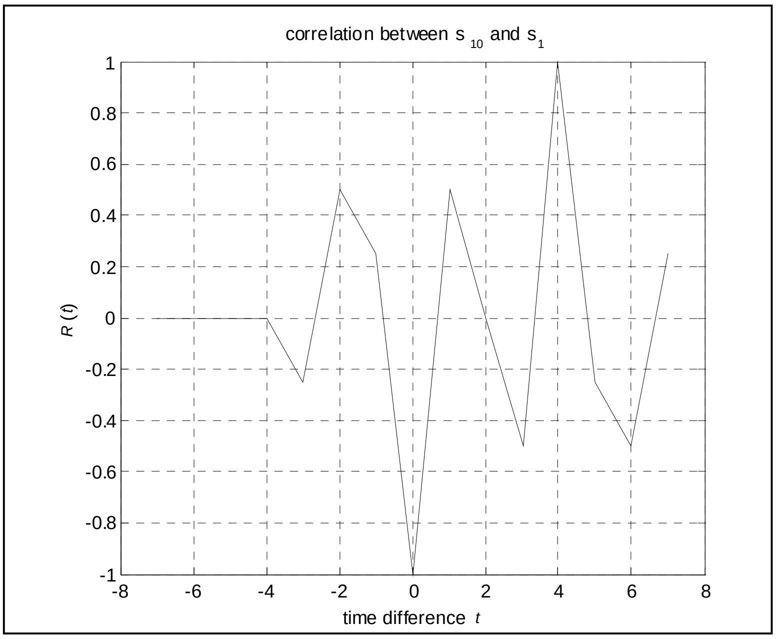
Between these values we use linear function

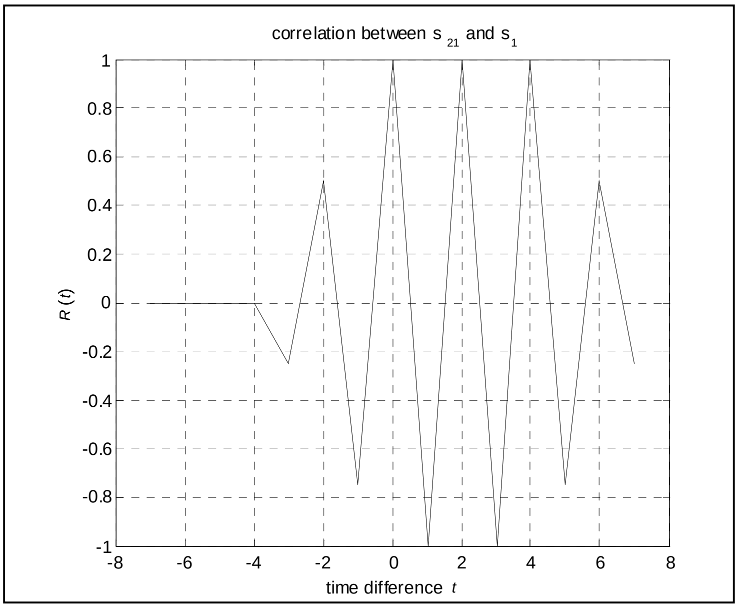
$$R(\tau) = \left(R(\lfloor\tau\rfloor) - R(\lceil\tau\rceil)\right)(\tau - \lfloor\tau\rfloor) + R(\lfloor\tau\rfloor).$$

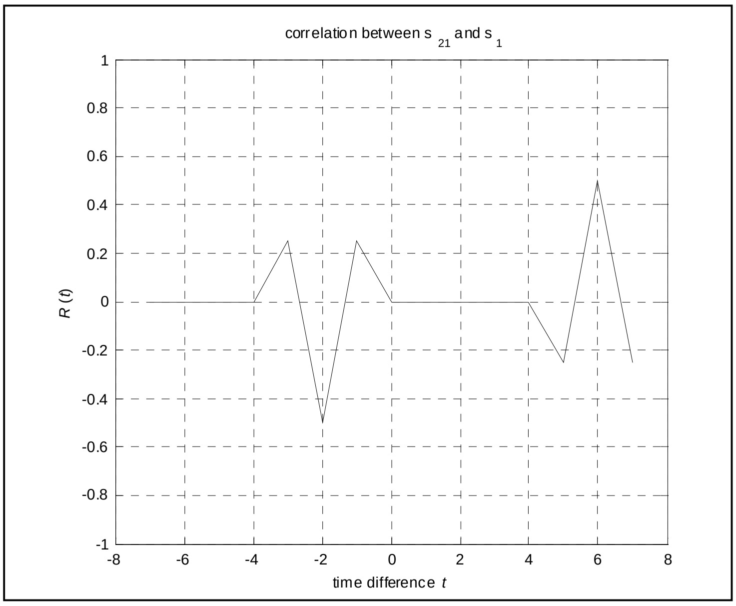
Where $\lfloor\ \rfloor, \lceil\ \rceil$ stand for the nearest smallest and highest integer respectively. We evaluate all the correlation functions with the "Matlab".

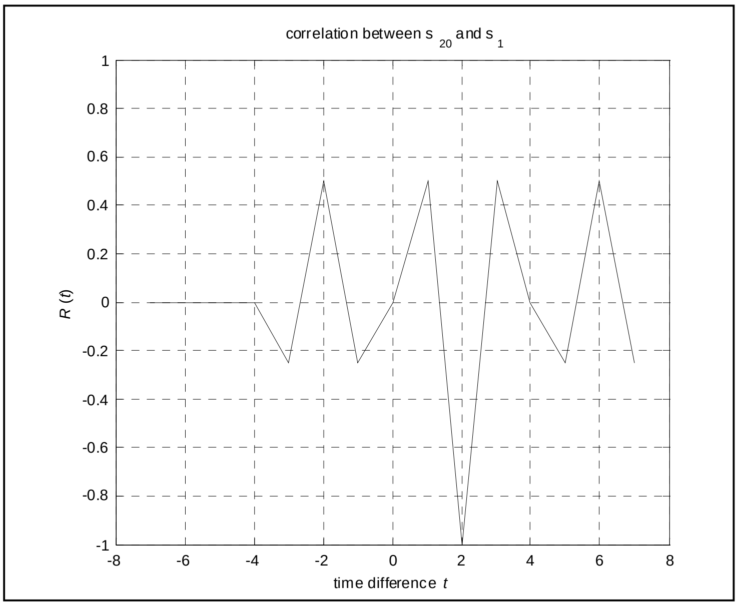
Figures from 1 to 8

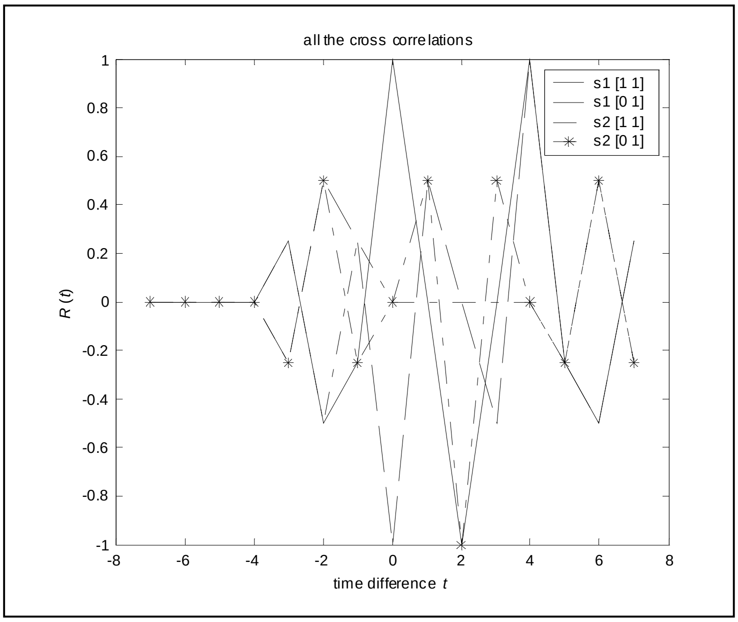





correlation between s 21 and s 1
R (t)
time difference t
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-8
-6
-4
-2
0
2
4
6
8


correlation between s 21 and s 1
R (t)
time difference t
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-8
-6
-4
-2
0
2
4
6
8


correlation between s 20 and s 1
R (t)
time difference t
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-8
-6
-4
-2
0
2
4
6
8


all the cross correlations
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
R (t)
-8
-6
-4
-2
0
2
4
6
8
time difference t
s1 [1 1]
s1 [0 1]
s2 [1 1]
s2 [0 1]

2.

The transmitted signal sequence is $a = [0\ 1\ 0\ 1]$. The spreading sequence is

$s_1 = [-1\ 1\ 1 -1]$.

Calculate amplitude of the signal and the interference for every bit if the channel response is

Tap amplitude	0.5	0.3	0.2
Delay τ in chips	0	1	2

And Rake receiver is tuned to the first channel tap.

2.

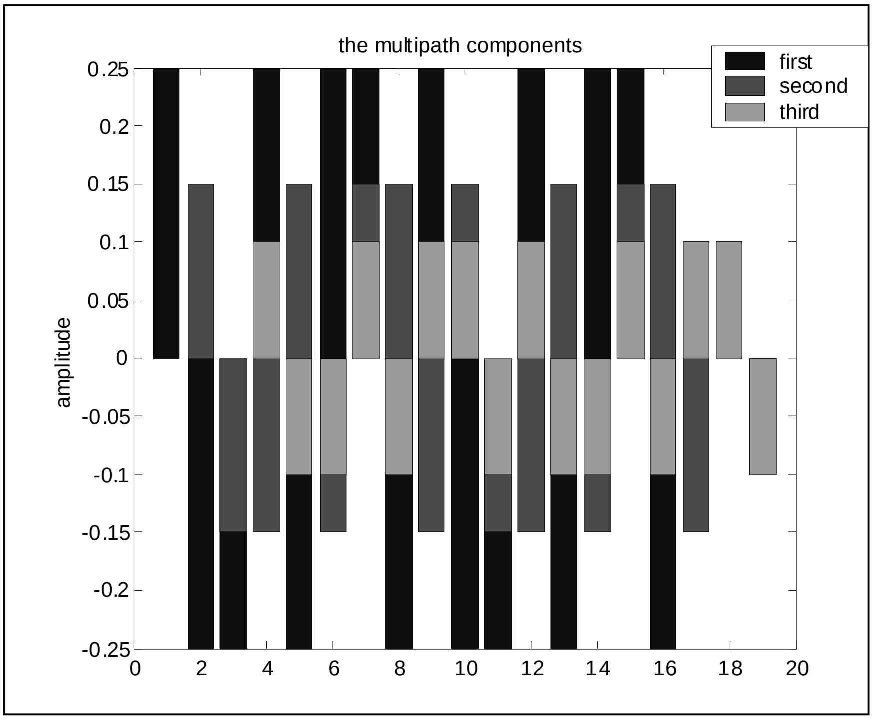
Since in the exercise there is no indication on Doppler components we assume that there is not any and describe the signal as a simple sum of delayed components of transmitted signal.

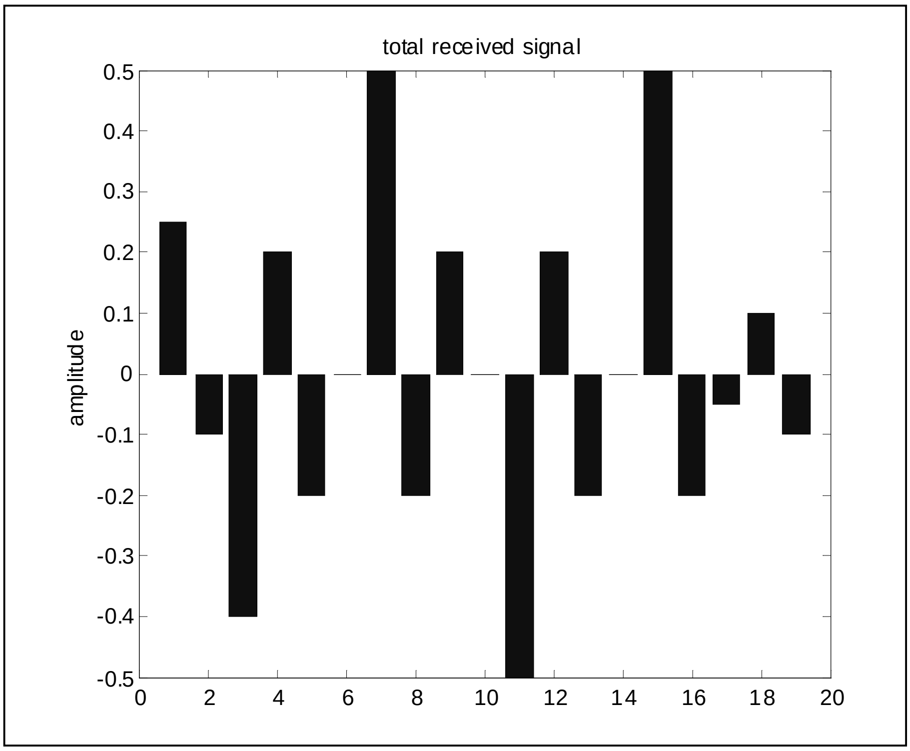
The received signal is described in the form

$$x(t) = \sum_L \alpha_l(t) s\left[t - u_l(t)\right]$$

Once again we can use notation for the spread signal as above:

$$s(t) = \sum_{n_b=0}^{1} a\left(n_b\right) p_b(t - n_b T_1)\left(\sum_{n_s=0}^{\tau_1} s_1\left(n_s\right) p_s\left(t - n_s \tau_1\right)\right)$$





The receiver correlates the received signal with reference spreading signal and integrates over it.

$$z_{m,k}(t) = \int_{T_i}^{T_{i+1}} \left(\alpha_l m_i s\left(t - u_l\right)\right) s_1(t)dt$$

$$+\sum_{\substack{L \\ l \neq 1}} \left(\int_{T_{-1}}^{u_{n,l}} \left(\alpha_l m_{i-1} s_1\left(t - u_l\right)\right) s_1\left(t\right) dt + \int_{u_{n,l}}^{T_0} \left(\alpha_l m_i s_1\left(t - u_l\right)\right) s_1(t)dt \right)$$

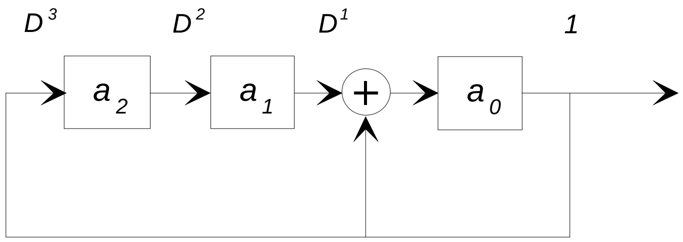
Where the first term describes the signal arriving along the path the receiver has synchronised on and the other term is sum of the signals from other paths.

Contributions of Taps\Symbols	1	2	3	4
1	-0.5	0.5	-0.5	0.5
2	0.075	-0.15	0.15	-0.15
3	-0.05	0.1	-0.1	0.1
Interference amp.	0.025	-0.05	0.05	-0.05
total amplitude	-0.475	0.45	-0.45	0.45

3.

Consider the feedback shift-register shown below with mod 2 calculations.

a) What is the polynomial that the shift register represents?



b) Determine the output of this circuit with the initial shift-register load
$a_0 = 1 \quad a_1 = 0 \quad a_2 = 0$.

c) Determine the output of this circuit with the initial shift register load
$a_0 = 1 \quad a_1 = 1 \quad a_2 = 0$.

d) Does this circuit generate a m-sequence?

e) Calculate autocorrelation of the output sequence.

f) Calculate power spectrum of the output sequence.

g) How much the interference from other user will be suppressed when the users have spreading code described above but the codes have different phases?

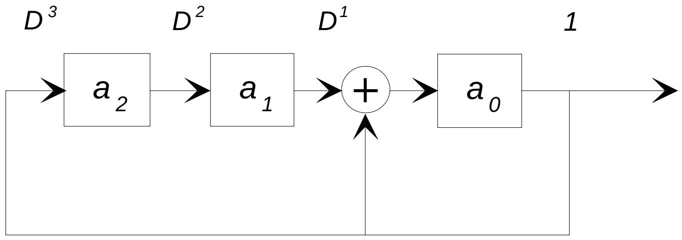
Reference:

R.L. Peterson, R. Ziemer, D. Borth: ”Introduction to spread spectrum communications” 1995. pages 695. chapter 3.

3.

a)

The shift register generating the spreading sequence is given on the figure below.



Each element introduces a delay D. The coefficient multiplying the delay element is 1 if the feedback is connected to it and 0 otherwise. The generator figure is described by a polynomial

$g(D) = 1 + D + D^3$

b)
At each step the output of the register is feed back to the register accordingly to the describing polynomial. By using output as input to the system we can calculate the registers state at each time step and from there the output sequence.

Cycle	Register State	$a(D)$			Output
0	001	1			1
1	101	1		D^2	1
2	111	1	D^1	D^2	1
3	110		D^1	D^2	0
4	011	1	D^1		1
5	100			D^2	0
6	010		D^1		0
7	001	1			1

The same output can also be calculated by dividing the generating polynomial $g(D)$ with the initial state $a_i(D) = 1$ $\left(\frac{g(D)}{a(D)}\right)$. For that at each state we multiply the polynomial $g(D)$ with the output of the generator and add to the state of the generator at that moment. This can be seen in the following table. Where the first line stands for the generator output at different steps D^t. Rest of the lines describe the polynomials of the system state and added polynomial $D^t g(D)$.

System output	1	D	D^2		D^4			D^7
$1+D+D^3$	1							
	1+	$D+$		D^3				
		$D+$		D^3				
		$D+$	D^2+		D^4			
			D^2+	D^3+	D^4			
			D^2+	D^3+		D^5		
					D^4+	D^5+		
					D^4+	D^5+		D^7
								D^7

Since the system is recursive such division may be continued to infinity.
However we see that at the delay D^7 the system state $a_7(D)$ is same as initial state and the division starts to repeat itself.

c)
The output sequence is calculated as above but for different initial state, 011.

Cycle	Register State	$a(D)$			Output
0	011	1	D^1		1
1	100			D^2	0
2	010		D^1		0
3	001	1			1
4	101	1		D^2	1

5	111	1	D^1	D^2	1
6	110		D^1	D^2	0
7	011	1	D^1		1

d)

Maximal length sequence or m-sequence is a shift-register sequence that has the maximal possible period for an r-stage shift register. For a shift register with 3 stages the maximal cycle period is $2^3 - 1 = 7$. From the output of our sequence we see that the period of the output sequence is 7 i.e. the shift register generates a m sequence.

e)

Let represent the sequence of the generator output with binary digits $\dots b_{-2}, b_{-1}, b_0, b_1, b_2 \dots$ from the alphabet $\{0,1\}$. Because the code is periodic with $b_n = b_{N+n}$ also the signal waveform $c(t)$ is periodic, with period $T = NT_c$ and is specified by $c(t) = \sum_{n=-\infty}^{\infty} a_n p(t - nT_c)$, where $a_n = (-1)^{b_n}$, and $p(t - nT_c)$ is an unit pulse beginning at 0 and ending at T_c. The waveform $c(t)$ is deterministic with auto-correlation function:

$$R_c(\tau) = \frac{1}{T}\int_0^T c(t)c'(t+\tau)dt,$$

$$R_c(\tau) = \frac{1}{T}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} a_n a_m \int_0^T p(t - mT_c)p(t+\tau-mT_c)dt.$$

The integral here is nonzero only when $p(t - mT_c)$ and $p(t+\tau-nT_c)$ overlap. The delay τ can be expressed as $\tau = kT_c + \tau_\varepsilon$, where $0 \le \tau_\varepsilon < T_c$. The pulses overlap only for $n = k + m$ and $n = k + m + 1$.

$$R_c(\tau) = R_c(k, \tau_\varepsilon) = \frac{1}{N}\sum_{m=0}^{N-1} a_m a_{k+m} \frac{1}{T}\int_0^{T_c - \tau_\varepsilon} p(\lambda)p(\lambda + \tau_\varepsilon)d\lambda$$

$$+\frac{1}{N}\sum_{m=0}^{N-1} a_m a_{k+m+1} \frac{1}{T}\int_{T_g - \tau_\varepsilon}^{T_c} p(\lambda)p(\lambda - T_c + \tau_\varepsilon)d\lambda$$

Let represent the binary sequence of b's as a vector $\mathbf{b}$.
The discrete periodic auto-correlation function is defined as:

$\Theta_b(k) = \frac{1}{N}\sum_{n=0}^{N-1} a_n a_{n+k} = \frac{N_A - N_D}{N}$, where N_A is a number of places in which $\mathbf{b}(0)$ agrees and N_D is a number of places where $\mathbf{b}(0)$ disagrees with $\mathbf{b}(k)$. Equivalently N_A is a number of zeros and N_D is a number of ones in modulo 2 sum of $\mathbf{b}(0)$ and $\mathbf{b}(k)$.

$$R_c(k,\tau) = \left(1 - \frac{\tau_\varepsilon}{T_c}\right)\Theta_b(k) + \frac{\tau_\varepsilon}{T_c}\Theta_b(k+1)$$

Property: a maximal-length sequence contains one more one than zero. The number of ones in the sequence is $\frac{1}{2}(N+1)$.

For an m-sequence the periodic auto-correlation function is two-valued and is given by:

$$\Theta_b(k) = \begin{cases} 1.0 & k = lN \\ -\frac{1}{N} & k \neq lN \end{cases}$$

$$0 \le \tau_\varepsilon \le T_c$$

$$R_c(\tau) = \left(1 - \frac{\tau}{T_c}\right) - \frac{1}{N}\frac{\tau}{T_c} = 1 - \frac{\tau}{T_c}\left(1 + \frac{1}{N}\right)$$

$$T_c \le \tau_\varepsilon \le (N-1)T_c$$

$$R_c(\tau) = \left(1 - \frac{\tau}{T_c}\right)\left(-\frac{1}{N}\right) - \frac{1}{N}\frac{\tau}{T_c} = -\frac{1}{N}$$

$$(N-1)T_c \le \tau_\varepsilon \le T_c$$

$$R_c(\tau) = \frac{\tau - (N-1)T_c}{T_c}\left(1 + \frac{1}{N}\right) - \frac{1}{N}.$$

f)

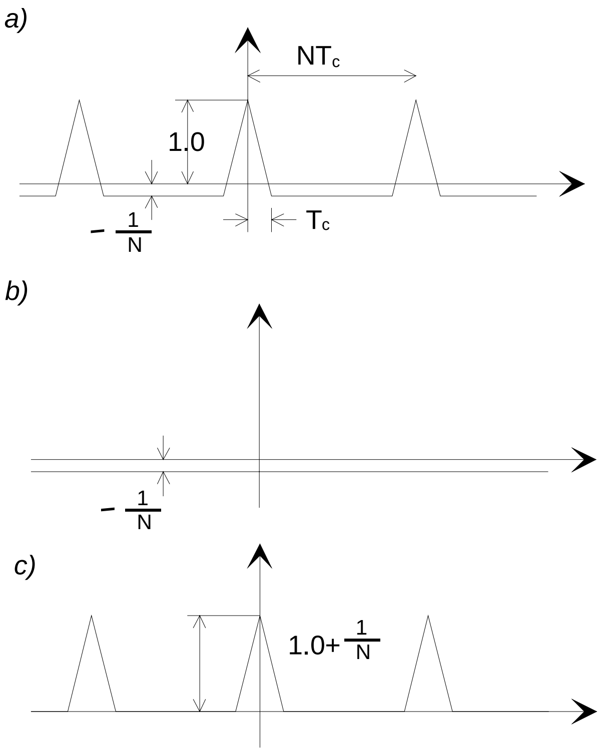
The power spectrum of the sequence is the Fourier transform of its auto-correlation function $R_C(\tau)$.

$$S_c(f) = \sum_{m=-\infty}^{\infty} P_m \delta(f - mf_0)$$

Because we have periodic signal the spectrum of it is discrete with values at locations $f_o = n\frac{1}{NT_c}$ where n is integer.

We are dealing with a periodic signal that can be described as sum of two signals. This is illustrated on the figure where one of the signals is constant at the level $-\frac{1}{N}$ and the other one is the correlation function to which is added a constant $\frac{1}{N}$. This other signal has values only in interval $-T_c < \tau < T_c$.

For calculating the signal spectrum we use the correlation function in the interval $-\frac{NT_c}{2} < \tau < \frac{NT_c}{2}$. The constant function in this interval has value $-\frac{1}{N}$ and the other functions can be described as two functions one interval $-T_c < \tau < 0, \boldsymbol{f_1}(\tau)$ and other in interval $0 < \tau < T_c, \boldsymbol{f_2}(\tau)$.



$$f_1(\tau) = \frac{\tau - T_c}{T_c}\left(1 + \frac{1}{N}\right) - \frac{1}{N} + \frac{1}{N} = \left(\frac{N+1}{N}\right)\frac{\tau + T_c}{T_c}$$

$$f_1(\tau) = 1 - \frac{\tau}{T_c}\left(1 + \frac{1}{N}\right) + \frac{1}{N} = \left(\frac{N+1}{N}\right)\left(1 - \frac{\tau}{T_c}\right)$$

First we calculate the spectrum value at frequency 0 - that is the mean of the signal. Because the correlation function is described as a sum of two functions the mean will be sum of the means of these two.

$$\begin{aligned} P_0 &= \frac{1}{NT_c}\left(\int_{-T/2}^{T/2}\left(-\frac{1}{N}\right)dt + \int_{-T_c}^{0} \boldsymbol{f}_1(t)dt + \int_{0}^{T} \boldsymbol{f}_2(t)dt\right) \\ &= (-1)\frac{1}{N} + \frac{1}{2}\frac{N+1}{N^2} + \frac{1}{2}\frac{N+1}{N^2} \\ &= \frac{N+1}{N^2} - \frac{N}{N^2} = \frac{1}{N^2} \end{aligned}$$

In order to calculate the spectrum at other frequencies we consider a fact that; by adding any constant value to the signal will change only mean of the signal, spectral component P_0, the value of other spectral components will not change. Accordingly to that we can calculate the spectrum of the signal from the signal 2 in the figure.

In order to calculate the spectrum of the signal we consider the relationship between the differentiation and Fourier transform

$$\frac{d}{dt}f_1(t) \Leftrightarrow 2\pi i f \mathfrak{F}(f)$$

$$\frac{1}{NT_c}\left(\int_{-T_c}^{0}\frac{1}{2\pi f_0 i}\frac{df_1(\tau)}{dt}e^{-i2\pi f_0 t}dt+\int_{0_c}^{T_c}\frac{1}{2\pi f_0 i}\frac{df_2(\tau)}{dt}e^{-i2\pi f_0 t}dt\right)$$

$$=\frac{1}{NT_c}\frac{1}{2\pi f_0 i}\frac{N+1}{N}\frac{1}{T_c}\left(\int_{-T_c}^{0}e^{-i2\pi f_0 t}dt+\int_{0_c}^{T_c}e^{-i2\pi f_0 t}dt\right)$$

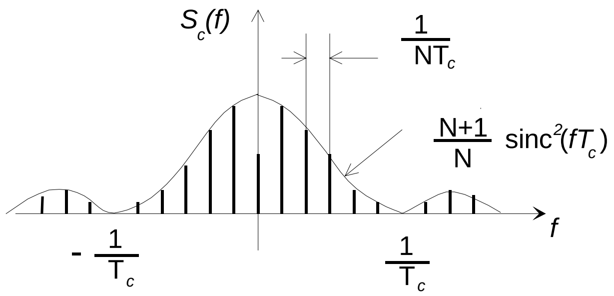
$$=\frac{N+1}{(NT_c)^2}\frac{1}{2\pi f_0 i}\left(\frac{1}{-2\pi f_0 i}\left(e^0-e^{i2\pi f_0 T_c}-e^{i2\pi f_0 T_c}+e^0\right)\right)$$

$$=\frac{N+1}{(NT_c)^2}\frac{1}{(-1)\left(2\pi f_0 i\right)^2}\left(2-\frac{2\left(e^{i2\pi f_0 T_c}+e^{-i2\pi f_0 T_c}\right)}{2}\right)$$

$$=\frac{N+1}{(NT_c)^2}\frac{1}{(-1)\left(2\pi f_0 i\right)^2}\left(2-2\cos(2\pi f_0 T_c)\right)$$

$$=\frac{N+1}{(NT_c)^2}\frac{1}{\left(2\pi f_0\right)^2}4\sin(2\pi f_0 T_c)$$

$$=\frac{N+1}{N^2}\operatorname{sinc}^2(2\pi f_0 T_c)$$



g)

Because other signals use the same spreading code and are separated only by phase they will be suppressed by the correlation factor that is to the level $-\frac{1}{N}$.