S72-238 WCDMA systems

Tutorial 6 22.01.2002.

Solutions

1.

Let assume a WCDMA receiver operating in a (static) multipath environment. The relative strength a_i and delays τ_i of the propagation paths (with respect to the line of sight (LOS) path) are as follows

a_i	1	0.3	0.6	0.2	0.5	0.2
τ_i $[\mu s]$	0.0	0.2	0.3	0.7	2.4	3.1

We assume that - when perfectly in code synchronisation - a signal processing branch is the receiver produces the relative power 1 (unit) for relative path strength 1 (unit) at the output of the matched filter before eventual path combining. When not in code synchronisation, a path is assumed to produce $\frac{1}{N}$ of this power, where N is code gain of the WCDMA system.

Calculate the SIR ratio (signal power to multipath interference ratio) for the following cases (other interfering signals and noise is not taken into account):

a) No RAKE reception; the receiver synchronises only to the LOS signal component. The channel bit rate is 32 $kbps$ (Note user bit rate undefined), N=128.
b) 4 finger RAKE reception with Equal Gain Combining, N=128.
c) 4 finger RAKE reception with Maximum Ratio Combining, N=128.
d) 4 finger RAKE reception with Maximum Ratio combining. In this case, however, the channel bit rate is 512 $kbps$, N=8.
e) 6 finger RAKE reception with Maximum Ratio Combining, N=128.

Calculate the percentage of the arrived signal energy that the 4 finger RAKE receiver utilise for decoding.

1.

The RAKE receiver consist a set of fingers, receivers, and each of them is synchronised to one of the channel taps. The outputs of fingers are combined accordingly to applied policy most common of which are Equal Gain and Maximum Ratio Combining.

Assume that the matched filter of a signal processing branch consist of a correlator and integrator. Correlator is between the signal code with strength a_i (received via propagation path i and the reference code with strength b_i where amplitude of b depends on combining policy.

The received signal is described by signal to noise ratio SNR what is power of the mean of a received signal divided by the second moment of the signal. In code synchronisation the relative average matched filter output power is $\left(a_i \cdot b_i\right)^2$ and otherwise $\frac{\left(a_i \cdot b_i\right)^2}{N}$.

a)

The relative signal power is $\left(a_i \cdot b_i\right)^2 = 1$ (power units) $\rightarrow b_i = 1$. The multipath interference is variance of the sum of the amplitudes from the paths that are not synchronised to the reference signal. The variance of the sum of the independent signals is sum of variances. Because the other multipath are not synchronous to the first path they are scaled down by N.

$$i = \frac{1}{N}\sum_{i=1}^{5}\left(a_i \cdot b_0\right)^2 = \frac{0.78}{N}$$

The SIR ratio is thus $\frac{N}{0.78} = 164.1 \Rightarrow 22.15\ dB$.

b)

We first investigate if there is inter symbol interference (ISI) in the receiver. ISI is generated whenever the channel delay spread D is larger than the bit duration T. Since $T = \left(\text{bit rate}\right)^{-1} = 31\ \mu s \gg D\ \mu s$ there is no ISI.

The signal power after Equal Gain Combining (EGC) is

$$S = \left(a_0 \cdot 1 + a_1 \cdot 1 + a_2 \cdot 1 + a_4 \cdot 1\right)^2 = 5.76$$

Since in EECG the reference code strength b_i are assumed equal to be $b_0 = 1$.

When we calculate the interference power we have to recall that it is the variance of the signal at the finger. When we consider the interference from different RAKE fingers to be independent the variance of the signal from each finger is sum of the variances at each finger. The interference power is correspondingly

$$I = \frac{1}{N}\begin{pmatrix} \sum_{\substack{i=0\\ i\neq 0}}^{5}\left(a_i \cdot 1\right)^2 + \sum_{\substack{i=0\\ i\neq 1}}^{5}\left(a_i \cdot 1\right)^2 \\ +\sum_{\substack{i=0\\ i\neq 2}}^{5}\left(a_i \cdot 1\right)^2 + \sum_{\substack{i=0\\ i\neq 4}}^{5}\left(a_i \cdot 1\right)^2 \end{pmatrix} = \frac{5,47}{N},$$

and the SIR ratio is $5.76 \cdot \frac{N}{5.46} = 134.82 \Rightarrow 21.29$

c)

Again, there is no ISI. In Maximum Ratio Combining (MRC) we assume $b_i = a_i$. The signal power is

$$S = \left(a_0 \cdot a_0 + a_1 \cdot a_1 + a_2 \cdot a_2 + a_4 \cdot a_4\right)^2 = 2.89,$$

the interference power is

$$I = \frac{1}{N}\begin{pmatrix} \sum_{\substack{i=0\\ i\neq 0}}^{5}\left(a_i \cdot a_0\right)^2 + \sum_{\substack{i=0\\ i\neq 1}}^{5}\left(a_i \cdot a_1\right)^2 \\ +\sum_{\substack{i=0\\ i\neq 2}}^{5}\left(a_i \cdot a_2\right)^2 + \sum_{\substack{i=0\\ i\neq 4}}^{5}\left(a_i \cdot a_4\right)^2 \end{pmatrix} = \frac{1.83}{N}$$

and SNR is $2.89 \cdot \frac{N}{1.83} = 202.6 \Rightarrow 23.07$

d)

In this case there is some ISI since $T = \left(\text{bit rate}\right)^{-1} = 1.95\ \mu s < d = 3.1\ \mu s$. The receiver either recognises that one multipath component is spread to the following information bit or it does not recognise that. In case the receiver does not recognise that it attempts to synchronise one finger to the multipath component that is in the interval of the current bit and carries information of a previous bit. In our case this means that three branches of the RAKE circuit, with relative powers

$S^2 = (1 \cdot 1 + 0.3 \cdot 0.3 + 0.6 \cdot 0.6)^2 = 2.10$

will synchronise to the right bit and the fourth branch with relative power S_2 to the previous bit This fourth branch is thus producing serious interference. A kind of worst case estimate of the SNR ratio is

$$I = \left(\left(a_4 \cdot a_4\right)^2 + \frac{1}{N}\begin{pmatrix} \sum_{\substack{i=0\\ i\neq 0}}^{5}\left(a_i \cdot a_0\right)^2 + \sum_{\substack{i=0\\ i\neq 1}}^{5}\left(a_i \cdot a_1\right)^2 \\ +\sum_{\substack{i=0\\ i\neq 2}}^{5}\left(a_i \cdot a_2\right)^2 + \sum_{\substack{i=0\\ i\neq 4}}^{5}\left(a_i \cdot a_4\right)^2 \end{pmatrix}\right) = 0.29$$

and SNR is $2.10 \cdot \dfrac{1}{0.29} = 7.23 \Rightarrow 8.59$.

In contrast when the receiver recognises that the channel spread exceeds the symbol length it can synchronise to the path that arrives in next symbol interval in what case the signal power will be.

$S^2 = (1 \cdot 1 + 0.3 \cdot 0.3 + 0.6 \cdot 0.6 + 0.4 + 0.4)^2 = 2.89$

and interference is

$$I = \frac{1}{N}\begin{pmatrix} \sum_{\substack{i=0\\ i\neq 0}}^{5}\left(a_i \cdot a_0\right)^2 + \sum_{\substack{i=0\\ i\neq 1}}^{5}\left(a_i \cdot a_1\right)^2 \\ +\sum_{\substack{i=0\\ i\neq 2}}^{5}\left(a_i \cdot a_2\right)^2 + \sum_{\substack{i=0\\ i\neq 4}}^{5}\left(a_i \cdot a_4\right)^2 \end{pmatrix} = \frac{1.83}{N}$$

and SNR is $2.89 \cdot \frac{N}{1.83} = 12.63 \Rightarrow 11.07$.

e)

Again, there is no ISI. In Maximum Ratio Combining (MRC) we assume $b_i = a_i$.

The signal power is

$$S = \left(a_0 \cdot a_0 + a_1 \cdot a_1 + a_2 \cdot a_2 + a_3 \cdot a_3 + a_4 \cdot a_4 + a_5 \cdot a_5\right)^2 = 3.17,$$

the interference power is

$$I=\frac{1}{N}\left(\begin{array}{l}\sum_{\substack{i=0\\ i\neq 0}}^{5}\left(a_i\cdot a_0\right)^2+\sum_{\substack{i=0\\ i\neq 1}}^{5}\left(a_i\cdot a_1\right)^2\\ +\sum_{\substack{i=0\\ i\neq 2}}^{5}\left(a_i\cdot a_2\right)^2+\sum_{\substack{i=0\\ i\neq 3}}^{5}\left(a_i\cdot a_3\right)^2\\ +\sum_{\substack{i=0\\ i\neq 4}}^{5}\left(a_i\cdot a_4\right)^2+\sum_{\substack{i=0\\ i\neq 5}}^{5}\left(a_i\cdot a_5\right)^2\end{array}\right)=\frac{1.96}{N}$$

and SNR is $3.17\cdot\frac{N}{1.96}=206.39\Rightarrow 23.14$.

2.

In a spread spectrum system are three users. For all users the spreading factor N is 128 and the system uses a spread bandwidth 3.84 MHz. The cross correlation factor between the spreading codes is $\frac{1}{\sqrt{N}}$. We compare the system performance for different received powers for different users.

a) If in the system are three users what is the received signal to noise ratio for each user when no multi-user detection is applied and:

- The received signal powers are $P=\begin{bmatrix}0.372 & 0.209 & 0.0233\end{bmatrix}\cdot 10^{-13}\ [W]$.
- The received signal powers are $P=\begin{bmatrix}0.233 & 0.233 & 0.233\end{bmatrix}\cdot 10^{-14}\ [W]$.

b) Assume now that a decorrelating receiver is used what is the received SNR for both cases presented above.

2.

SNR is calculated as power of the mean of the received signal divided by the variation of the received signal. In order to evaluate this values we have to describe our system.

Let us describe the received information bits as $\mathbf{b}=\begin{bmatrix}b_1 & b_2 & b_3\end{bmatrix}$ where $b_1=\pm 1$.

Received signal amplitudes by a matrix

$$\mathbf{A}=\begin{bmatrix}A_1 & 0 & 0\\ 0 & A_2 & 0\\ 0 & 0 & A_3\end{bmatrix},$$

and correlation among the received signals by the correlation matrix

$$\mathbf{R}=\begin{bmatrix}1 & \frac{1}{\sqrt{N}} & \frac{1}{\sqrt{N}}\\ \frac{1}{\sqrt{N}} & 1 & \frac{1}{\sqrt{N}}\\ \frac{1}{\sqrt{N}} & \frac{1}{\sqrt{N}} & 1\end{bmatrix}.$$

The noise after correlation is described by a vector $\mathbf{n} = \begin{bmatrix} n_1 & n_2 & n_3 \end{bmatrix}$. The received signal for user 1 is given as

$$y_1 = A_1 b_1 + \frac{1}{\sqrt{N}} A_2 b_2 + \frac{1}{\sqrt{N}} A_3 b_3 + n_1 .$$

This kind of equation can be written for each user and by combining them the received signals can be described in a matrix form

$$\mathbf{y} = \begin{matrix} y_1 \\ y_2 \\ y_3 \end{matrix} = \mathbf{RAb} + \mathbf{n} .$$

a)

The SNR for user i is given by relationship

$$SIR_1 = \frac{\left(E\left\{y_1\right\}\right)^2}{E\left\{y_1^2\right\} - \left(E\left\{y_1\right\}\right)^2} .$$

The mean value of the interfering signals is zero. This is because the interfering signals have random phase compared to the informational signal and amplitude alters between ± 1. Also the mean of the thermal noise is zero.

$E\left\{y_1\right\} = \pm A_1$

Assume that user signal, noise, and interfering signals are independent.
Because of that the variance can be calculated as sum of variances.

$$E\left\{y_1^2\right\} = E\left\{\left(A_1 b_1\right)^2\right\} + E\left\{\left(\frac{1}{\sqrt{N}} A_2 b_2\right)^2\right\} + E\left\{\left(\frac{1}{\sqrt{N}} A_3 b_3\right)^2\right\} + E\left\{n_1^2\right\}$$

From there we get for the first user

$$SIR_1 = \frac{A_1^2}{A_1^2 + \frac{A_2^2}{N} + \frac{A_3^2}{N} + n_1^2 - A_1^2} .$$

This kind of equation can be derived for each user.

In order to evaluate that equation we have to calculate the noise power n_1^2. This is equal to the noise spectral density multiplied with the signal bandwidth after decorrelation.

Noise for the signal is $4 \cdot 10^{-21} \cdot \frac{W}{N}$ where $\frac{W}{N}$ is the spread bandwidth divided by the spreading factor.

$$n_1^2 = 4 \cdot 10^{-21} \frac{3.84 \cdot 10^6}{128} = 1.2 \cdot 10^{-16}$$

The noise power is same for all users.

We are given the received signal power. By inserting this into the Equation

$$SIR_1 = \frac{P_1}{\frac{1}{N}\left(P_2 + P_3\right) + n_1^2}.$$

- By inserting the received signal powers $P = \left[0.372 \quad 0.209 \quad 0.0233\right] \cdot 10^{-13}\ [W]$ into this equations a SIR vector with element for each user

$$SIR = \begin{matrix} 123.3 \\ 48.8 \\ 4 \end{matrix} \Rightarrow \begin{matrix} 20.9 \\ 16.9 \\ 6 \end{matrix}\ dB$$

- If the received signal powers are $P = \left[0.233 \quad 0.233 \quad 0.233\right] \cdot 10^{-14}\ [W]$ we get SIR vector with element for each user

$$SIR = \begin{matrix} 14.9 \\ 14.9 \\ 14.9 \end{matrix} \Rightarrow \begin{matrix} 11.7 \\ 11.7 \\ 11.7 \end{matrix}\ dB$$

b) We described the received signal in matrix form as $\mathbf{y} = \mathbf{RAb} + \mathbf{n}$.
After decorrelation the received signal $\mathbf{y}$ will be multiplied with the inverse of the correlation matrix and the resulting signal vector is

$$\tilde{\mathbf{y}} = \mathbf{R}^{-1}\mathbf{RAb} + \mathbf{n} = \mathbf{Ab} + \mathbf{R}^{-1}\mathbf{n}.$$

SIR is still calculated as

$$SIR_1 = \frac{\left(E\left\{\tilde{y}_1\right\}\right)^2}{E\left\{\tilde{y}_1^2\right\} - \left(E\left\{\tilde{y}_1\right\}\right)^2}.$$

For first user the mean will again be $E\left\{\tilde{y}_1\right\} = \pm A_1$.
The received signal contains no interference but only user signal and weighted sum of noise from all the users. The noise to different users is independent and variance can be calculated as sum of these noise variances

$$E\left\{y_1^2\right\} = E\left\{\left(A_1 b_1\right)^2\right\} + n_1^2 \sum_{i=1}^{3} E\left\{\left(R_{1j}^{-1}\right)^2\right\},$$

$$SIR_1 = \frac{A_1^2}{n_1^2 \sum_{i=1}^{3} E\left\{\left(R_{1j}^{-1}\right)^2\right\}} = \frac{P_1}{n_1^2 \sum_{i=1}^{3} E\left\{\left(R_{1j}^{-1}\right)^2\right\}}.$$

- The received signal powers are $P = \left[0.372 \quad 0.209 \quad 0.0233\right] \cdot 10^{-13}\ [W]$. We get a SIR vector with element for each user

$$SIR = \begin{matrix} 262.6 \\ 147.7 \\ 16.4 \end{matrix} \Rightarrow \begin{matrix} 24.2 \\ 21.7 \\ 12.2 \end{matrix} \ dB$$

- If the received signal powers are $P = \begin{bmatrix} 0.233 & 0.233 & 0.233 \end{bmatrix} \cdot 10^{-14} \ [W]$ we get SIR vector with element for each user

$$SIR = \begin{matrix} 16.4 \\ 16.4 \\ 16.4 \end{matrix} \Rightarrow \begin{matrix} 12.2 \\ 12.2 \\ 12.2 \end{matrix} \ dB$$

3.

The WCDMA radio network has been planned for $75\ \%$ load. In case the system does not have accurate information about the external interference level there can be significant capacity loss. Assume that $15\ \%$ of the loading is caused by external interference, how much capacity is reduced in terms of number of speech users:
Assume single link $SIR = 6\ \ dB$.
$R = 15\,\frac{kbit}{s}$ with voice activity factor 0.67.
Other to own cell interference ratio $i = 65$.
Chip rate is $3.84\ \frac{Mchip}{s}$.

3.

The total capacity is pole capacity of the system that is described by the value where the $load\ factor$ reaches 1. Theoretical estimate of the number of simultaneous users in a cell (provided that there is uniform user profile):

$$\eta_{UL} = N(1+i)L_j = N(1+i)\frac{1}{1+\frac{W}{\frac{E_b}{N_0}R}}.$$

The capacity is reached for $\eta_{UL} = 1$,

$$N = \left(1 + \frac{W}{R\frac{E_b}{N_0}\nu}\right)\frac{1}{(1+i)}.$$

The "own" loading in case of external interference is $75\,\% - 15\,\% = 60\,\%$.

There will be $\frac{60}{75}\ \%$ less users in the network, $\left(80\,\%\right)$.

Calculating for given parameters we get $N = 58$ with $75\ \%$ loading 44 users, and with $60\ \%$ loading 35 users.

4.

Diversity reception is used to improve the sensitivity of a basestation receiver. Assume, that in average the diversity gain is $2\ dB$, how much is the UL loading reduced when comparing non diversity and diversity case? Assume that the diversity is applied at all the cells in the network. What is the gain in the radio link budget?

Assume single link $SIR = 6\ \ dB$.
$R = 15\frac{kbit}{s}$.
Other to own cell interference ratio $i = 65$.
Chip rate is $3.84\ \frac{Mchip}{s}$.
In the cell are $35\ 100\ \%$ active users.

4.

Solution: EbNo with diversity $6 - 2 = 4\ \ dB$.
If the diversity is uniformly applied we can assume that other to own cell interference ratio stays in $65\ \ \%$.

Loading can be estimated with:

$$\eta_{UL} = N(1+i)L_j = N(1+i)\frac{1}{1+\dfrac{W}{\frac{E_b}{N_0}R}}.$$

With 6 dB this gives 0.88 , and with 4 dB this gives 0.56 ,

The loss in RLB for each loading can be estimated with $L = 10\log_{10}\left(1-\eta_{UL}\right)$. The calculated gain in radio link budget (difference between the two η_{UL}) is $5.74\ dB$.

5.

How big is the handoff region of a cell, assuming that the handoff may occur when the signals from two base stations to mobile station differ less than a) $6\ dB$ b) $3\ dB$. Consider a hexagonal cell structure and radio wave attenuation exponent a) 3.5 b) 4 c) 4.5.

5.

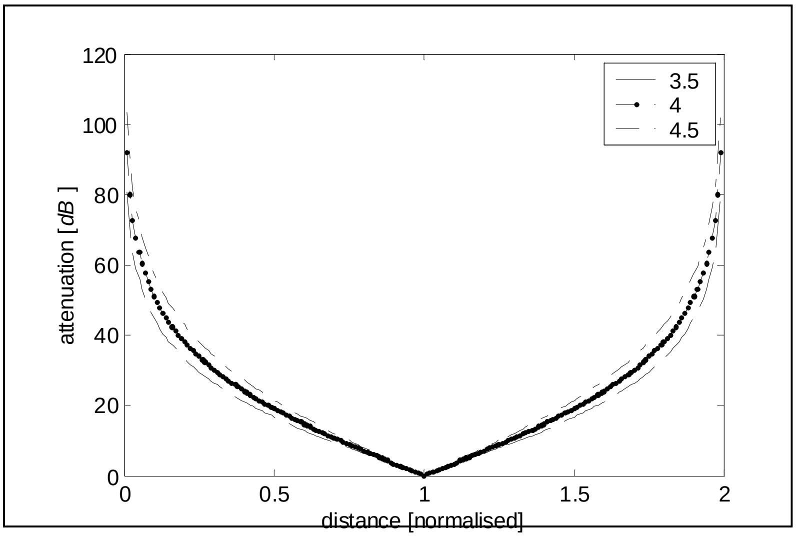
We have to calculate in how big area of the cell the signal powers from neighbouring BS differ 6 or 3 dB. First we assume that the cells are circular and the crude estimate for the area that we look for can be calculated as

$$a = \frac{\pi\left(d^2 - d_{ho}^2\right)}{\pi d^2}.$$

Where d_{ho} is the distance from the BS where the signal strengths from the neighbouring BS differ by given factor and the d is the cell radius.

Let, in form of co-ordinates (x, y), the position of $\mathrm{BS}\ 1$ be (0.0) and the position of $\mathrm{BS}\ 2$ be $(2,0)$.

When MS moves from $\mathrm{BS}\ 1$ towards $\mathrm{BS}\ 2$ one signal is increasing and the other is decreasing, as in the figure below.



The signal power attenuation as a function of distance d is $a = kd^m$ or $A = m\log(d) + K\ \ [dB]$. Let us assume $k = 1\ \left(K = 0\ \ [dB]\right)$. The difference between the signal powers in dB is $\Delta A = \left|m \cdot 10\log_{10}(d) - m \cdot 10\log_{10}(2-d)\right|\ \ [dB]$ where m is the attenuation factor.

When the MS is located between two BS, the 6 dB power difference region covers approximately the distance between the BS.

A crude estimation for the handover area would thus be about

$$a = \frac{\pi\left(d^2 - d_{ho}^2\right)}{\pi d^2}$$

	HO level 3 dB		HO level 6 dB	
	dh	a	dh	a
3.5	0.9016	0.1871	0.8051	0.3518
4.0	0.9138	0.1650	0.8290	0.3276
5.5	0.9233	0.1475	0.8476	0.2944