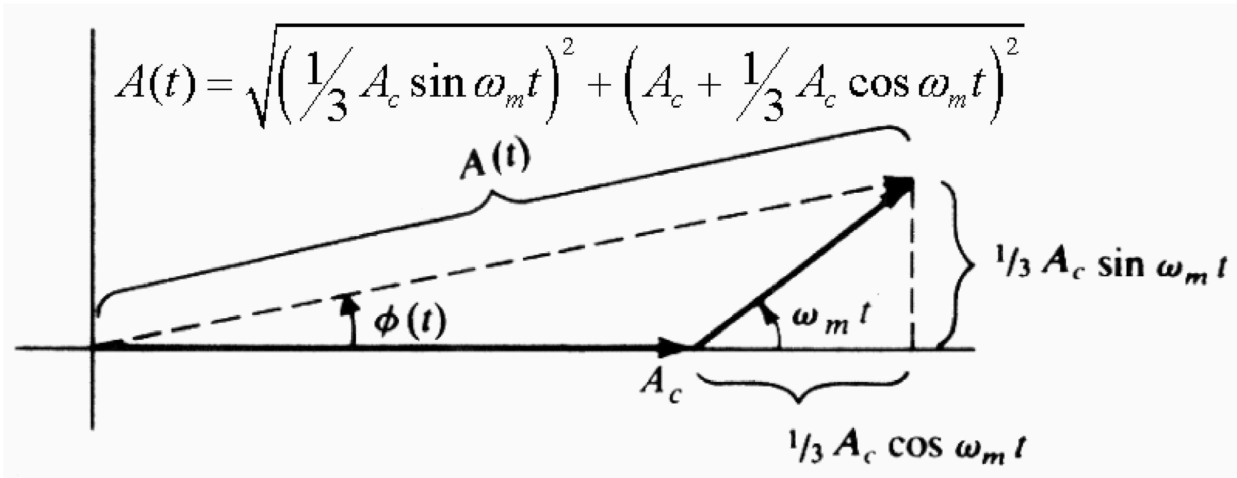
Questions

- *Q: Envelope detection was pretty hard to comprehend. Why doesn't the envelope detection work on SSB modulation?*



$$a^x \approx 1 + x\ln a + \frac{1}{2!}(x\ln a)^2 + \frac{1}{3!}(x\ln a)^3 ...$$

- Substituting the expression of the envelope to the series expression of square root reveals that it can not be simplified to the form

$$A(t) = KA_m \sin(\omega_m t)$$

Mathematica® output

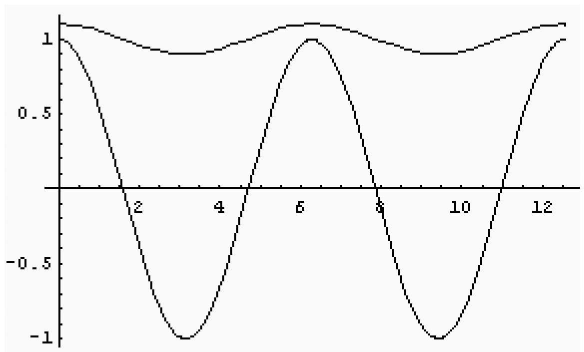
```
In[21]:= f[x_, m_] = Sqrt[(m Sin[x] / 2)^2 + (1 + m Cos[x] / 2)^2]
```

Out[21]= $\sqrt{\left(1+\frac{1}{2}\,m\,\mathrm{Cos}[x]\right)^2+\frac{1}{4}\,m^2\,\mathrm{Sin}[x]^2}$

```
In[30]:= f[2, 0.2] // N

Out[30]= 0.962689
```

```
In[26]:= Plot[{f[x, .2], Cos[x]}, {x, 0, 4 Pi}]
```

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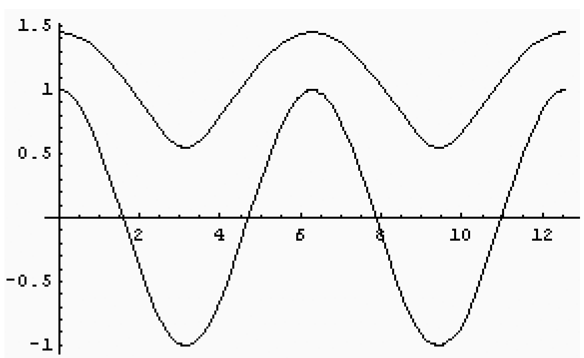
Out[26]= - Graphics -
```

```
In[29]:= Plot[{f[x, .9], Cos[x]}, {x, 0, 4 Pi}]
```

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Out[29]= - Graphics -
```

Questions (cont.)

- *Q: ... What puzzles me still is how the synchronous detection works. Must be something about autocorrelation…*

  ANS: Multiplication with the carrier frequency produces twice the carrier and around DC-converted components. The latter one is the detected wave

- … it would be nice to hear about real life applications between all those theoretical slides …

  *ANS: have a look on interesting demos! (*http://www.williamson-labs.com/home.htm*)*

- *For me, the most important lesson is that the signals that look awkward and strange at first sight, can be oftentimes divided in smaller parts and then make some sense of them. For example phasor presentation visualizes well what is really happening in amplitude or phase modulation.*

  Comment: It is easier to understand if one can consider topics from various point of views

Analysis of de-tuned resonant circuit

- Capacitance part of a resonant circuit can be made to be a function of modulation voltage *m(t).*

$f_{CC} = 1/(2\pi\sqrt{LC})$ Resonance frequency

$f_{CC}[x(t)] = 1/\{2\pi\sqrt{LC[x(t)]}\}$ De-tuned resonance frequency

$C[x(t)] = C_0 + Cx(t)$ Capacitance diode

$$f_{CC}[x(t)] = f_C(1 - Cx(t)/C_0)^{-1/2},\ f_C = 1/(2\pi\sqrt{LC_0})$$

- That can be simplified by the series expansion

$$(1\text{-}kx)^{-1/2} = 1 + \frac{kx}{2} + \frac{3k^2x^2}{8}\ldots|kx| << 1$$

$$f_{CC}[x(t)] = f_C(1 - Cx(t)/C_0)^{-1/2}$$

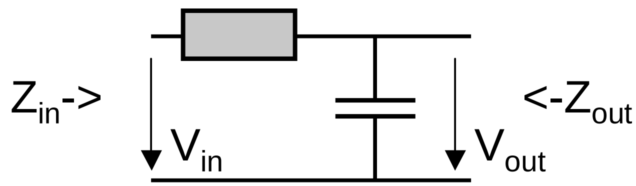
$$\approx f_C\left[1 + \frac{1}{2}\frac{Cx(t)}{C_0}\right] = \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

$$\phi(t) = 2\pi f_C t + 2\pi\underbrace{\frac{C}{2C_0}f_C}_{f_\Delta}\int_t x(\lambda)d\lambda$$

Note that this applies for a relatively small modulation index

Remember that the instantaneous frequency is the derivative of the phase

LP-filter is an approximation of the ideal integrator



$$H(f)V_{in}(f) = V_{out}(f)$$

$$H(f) = \cancel{I(f)}Z_{out}(f)/[\cancel{I(f)}Z_{in}(f)] = Z_{out}(f)/Z_{in}(f)$$

$$H(f) = \frac{(j\omega C)^{-1}}{R+(j\omega C)^{-1}} = \frac{1}{j\omega RC+1}$$

$$H(f) \approx \frac{1}{j\omega RC}, \omega >> 1$$

Ideal integrator is defined by

$$_{out}(f) = \ldots\left[\int_{-\infty}^{t} v_{in}(\lambda)d\lambda\right] = \frac{V_{in}(f)}{j2\pi f} + \frac{1}{2}V_{in}(0)\delta(f)$$

$$_{out}(f)\Big|_{V_{in}(t)=\delta(t)} = \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$$

$$\ldots\left[\delta(t-\tau_d)\right] = \exp(-j\omega\tau_d)$$

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