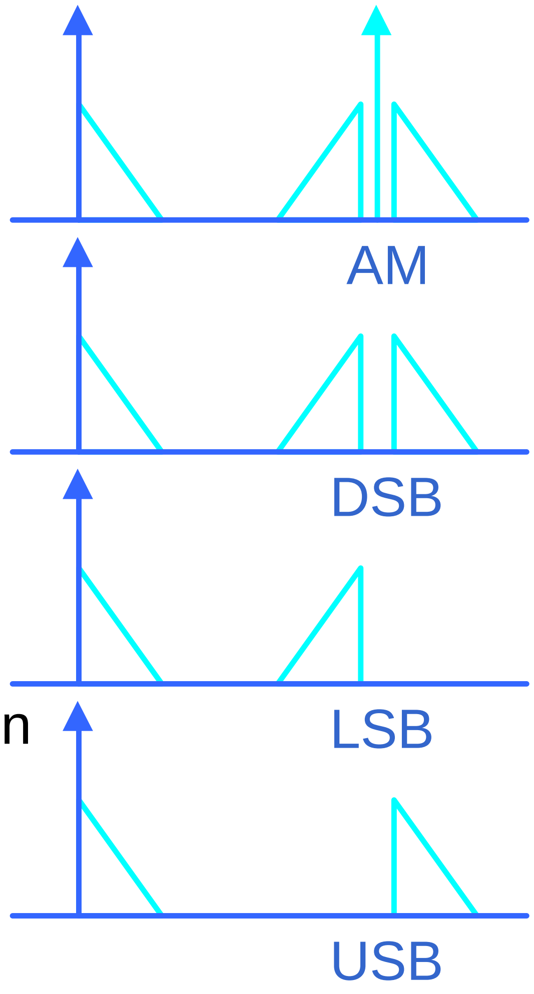
S-72.244 Modulation and Coding Methods

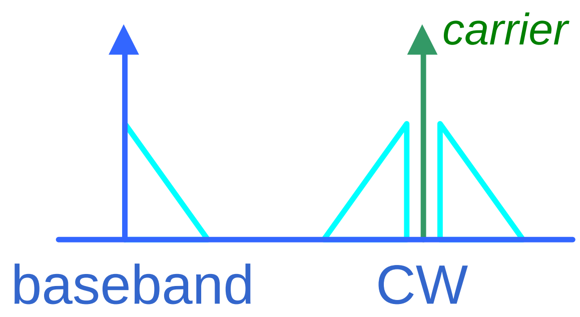
Linear Carrier Wave Modulation

Linear carrier wave (CW) modulation

- Bandpass systems and signals
- Lowpass (LP) equivalents
- Amplitude modulation (AM)
- Double-sideband modulation (DSB)
- Modulator techniques
- Suppressed-sideband amplitude modulation (LSB, USB)
- Detection techniques of linear modulation
  - Coherent detection
  - Noncoherent detection

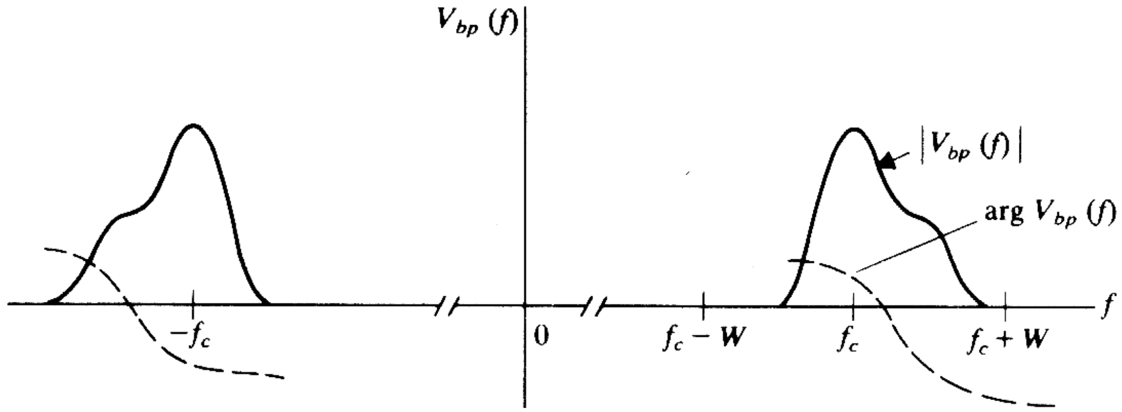


Baseband and CW communications



- Baseband communications is used in
  - PSTN local loop
  - PCM communications for instance between exchanges
  - (fiber-) optical communication
- Using carrier to shape and shift the frequency spectrum (eg CW techniques) enable modulation by which several advantages are obtained
  - different **radio bands** can be used for communications
  - **wireless** communications
  - **multiplexing** techniques become applicable
  - exchanging transmission bandwidth to received SNR

Defining bandpass signals



- The bandpass signal is band limited

$$V_{bp}(f) = 0, |f| < f_c - W \wedge |f| > f_c + W$$
$$V_{bp}(f) \neq 0, \text{otherwise}$$

- We assume also that (why?)

$$W << f_C$$

- In telecommunications bandpass signals are used to convey messages over medium
- In practice, transmitted messages are never strictly band limited due to
  - their nature in frequency domain (Fourier series coefficients may extend over very large span of frequencies)
  - non-ideal filtering

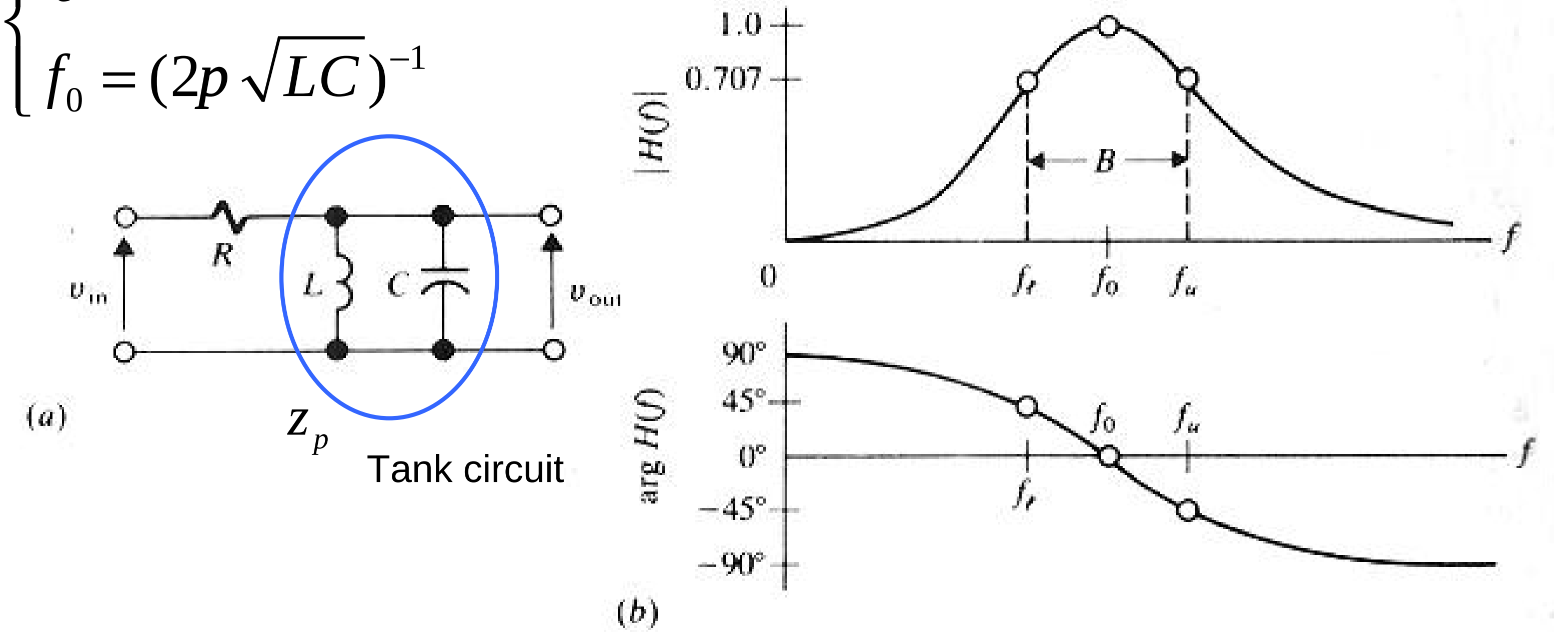
Example of a bandpass system

- Consider a simple bandpass system: a resonant (tank) circuit

$$z_p = \frac{jwL / jwC}{jwL + 1/ jwC} \quad z_i = R + z_p \quad V_{in}(w)H(w) = V_{out}(w)$$

$$H(w) = V_{out}(w)/V_{in}(w) = z_p / z_i \Rightarrow H(w) = 1/[1 + jQ(f / f_0 + f_0 / f)]$$

$$\begin{cases} Q = R\sqrt{C / L} \\ f_0 = (2p\sqrt{LC})^{-1} \end{cases}$$



Bandwidth and Q-factor

- The bandwidth is inversely proportional to Q-factor:

$$B_{3dB} = f_0 / Q \qquad \text{(for the tank circuit: } Q = R\sqrt{C/L}\text{)}$$

- System design is easier (next slide) if the fractional bandwidth *1/Q=B/f_0* is kept relatively small:

$$0.01 < B / f_0 < 0.1$$

- Some practical examples:

Frequency band	Carrier frequency	Bandwidth
Longwave radio	100 kHz	2 kHz
Shortwave radio	5 MHz	100 kHz
VHF	100 MHz	2 MHz
Microwave	5 GHz	100 MHz
Millimeterwave	100 GHz	2 GHz
Optical	5×10^{14} Hz	10^{13} Hz

Why system design is easier for smaller fractional bandwidths (FB)?

- Antenna and bandpass amplifier design is difficult for large FB:s:
  - one will have "**difficult to realize**" components or parameters in circuits as
    - too high Q
    - **too small or large values** for capacitors and inductors
- These structures have a bandpass nature because one of their important elements is the resonant circuit. Making them broadband means decreasing **resistive losses** that can be difficult

I-Q (in-phase-quadrature) description for bandpass signals

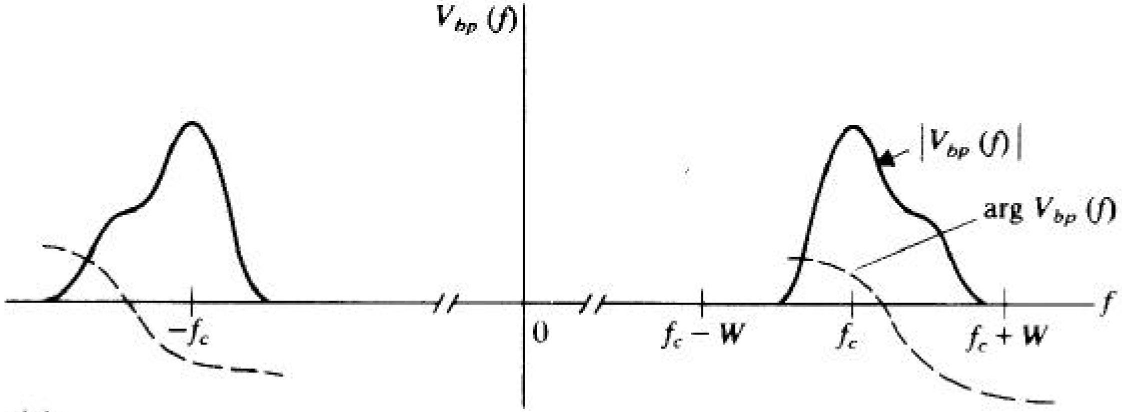
- In I-Q presentation bandpass signal carrier and modulation parts are separated into different terms

$$v_{bp}(t) = A(t)\cos[w_C t + f(t)]$$

$$v_{bp}(t) = v_i(t)\cos(w_C t) - v_q(t)\sin(w_C t)$$

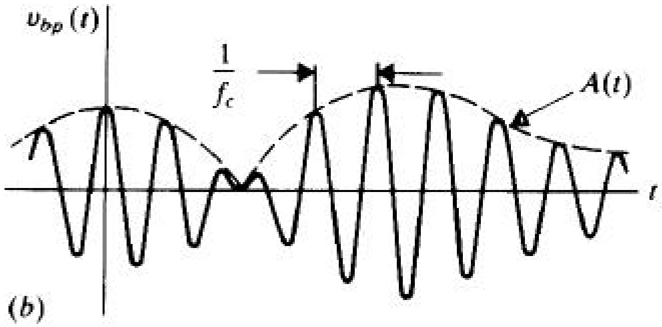
$$v_i(t) = A(t)\cos f(t),\ v_q(t) = A(t)\sin f(t)$$

Bandpass signal in frequency domain



(a)

Bandpass signal in time domain



$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

dashed line denotes envelope

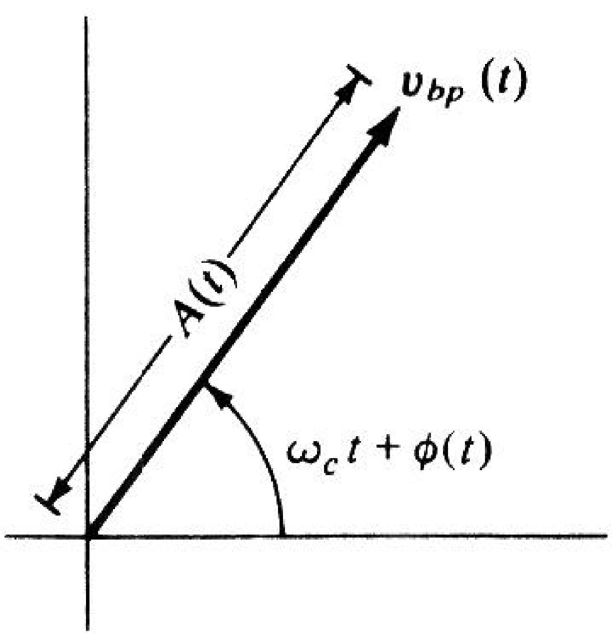
The phasor description of bandpass signal

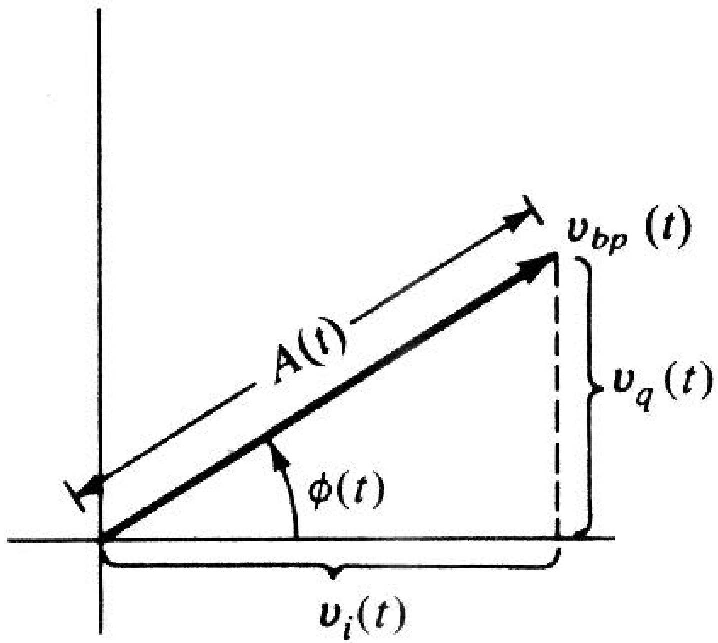
- Bandpass signal is conveniently represented by a phasor rotating at the angular carrier rate $\omega_C t + \phi(t)$:

$$v_{bp}(t) = v_i(t)\cos(\omega_C t) - v_q(t)\sin(\omega_C t)$$

$$v_i(t) = A(t)\cos\phi(t),\; v_q(t) = A(t)\sin\phi(t)$$

$$A(t) = \sqrt{v_i^2(t) + v_q^2(t)} \quad \phi(t) = \begin{cases} v_i(t) \geq 0, \arctan(v_q(t)/v_i(t)) \\ v_i(t) < 0, \pi + \arctan(v_q(t)/v_i(t)) \end{cases}$$





Lowpass (LP) signal

$$\begin{cases} v_{bp}(t) = v_i(t)\cos(w_c t) + v_q(t)\sin(w_c t) \\ v_i(t) = A(t)\cos f(t) \\ v_q(t) = A(t)\sin f(t) \end{cases}$$

- Lowpass signal is defined by $V_{lp}(f) \triangleq \frac{1}{2}\left[V_i(f) + jV_q(f)\right]$ yielding in time domain

$$v_{lp}(t) = \mathbb{F}^{-1}\left[V_{lp}(f)\right] = \frac{1}{2}\left[v_i(t) + jv_q(t)\right]$$

Taking rectangular-polar conversion yields then

$$v_{lp}(t) = A(t)\left[\cos f(t) + j\sin f(t)\right]/2$$

$$\left|v_{lp}(t)\right| = A(t)/2,\ \arg v_{lp}(t) = f(t)$$

$$= v_{lp}(t) = \frac{1}{2}A(t)\exp jf(t)$$

Transforming lowpass signals and bandpass signals

$$v_{bp}(t) = A(t)\cos[w_c t + f(t)]$$

$$v_{bp} = \mathrm{Re}\left\{A(t)\exp[jw_c t + f(t)]\right\}$$

$$v_{bp} = 2\mathrm{Re}\left\{\underbrace{\frac{A(t)}{2}\exp[jf(t)]}_{v_{lp}(t)}\exp[jw_c t]\right\}$$

$$v_{bp} = 2\mathrm{Re}\left\{v_{lp}(t)\exp[jw_c t]\right\}$$

- Physically this means that the lowpass signal is **modulated** to the carrier frequency w when it is transformed to bandpass signal. Bandpass signal can be transformed into lowpass signal by (tutorials). What is the physical meaning of this?

$$V_{lp}(f) = V_{bp}(f + f_C)u(f + f_C)$$

Amplitude modulation (AM)

- We discuss three linear mod. methods: (1) AM (amplitude modulation), (2) DSB (double sideband modulation), (3) SSB (single sideband modulation)
- AM signal:

$$x_C(t) = A_c[1 + m x_m(t)]\cos(w_c t + f(t))$$
$$= \underbrace{A_c \cos(w_c t + f(t))}_{\text{Carrier}} + A_c m \underbrace{x_m(t)\cos(w_c t + f(t))}_{\text{Information carrying part}} \qquad \begin{cases} 0 \le m \le 1 \\ |x_m(t)| \le 1 \end{cases}$$

- $f(t)$ is an arbitrary *constant*. Hence we note that no information is transmitted via the phase. Assume for instance that $f(t)=0$, then the LP components are

$$v_i(t) = A(t)\cos(f(t)) = A(t) = A_c[1 + m x_m(t)]$$
$$v_q(t) = A(t)\sin(f(t)) = 0$$

- Also, the carrier component contains no information-> Waste of power to transmit the unmodulated carrier, but can still be useful (how?)

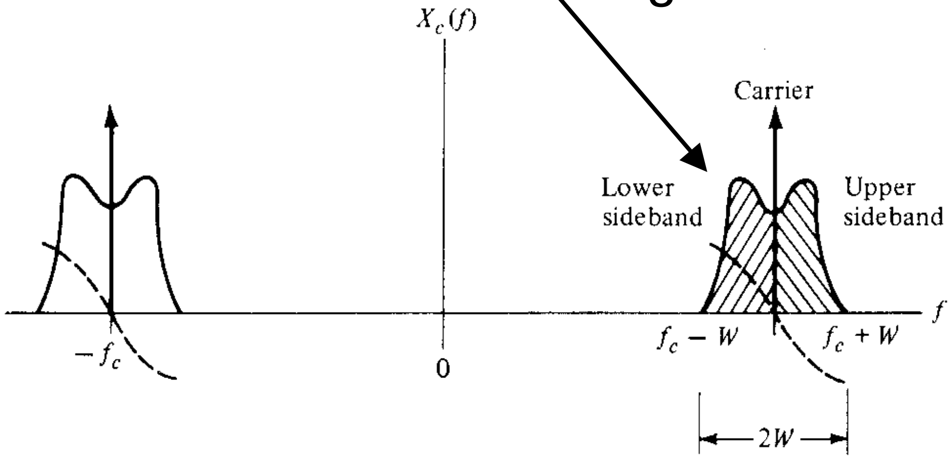
AM: waveforms and bandwidth

- AM in frequency domain:

$$x_c(t) = A_c[1 + m x_m(t)]\cos(w_c t)$$
$$= \underbrace{A_c \cos(w_c t)}_{\text{Carrier}} + m \underbrace{x_m(t)\cos(w_c t)}_{\text{Information carrying part}}$$

$$X_c(f) = \underbrace{A_c d(f - f_c)/2}_{\text{Carrier}} + m \underbrace{A_c X_m(f - f_c)/2}_{\text{Information carrying part}} \quad f > 0 \text{ (for brief notations)}$$

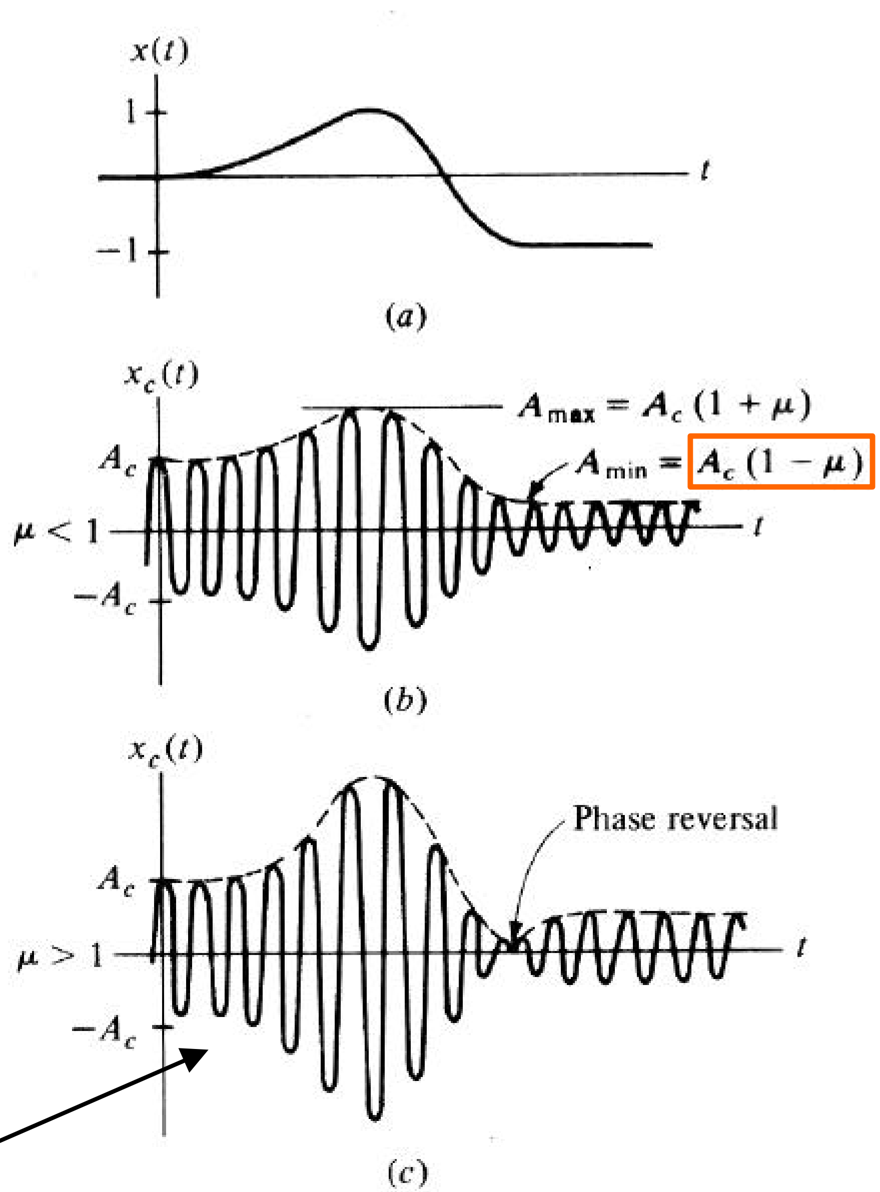
- AM bandwidth is twice the message bandwidth *W*:



$$v(t)\cos(w_c t + f) \leftrightarrow \tfrac{1}{2}[V(f - f_c)\exp jf + V(f + f_c)\exp - jf]$$

AM waveforms

(a): modulation
(b): modulated carrier with μ<1
(c): modulated carrier with μ>1



Envelope distortion!

(AM signal: $x_c(t) = A_c[1 + m x_m(t)]\cos(w_c t)$)

AM power efficiency

- AM wave total power consists of the idle carrier part and the useful signal part: $< x_c^2(t) >= \underbrace{< A_c^2 \cos^2(w_c t) >}_{\text{Carrier}}$

(AM signal: $x_c(t) = A_c[1 + m x_m(t)]\cos(w_c t)$)

$$+ < m^2 A_c^2 \underbrace{x_m^2(t)}_{\text{Power: } S_X} \cos^2(w_c t) >$$

$$= \underbrace{A_c^2/2}_{P_C} + \underbrace{m^2 A_c^2 S_X/2}_{2P_{SB}}$$

- Assume $A_C=1$, $S_X=1$, then for $m=1$ (the max value) the total power is

$$P_{T\max} = \underbrace{1/2}_{\text{Carrier power}} + \underbrace{1/2}_{\text{Modulation power}}$$

- Therefore at least half of the total power is wasted on carrier
- Detection of AM is simple by enveloped detector that is a reason why AM is still used. Also, sometimes AM makes system design easier, as in fiber optic communications

$$\frac{A^2}{T}\cdot\int_0^T \cos\left(2\cdot\frac{\pi\cdot t}{T}\right)^2 dt \rightarrow \frac{1}{2}\cdot A^2$$

DSB signals and spectra

- In DSB the wasteful carrier is suppressed:

$$x_c(t) = A_c x_m(t)\cos(w_c t)$$

- The spectra is otherwise identical to AM and the transmission BW equals again double the message BW

$$X_c(f) = A_c X_m(f - f_c)/2,\ f > 0$$

- In time domain **each** modulation signal zero crossing produces *phase reversals* of the carrier. For DSB, the total power S_T and the power/sideband P_{SB} have the relationship

$$S_T = A_c^2 S_X / 2 = 2P_{SB} \Rightarrow \ P_{SB} = A_c^2 S_X / 4\,(DSB)$$

- Therefore AM transmitter requires twice the power of DSB transmitter to produce the same coverage assuming S_X=1. However, in practice S_X is usually smaller than 1/2, under which condition at least four times the DSB power is required for the AM transmitter for the same coverage

$$\text{AM: } x_c(t) = A_c[1 + m x_m(t)]\cos(w_c t)$$

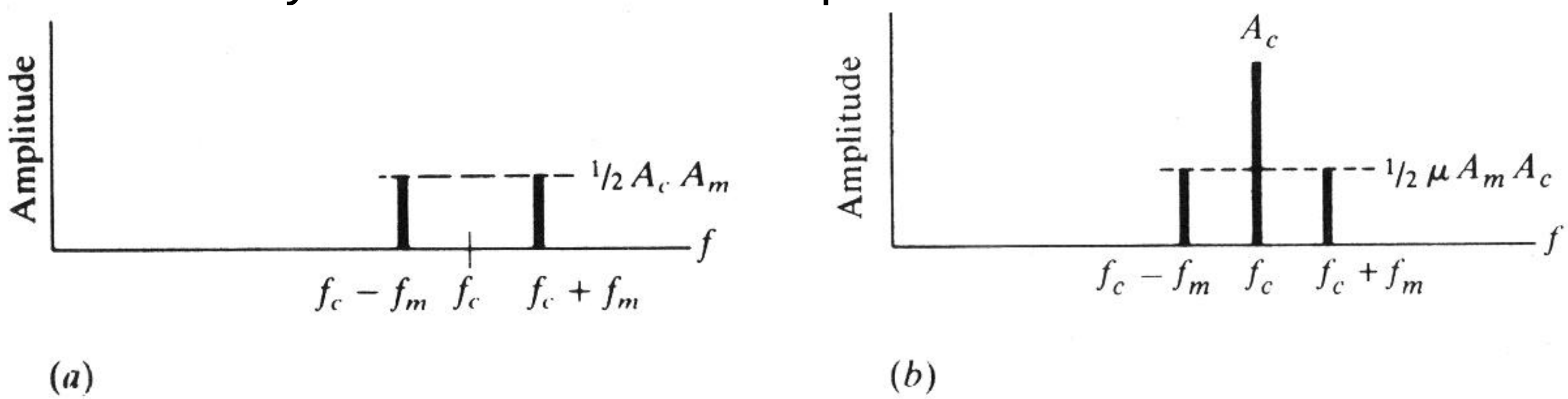
DSB and AM spectra

- AM in frequency domain with $x_m(t) = A_m \cos(w_m t)$

$$X_c(f) = \underbrace{A_c d(f - f_c)/2}_{\text{Carrier}} + m\underbrace{A_c X_m(f - f_c)/2}_{\text{Information carrying part}},\ f > 0 \text{ (general expression)}$$

$$X_c(f) = A_c d(f - f_c)/2 + m A_c A_m d(f_c \pm f_m)/2 \text{ (tone modulation)}$$

- In summary, difference of AM and DSB at frequency domain is the missing carrier component. Other differences relate to power efficiency and detection techniques.



(a) DSB spectra, (b) AM spectra

AM phasor analysis,tone modulation

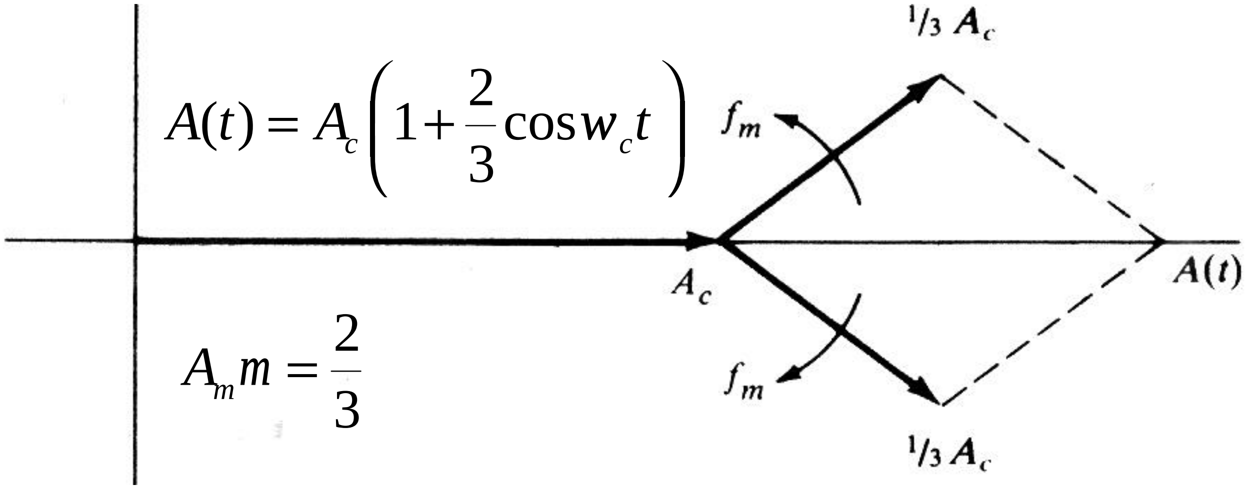
- AM and DSB can be inspected also by trigonometric expansion yielding for instance for AM

$$x_C(t) = A_C A_m m \cos(w_m t)\cos(w_C t) + A_C \cos(w_C t)$$

$$= \frac{A_C A_m m}{2}\cos(w_C - w_m)t + \frac{A_C A_m m}{2}\cos(w_C + w_m)t$$

$$+ A_C \cos(w_C t)$$

- This has a nice phasor interpretation;
  take for instance m=2/3, A_m=1:

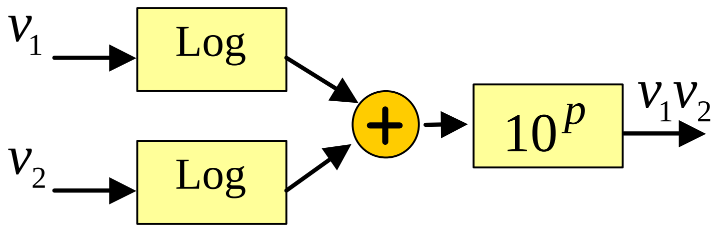


AM signal: $x_c(t) = \underbrace{A_c[1 + m x_m(t)]}_{A(t)}\cos(w_c t)$

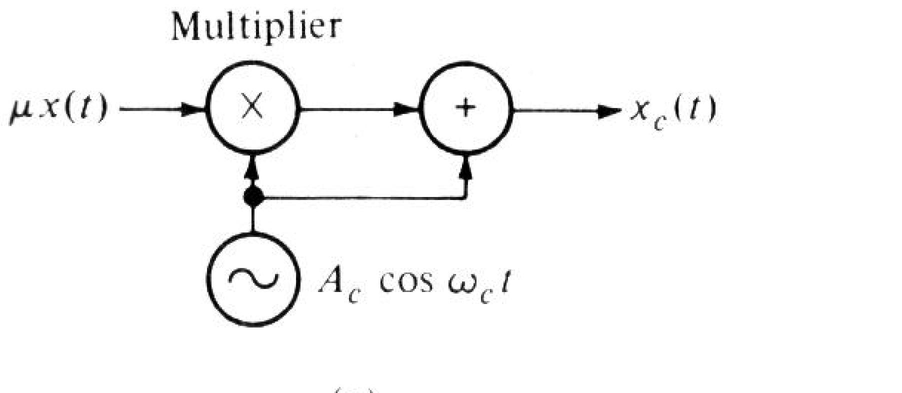
Linear modulators

- Note that AM and DSB systems generate new frequency components that were not present at the carrier or at the message.
- Hence modulator must be a nonlinear system
- Both AM and DSB can be generated by
  - analog or digital multipliers
  - special nonlinear circuits
    - based on semiconductor junctions (as diodes, FETs etc.)
    - based on analog or digital nonlinear amplifiers as
      - log-antilog amplifiers:

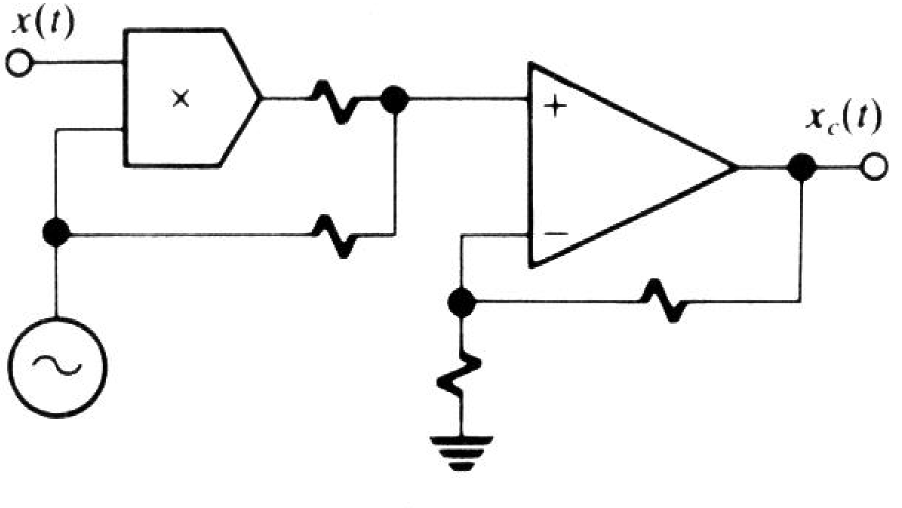
$$p = \log v_1 + \log v_2$$
$$10^p = v_1 v_2$$



- (a) Product modulator
  (b) respective schematic
  diagram
  =multiplier+adder



(a)

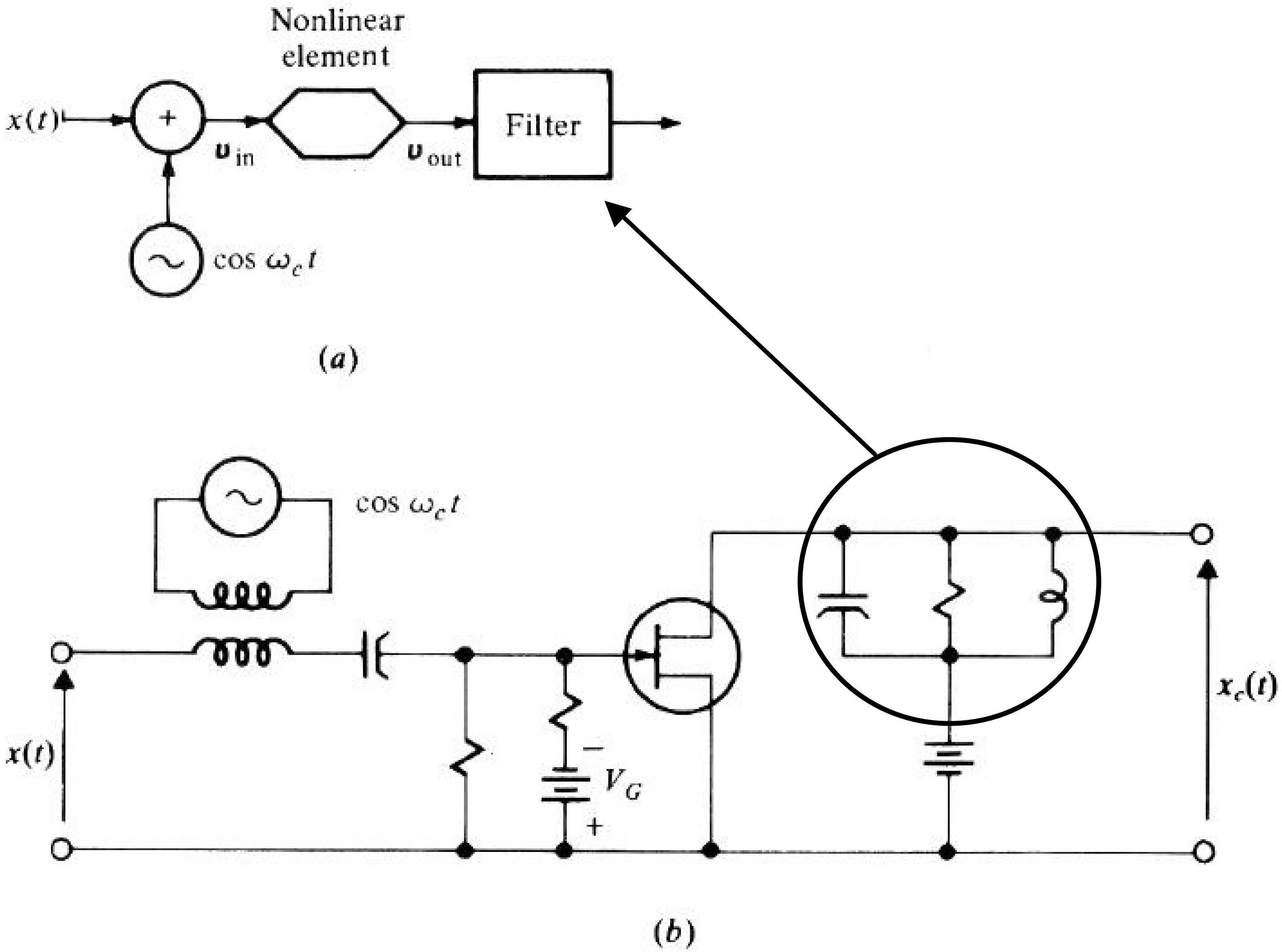


(b)

(AM signal: $x_c(t) = A_c[1 + m x_m(t)]\cos(w_c t)$)

Square-law modulator (for AM)

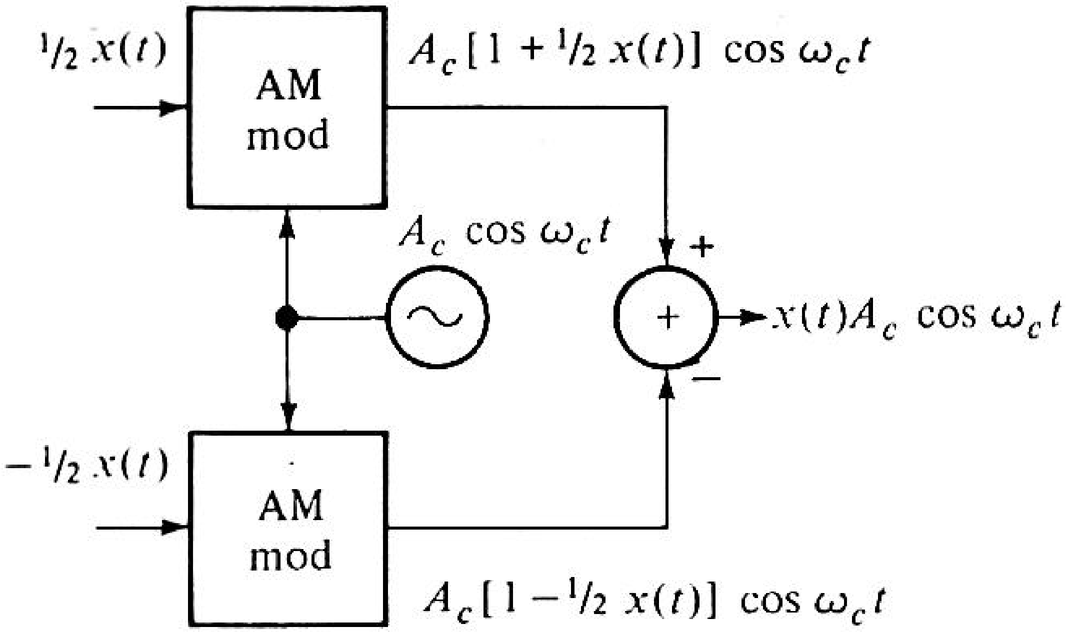
- Square-law modulators are based on nonlinear elements:



(a) functional block diagram, (b) circuit realization

Balanced modulator (for DSB)

- By using balanced configuration non-idealities on square-law characteristics can be compensated resulting a high degree of carrier suppression:



- Note that if the modulating signal has a DC-component, it is not cancelled out and will appear at the carrier frequency of the modulator output

Synchronous detection

- All **linear modulations** can be detected by synchronous detector
- Regenerated, **in-phase carrier replica** required for signal regeneration that is used to multiple the received signal
- Consider an universal*, linearly modulated signal:

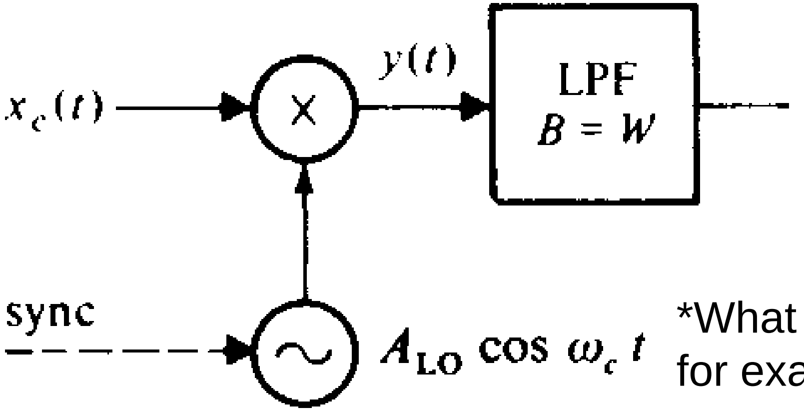
$$x_c(t) = [K_c + K_m x(t)]\cos(w_c t) + K_m x_q(t)\sin(w_c t)$$

- The multiplied signal *y(t)* is:

$$x_c(t) A_{LO}\cos(w_c t) = \frac{A_{LO}}{2}\left\{[K_c + K_m x(t)][1 + \cancel{\cos(2w_c t)}] - \cancel{K_m x_q(t)\sin(2w_c t)}\right\}$$

$$= \frac{A_{LO}}{2}[K_c + K_m x(t)]$$

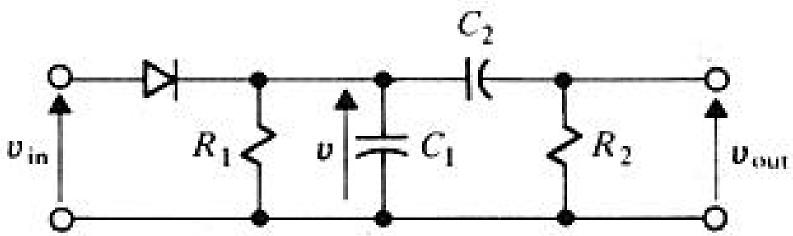
Synchronous detector



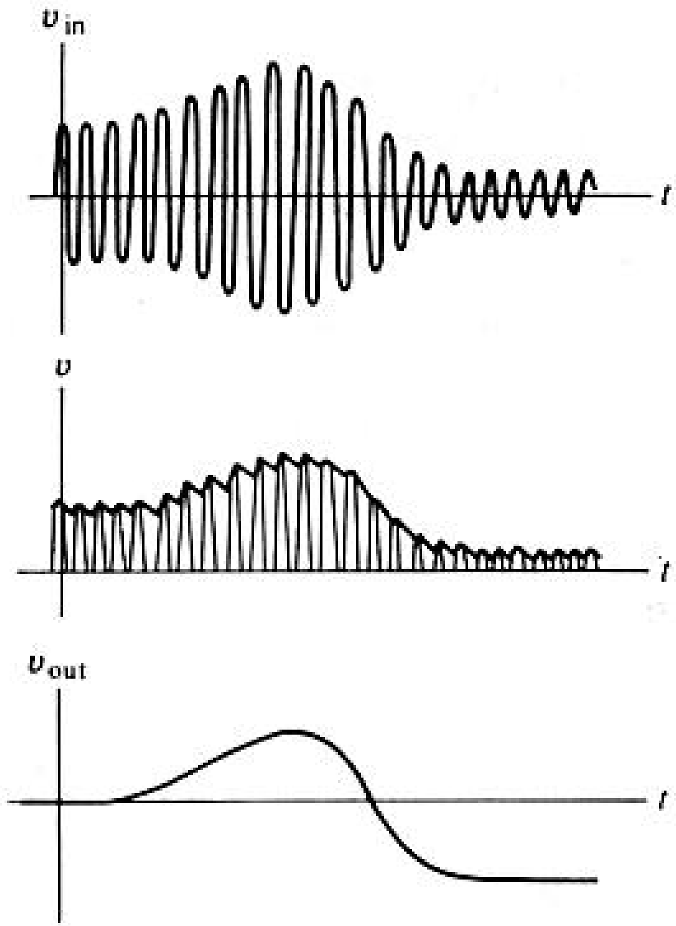
*What are the parameters for example for AM or DSB?

The envelope detector

- Important motivation for using AM is the possibility to use the envelope detector that
  - has a simple structure (also cheap)
  - needs no synchronization (e.g. no auxiliary, unmodulated carrier input in receiver)
  - no threshold effect (SNR can be very small and receiver still works)



(a)



(b)

Envelope detector analyzed

- Assume diode half-wave rectifier used to rectify AM-signal. Therefore after the diode AM modulation is in effect multiplied with the half-wave rectified sinusoidal signal *w(t)*

$$v_R = [A + m(t)]\cos w_C t \underbrace{\left[\frac{1}{2} + \frac{2}{p}\left(\cos w_C t - \frac{1}{3}\cos 3w_C t + \ldots\right)\right]}_{w(t)}$$

$$v_R = \frac{1}{p}[A + m(t)] + \text{other higher order terms}$$

- The diode detector is then followed by a lowpass circuit to remove the higher order terms
- The resulting DC-term may also be blocked by a capacitor
- Note the close resembles of this principle to the synchronous-detector (why?)

$$\cos^2(x) = \frac{1}{2}[1 + \cos(2x)]$$