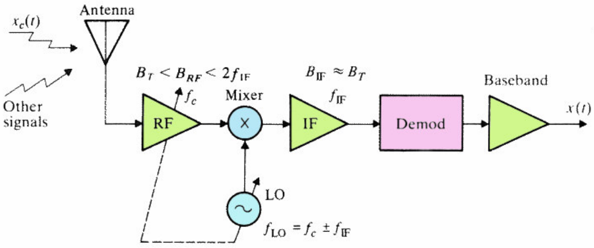
S-72.245 Transmission Methods in Telecommunication Systems (4 cr)

Carrier Wave Modulation Systems

Analog Carrier Wave Systems

- Carrier wave techniques form a bases for telecommunication systems
- Topics today in CW-applications:
  - Single conversion radio receiver
    - FM radio (analog) stereo multiplexing
  - Measurement equipment
    - Spectrum analyzer
  - Multiplexing techniques
    - Frequency Division Multiplexing (FDM)
    - Quadrature-carrier multiplexing
  - Phase-locked loop (PLL)
    - FM-demodulator
    - frequency synthesis
    - Costas loop

Single -conversion receiver*



- Assume reception of a bandpass signal

$$x_c(t) = A(t)\cos[\omega_c t + \phi(t)]$$

- Multiplication at the receiver with the local oscillator signal having frequency of f_{LO} yields signals at two CW-bands

$$\begin{aligned} x_{IF}(t) &= x_{LO}(t)x_c(t) \\ &= A(t)\cos(\omega_{LO}t)\cos[\omega_c t + \phi(t)] \\ &= A(t)\cos[(\omega_{LO} - \omega_c)t + \phi(t)]/2 + A(t)\cos[(\omega_{LO} + \omega_c)t + \phi(t)]/2 \end{aligned}$$

- Therefore, IF can be selected as $f_{IF} = f_{LO} \pm f_c$ or LO can be selected as $f_{LO} = f_c \pm f_{IF}$

*also called as heterodyne-receiver

Mirror frequency

- Select for IF for instance $A_m(t)\cos[(\omega_{LO} - \omega_c)t + \phi(t)]/2$
- For the reason that cos is even function there are two frequency bands that convert to intermediate frequency namely

$$\omega_{IF} = |\omega_{LO} - \omega_c| = \omega_{LO} - \omega_c \vee \omega_c' - \omega_{LO}$$

$$\Rightarrow \omega_c' = \omega_{LO} + \omega_{IF}, \omega_c = \omega_{LO} - \omega_{IF}$$

This means that both bandpass signals at the received frequencies $\omega_{LO} \pm \omega_{IF}$ are converted to the intermediate frequency.

- Example: Assume we set

$$f_{LO} = 110\,\mathrm{MHz}, f_{IF} = 10\,\mathrm{MHz}$$

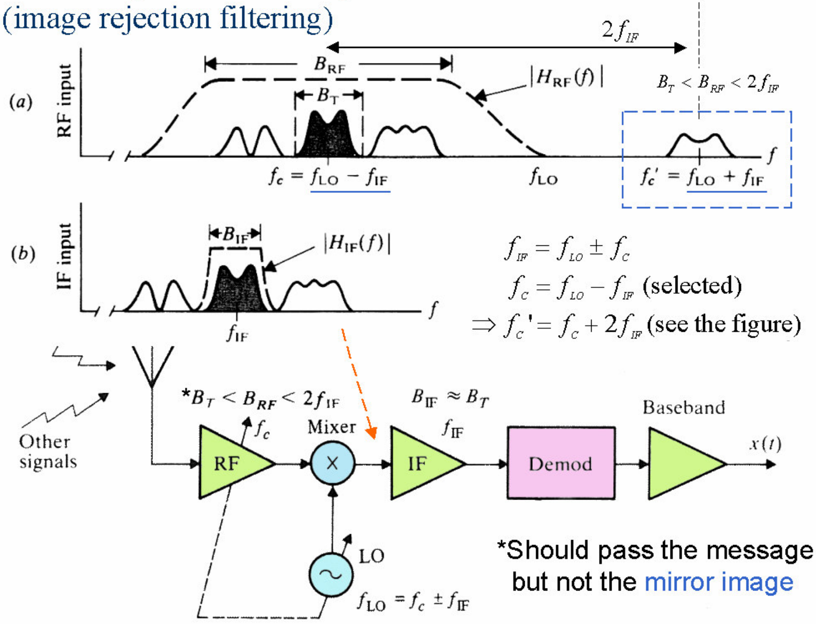
therefore receiver picks signals at the bands of

$$\begin{aligned} f_c = f_{LO} \pm f_{IF} &= 110\,\mathrm{MHz} \pm 10\,\mathrm{MHz} \\ &= 120\,\mathrm{MHz} \wedge 100\,\mathrm{MHz} \end{aligned}$$

- However, this is usually not wanted, and the other band must be filtered away by the first bandpass filter at the receiver

Filtering mirror frequencies

(image rejection filtering)



$$f_{IF} = f_{LO} \pm f_c$$

$$f_c = f_{LO} - f_{IF} \text{ (selected)}$$

$$\Rightarrow f_c' = f_c + 2f_{IF} \text{ (see the figure)}$$

*Should pass the message but not the mirror image

SC basic characteristics

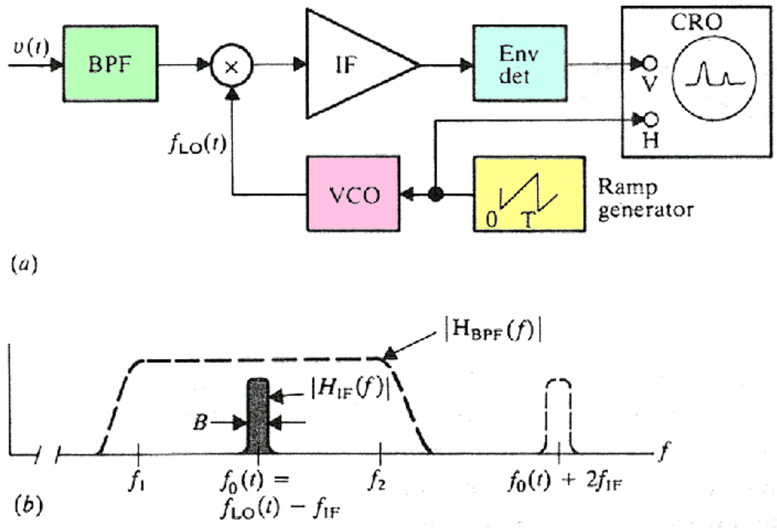
- SC can be used with all CW methods
- The RF stage provides image rejection
- The IF stage provides gain and interference rejection
  - note that the fractional BW= B_T/f_{IF} is selected by adjusting f_{IF}
- Remember from the second lecture that system design is easier if the fractional bandwidth is kept relatively small: For analog FM broadcasting:

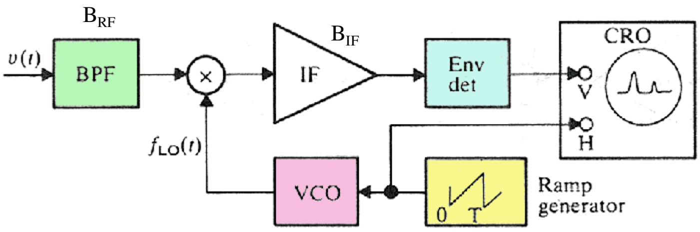
$$B_{IF}/f_{IF} = 200\,\text{kHz}/10.6\,\text{MHz} \approx 0.02$$

when it was required $0.01 < B/f_0 < 0.1$

- Tuning of the receiver to a desired band is easy by adjusting the local oscillator. (Often B_{RF} is selected to be so wide and f_{LO} so high that the first bandpass filter (amplifier) center frequency requires no tuning, as usually in FM radios)

Scanning spectrum analyzer





- VCO, B_{RF} and B_{IF} filters form together a scanning bandpass filter (SBF)
- Ramp generator takes care of sweeping SBF
- After the IF filter the envelope detector yields signal whose power is comparable to the power that has passed the SBF
- Sweep rate and B_{IF} determine system resolution. High resolution->small B_{IF} and sweep rate as discussed soon
- When larger sensitivity is desired sweep rate must be decreased
- Spectrum analyzer includes often integrator (or averaging function) to improve SNR via inclusion of multiple sweep data

Example: averaging improves SNR

- Assume white Gaussian noise with variance σ^2 (random part) is added into a sinusoidal signal (deterministic part) and thus the signal SNR is

$$SNR_1 = \frac{P_S}{P_N} = \frac{(V/\sqrt{2})^2}{\sigma_N^2} = \frac{V^2}{2\sigma_N^2}$$

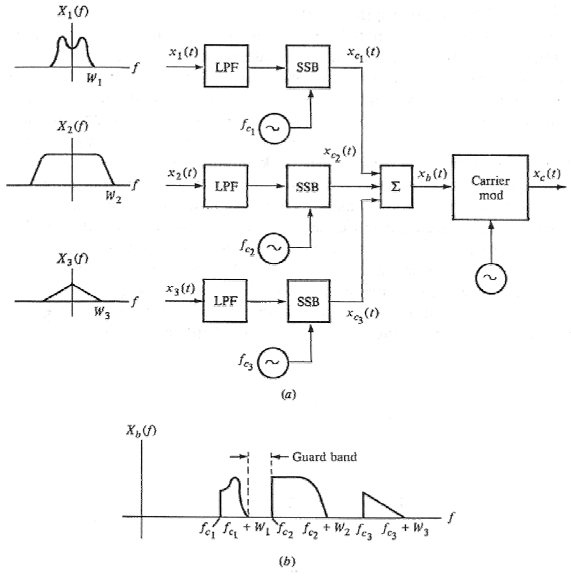
How much SNR can be expected to improve by *n*-fold averaging?

Ans: $$SNR_n = \frac{(nV/\sqrt{2})^2}{n\sigma_N^2} = \frac{nV^2}{2\sigma_N^2} = nSNR_1$$

or in dB:

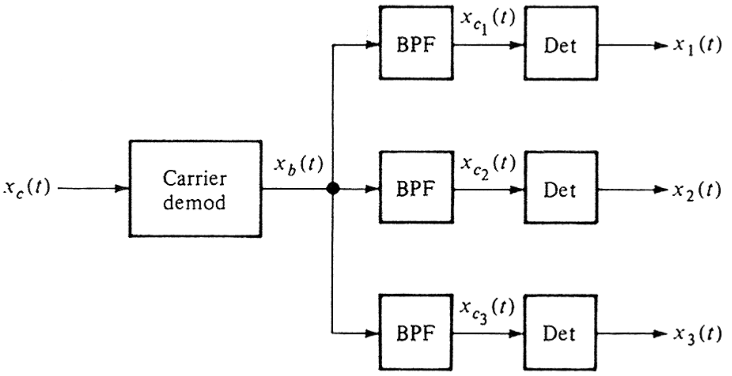
$$\begin{aligned}\Delta_{SNR} &= 10\log_{10}(SNR_n) - 10\log_{10}(SNR_1)\\ &= 10\log_{10}(SNR_n / SNR_1) = 10\log_{10}(n)\end{aligned}$$

Frequency-division multiplexing (FDM)

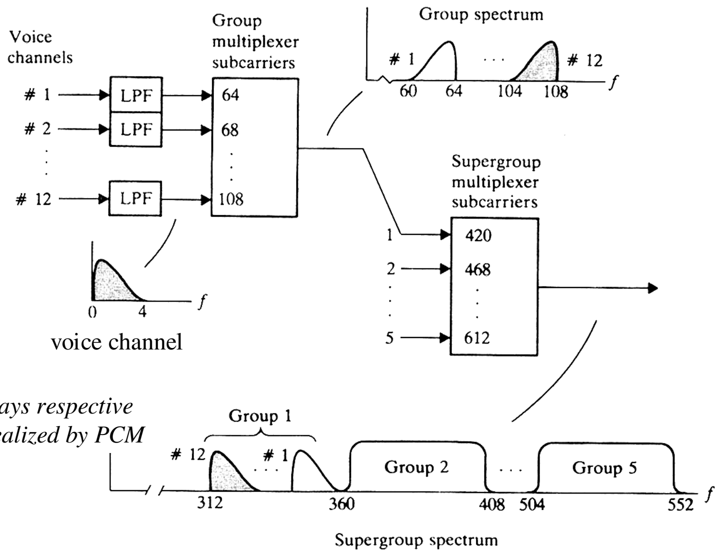


FDM receiver

First the FDM wave is demodulated. Then each subcarrier is detected by using separate bandpass filters and detectors.



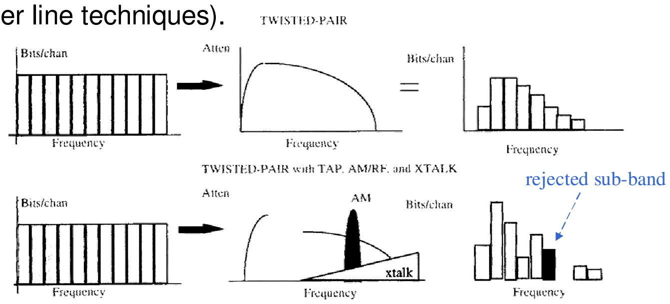
AT & T FDM hierarchy in PSTN



Note: nowadays respective combining realized by PCM

Advanced FDM: xDSL with OFDM

- Conventional FDM:
  - Each channel occupies accurately certain frequency band
  - Bandwidth efficiency increased by using SSB modulation
  - Usage of guard bands wastes resources
  - A lot of filtering functions (complex circuitry)
- Modern FDM: OFDM (orthogonal frequency division multiplexing) and DMT (discrete multitone modulation) yield increased spectral adaptation. Applied in xDSL (digital subscriber line techniques).



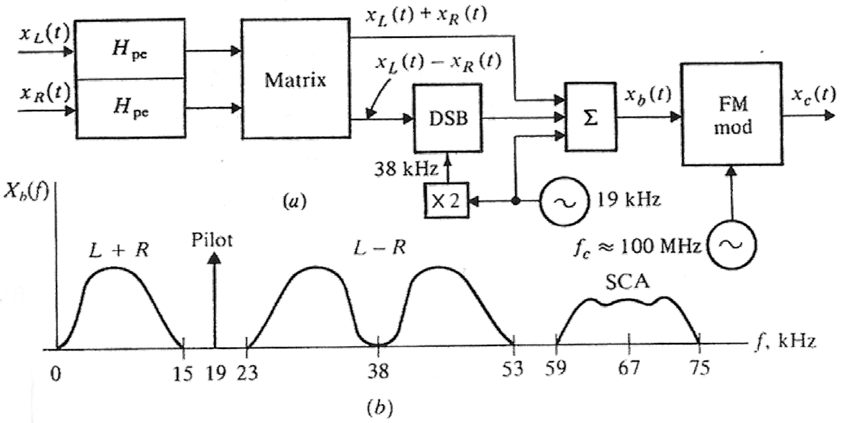
DMT with cable attenuation only

DMT with cable attenuation, interference and cross-talk

rejected sub-band

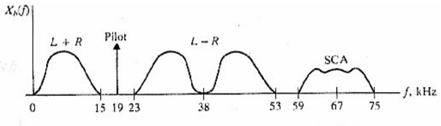
FM stereo multiplexing (MPX-system)

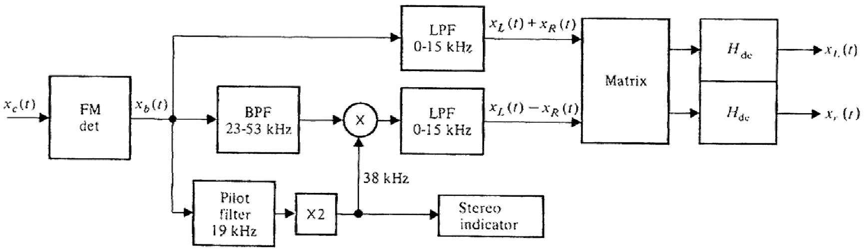
- The MPX encoder utilizes various linear modulation methods
- L+R and L-R signals are transmitted on different channels
- SCA (Subsidiary Communication Authorization) is used to transmit background music for selected subscribers



FM stereo decoder

- System is based on detecting both L+R and L-R signals from which the R and L can be calculated
- Compatibility to mono-phonic transmission is granted by using the unmodulated L+R and DSB modulated L-R signal at 23-53 kHz that is automatically filtered out in mono-phonic reception



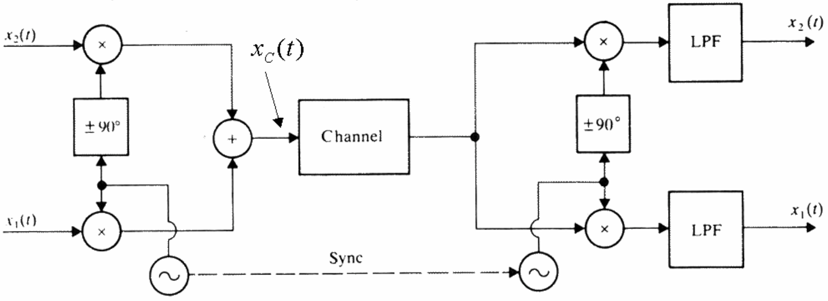


Quadrature-carrier multiplexing

- Two signals x_1 and x_2 are transmitted via same channel

$$x_c(t) = A_c\left[x_1(t)\cos(\omega_c t) \pm x_2(t)\sin(\omega_c t)\right]$$

- Signals can be analog or digital CW or baseband signals (QPSK, DSB, SSB ...)



Task: show that the signals x_1 and x_2 can be detected ***independently*** *at the receiver!*

Quadrature-carrier reception

- In order to detect the x_1 component multiply by the cos-wave:

$$\cos(\omega_c t)\left[x_1(t)\cos(\omega_c t) \pm x_2(t)\sin(\omega_c t)\right]$$
$$= x_1(t)\left[1 - \cos(2\omega_c t)\right]/2 \pm x_2(t)\sin(2\omega_c t)/2$$

- In order to detect the x_2 component multiply by sin-wave:

$$\sin(\omega_c t)\left[x_1(t)\cos(\omega_c t) \pm x_2(t)\sin(\omega_c t)\right]$$
$$= x_2(t)\left[1 - \cos(2\omega_c t)\right]/2 \pm x_1(t)\sin(2\omega_c t)/2$$

- Note
  - Second-order frequency must be filtered away
  - The local oscillator must be precisely in-phase to the received signal, otherwise cross-talk will follows

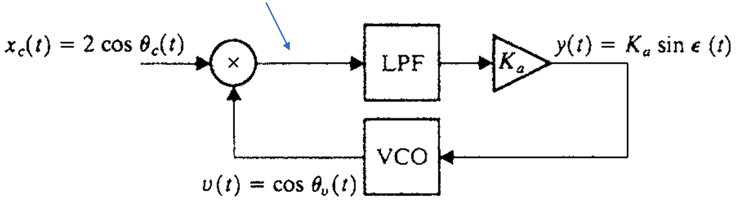
Phase-locked loops (PLLs)

- Phase-locked loop is a feedback arrangement capable to synchronize itself to a noisy external reference
- The output signals of the loop can be used to produce for instance multitude of locked frequencies
- PLL application areas include...
  - modulators
  - demodulators
  - frequency synthesis
  - multiplexers
  - signal processors

The PLL principle

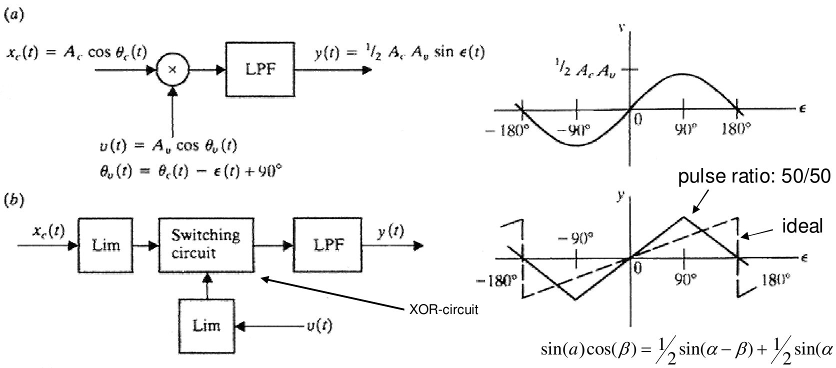
- The PLL circuit consists of
  - phase comparator (in the figure below the multiplier)
  - lowpass filter
  - feedback amplifier
  - VCO (voltage controlled oscillator), whose **output frequency is linearly proportional to input amplitude**
- Principle: phase difference of $X_c(t)$ and $v(t)$ adjusts VCO

Phase comparator output is comparable to phase difference of input signals



PLL phase comparator realizations

- Circuits: (a) analog and (b) digital phase comparator circuit
- Note that for (a) output is proportional to
  - input signal phase difference
  - input signal amplitudes (unintended AM thus harmful)
- In (b) AM effects are compensated and response is more linear



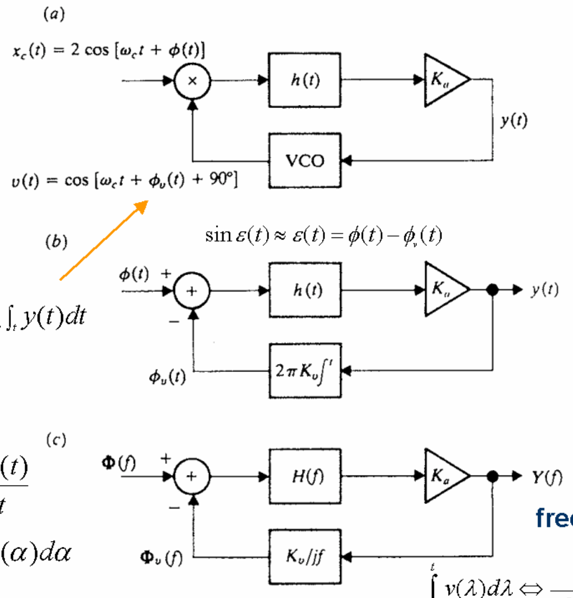
pulse ratio: 50/50

ideal

XOR-circuit

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}\sin(\alpha-\beta) + \frac{1}{2}\sin(\alpha+\beta)$$

FM detection by PLL



time domain

$$\sin\varepsilon(t) \approx \varepsilon(t) = \phi(t) - \phi_v(t)$$

$$\phi_v(t) = 2\pi K_v \int_t y(t)dt$$

phase domain

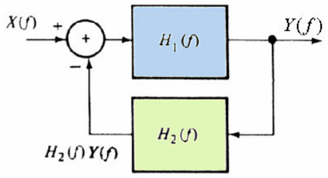
$$\begin{cases} \omega(t) = \dfrac{d\phi_v(t)}{dt} \\ \phi_v(t) = \int_{-\infty}^{t} \omega(\alpha)d\alpha \end{cases}$$

frequency domain

$$\int_{-\infty}^{t} v(\lambda)d\lambda \Leftrightarrow \frac{1}{j2\pi f}V(f) + \tfrac{1}{2}V(0)\delta(f)$$

PLL FM-demodulator: the feedback analysis

Solve transfer function with feedback:

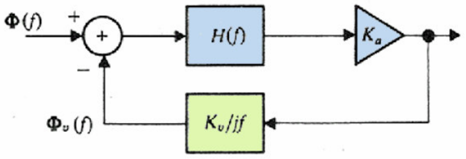


$$Y(f) = (X(f) - H_2(f)Y(f))H_1(f)$$

$$Y(f) + H_1(f)H_2(f)Y(f) = X(f)H_1(f)$$

$$Y(f) = \frac{H_1(f)}{1 + H_1(f)H_2(f)}X(f)$$

This is applied to the **linearized PLL** yielding relationship between the input phase and output voltage:



$$Y(f) = \frac{K_a H(f)}{1 + K_a H(f) K_v / jf}\Phi(f)$$

$$= \frac{1}{K_v}\frac{jfKH(f)}{jf + KH(f)}\Phi(f)$$

$$(K = K_a K_v)$$

Applying the FM signal to the linearized PLL model

- Remember the FM wave:

  $$d\phi(t)/dt = 2\pi f_\Delta x(t)$$

  where the modulating signal is denoted by *x(t).* The input FM phase to the system is thus

  $$\phi(t) = 2\pi f_\Delta \int_t x(\lambda)d\lambda$$

- This is in frequency domain: $\Phi(f) = 2\pi f_\Delta X(f)/(j2\pi f)$

  assuming no DC component or *V*(0) = 0, or

  $$\int_t v(\lambda)d\lambda \Leftrightarrow \frac{1}{j2\pi f}V(f) + \underbrace{\tfrac{1}{2}V(0)\delta(f)}_{=0}$$

Applying FM signal to the detector... (cont.)

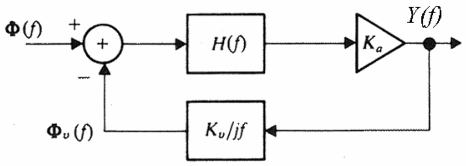
- Thus the input is $\Phi(f) = f_\Delta X(f)/(jf)$ and the output is

  $$Y(f) = \frac{1}{K_v}\frac{jfKH(f)}{jf + KH(f)}\Phi(f) = \frac{f_\Delta X(f)}{K_v}H_L(f)$$

  where the loop equivalent transfer function is

$$H_L(f) = \frac{H(f)}{H(f) + j(f/K)}$$

$$K = K_a K_v$$



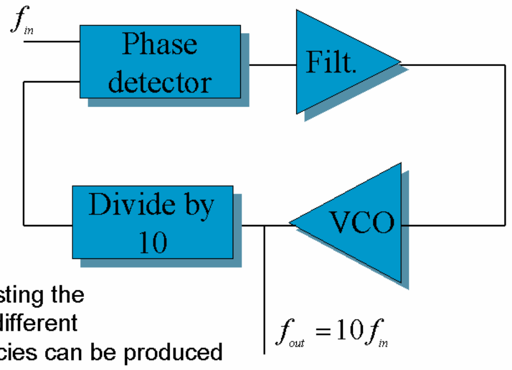
- Assume that the first order LP function is used or

$$H_L(f) = \frac{1}{1 + j(f/K)} \Rightarrow Y(f) \approx \frac{f_\Delta}{K_v}\frac{X(f)}{1 + j(f/K)} \approx \frac{f_\Delta}{K_v}X(f), \frac{W}{K} << 1$$

$$\Rightarrow y(t) \approx \frac{f_\Delta}{K_v}x(t)$$

PLL based frequency synthesizer

Reference signal f_{in} is locked for instance to the fundamental frequency of a crystal oscillator



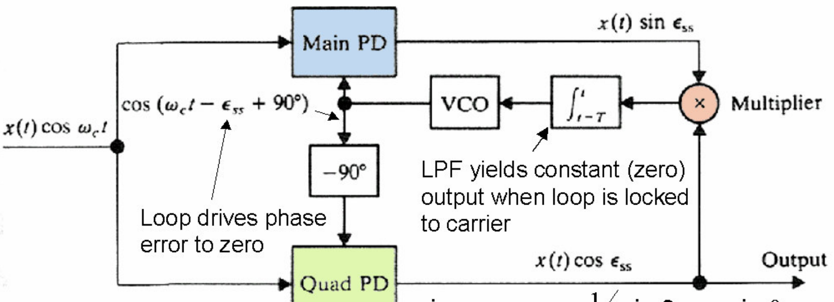
By adjusting the divider different frequencies can be produced whose phase is locked into f_{in}

Detecting DSB using PLL-principle

- An important application for PLLs is in **synchronization** of receiver local oscillator in synchronous detection
- In the **Costas PLL** (below) two phase discriminators are used to:
  - cancel out DSB modulation *x(t)* in the driving signal
  - synchronize the output frequency to the center frequency of the DSB spectra (the suppressed carrier)
  - to detect the DSB signal

Costas PLL detector for DSB

PD: phase detector (=multiply+LPF)



Loop drives phase error to zero

LPF yields constant (zero) output when loop is locked to carrier

$$\sin\varepsilon_{ss}\cos\varepsilon_{ss} = \tfrac{1}{2}\sin 2\varepsilon_{ss} + \sin 0 \approx \varepsilon_{ss}$$