## S-72.245 Transmission methods in Telecommunication Systems

Tutorial 4

## Objectives

- Getting familiar with exponential modulation: line spectrum, phasor diagram and bandwidth estimation. To understand instantaneous phase and frequency.
- To investigate FM modulators and demodulators.


## Quizzes

Q4.1 Tone modulation is applied simultaneously to a frequency modulator and phase modulator and the two output spectra are identical. Describe how these two spectra will change when:
a) The tone amplitude is increased or decreased
b) The tone frequency is increased or decreased

Q4.2 Let us consider an FM system with $f_{\Delta}=30 \mathrm{kHz}$. What is the required transmission bandwidth $B_{T}$ when the modulating signal is an unit-amplitude tone with the tone frequency of $f_{m}=0.1,1.0$, or 5.0 Hz ? Repeat your inspection for PM system with $\phi_{\Delta}=3 \mathrm{rad}$.

Q4.3 Show (in the time domain), that the system below works as an FM-modulator when A $\ll 1$.


Q4.4 An angle-modulated signal, with the carrier frequency $\omega_{c}=2 \pi \cdot 10^{6}$ is described by the equation

$$
\varphi_{E M}(t)=10 \cos \left[\omega_{c} t+0.1 \cdot \sin (2000 \pi t)\right]
$$

a) Find the average power of the signal
b) Find the respective frequency deviation $\Delta f$
c) Find the respective phase deviation $\Delta \phi$
d) Estimate the required transmission bandwidth

Q4.5 Carry out the details to prove the expressions for envelope and phase of unmodulated carrier with additive interference (slide 25).

## Matlab assignments

M4.1 A message signal is defined by

$$
m(t)= \begin{cases}\operatorname{sinc}(100 t), & |t| \leq t_{0} \\ 0, & \text { otherwise }\end{cases}
$$

where $t_{0}=0.1 \mathrm{sec}$. This message modulates the carrier $c(t)=\cos \left(2 \pi f_{c} t\right)$, where $f_{c}=250 \mathrm{~Hz}$. The deviation constant is $k_{f}=100$. Plot the modulated signal in time and in frequency domain by using Matlab.

M4.2 Demodulate the modulated carrier of M4.1 by using the phase shift FM-detector discussed in the Appendix and sketch the related waveforms in Matlab. Compare the demodulated signal with the message signal and explain possible differences.

## Phase-shift FM detector

- An approximation for any derivative:

$$
d \phi(t) / d t=[\phi(t)-\phi(t-\Delta t)] / \Delta t
$$

- Derivative of an FM wave contains message:

$$
d \phi(t) / d t=2 \pi f(t)=2 \pi\left[f_{C}+f_{\Delta} x(t)\right]
$$

- FM-detector can be realized by approximating the derivative:

$$
\begin{aligned}
& \phi(t)-\phi(t-\Delta t)=\Delta t d \phi(t) / d t \\
& =2 \pi \Delta t\left[f_{C}+f_{\Delta} x(t)\right]
\end{aligned}
$$

## Phase-shift FM detector: circuit realization



- Limiter removes extra amplitude variations
- BPF removes out-of-band components generated in the limiter
- After delay-and-multiply detection carrier components are removed by the last LPF

$$
\cos \left[\omega_{c} t+\phi(t)\right] \sin \left[\omega_{c} t+\phi\left(t-t_{1}\right)\right] \quad=2 \pi \Delta t\left[f_{C}+f_{\Delta} x(t)\right]
$$

$$
=\frac{1}{2}\left\{\sin \left[2 \omega_{c} t+\phi(t)+\phi\left(t-t_{1}\right)\right]-\sin \left[\phi(t)-\phi\left(t-t_{1}\right)\right]\right\}
$$

$$
\sin \underbrace{\left[\phi(t)-\phi\left(t-t_{1}\right)\right.}_{\text {small }}) \approx \phi(t)-\phi\left(t-t_{1}\right)=2 \pi t_{1}\left[f_{c}+f_{\Delta} x(t)\right]
$$

