S-72.245 Transmission Methods in Telecommunication Systems (4 cr)

Digital Baseband Transmission

Digital Baseband Transmission

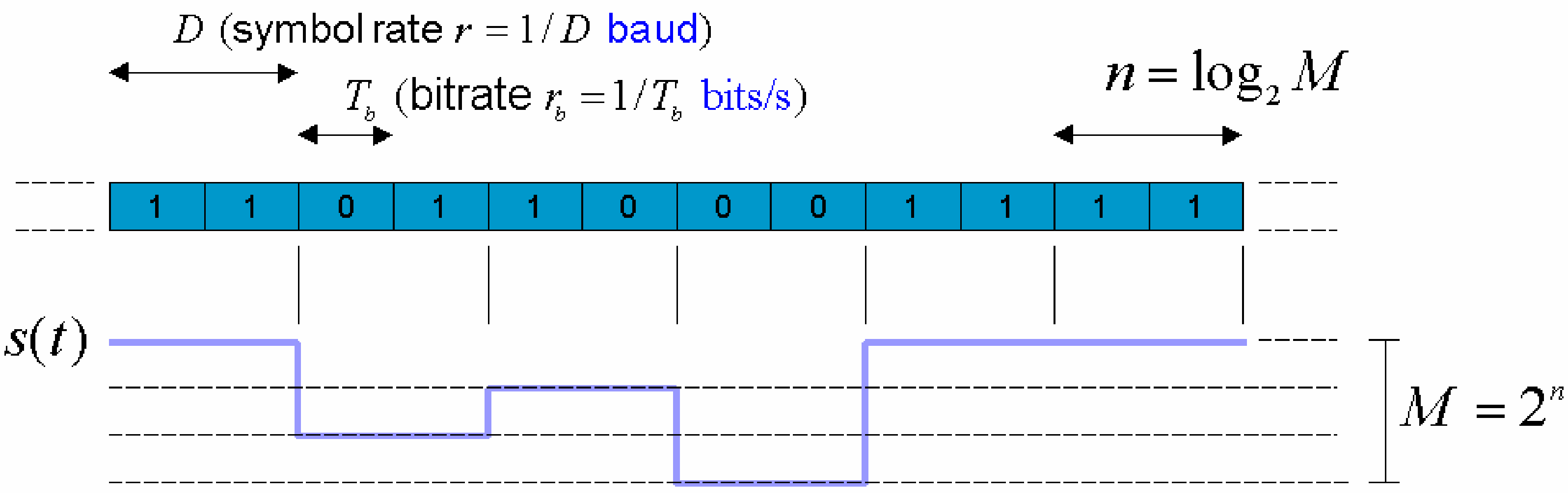
- Why to apply digital transmission?
- Symbols and bits
- Baseband transmission
  - Binary error probabilities in baseband transmission
- Pulse shaping
  - minimizing ISI and making bandwidth adaptation - cos roll-off signaling
  - maximizing SNR at the instant of sampling - matched filtering
  - optimal terminal filters
- Determination of transmission bandwidth as a function of pulse shape
  - Spectral density of Pulse Amplitude Modulation (PAM)
- Equalization - removing residual ISI - eye diagram

Why to Apply Digital Transmission?

- Digital communication withstands channel noise, interference and distortion better than analog system. For instance in PSTN inter-exchange STP*-links NEXT (Near-End Cross-Talk) produces several interference. For analog systems interference must be below 50 dB whereas in digital system 20 dB is enough. With this respect digital systems can utilize lower quality cabling than analog systems
- Regenerative repeaters are efficient. Note that cleaning of analog-signals by repeaters does not work as well
- Digital HW/SW implementation is straightforward
- Circuits can be easily reconfigured and preprogrammed by DSP techniques (an application: software radio)
- Digital signals can be coded to yield very low error rates
- Digital communication enables efficient exchanging of SNR to BW-> easy adaptation into different channels
- The cost of digital HW continues to halve every two or three years

STP: Shielded twisted pair

Symbols and Bits



Generally: $s(t) = \sum_k a_k p(t - kD)$ (a PAM* signal)

For *M*=2 (binary signalling): $s(t) = \sum_k a_k p(t - kT_b)$

For non-Inter-Symbolic Interference (ISI), *p(t)* must satisfy:

$$p(t) = \begin{cases} 1, t = 0 \\ 0, t = \pm D, \pm 2D \ldots \end{cases}$$

unipolar, 2-level pulses

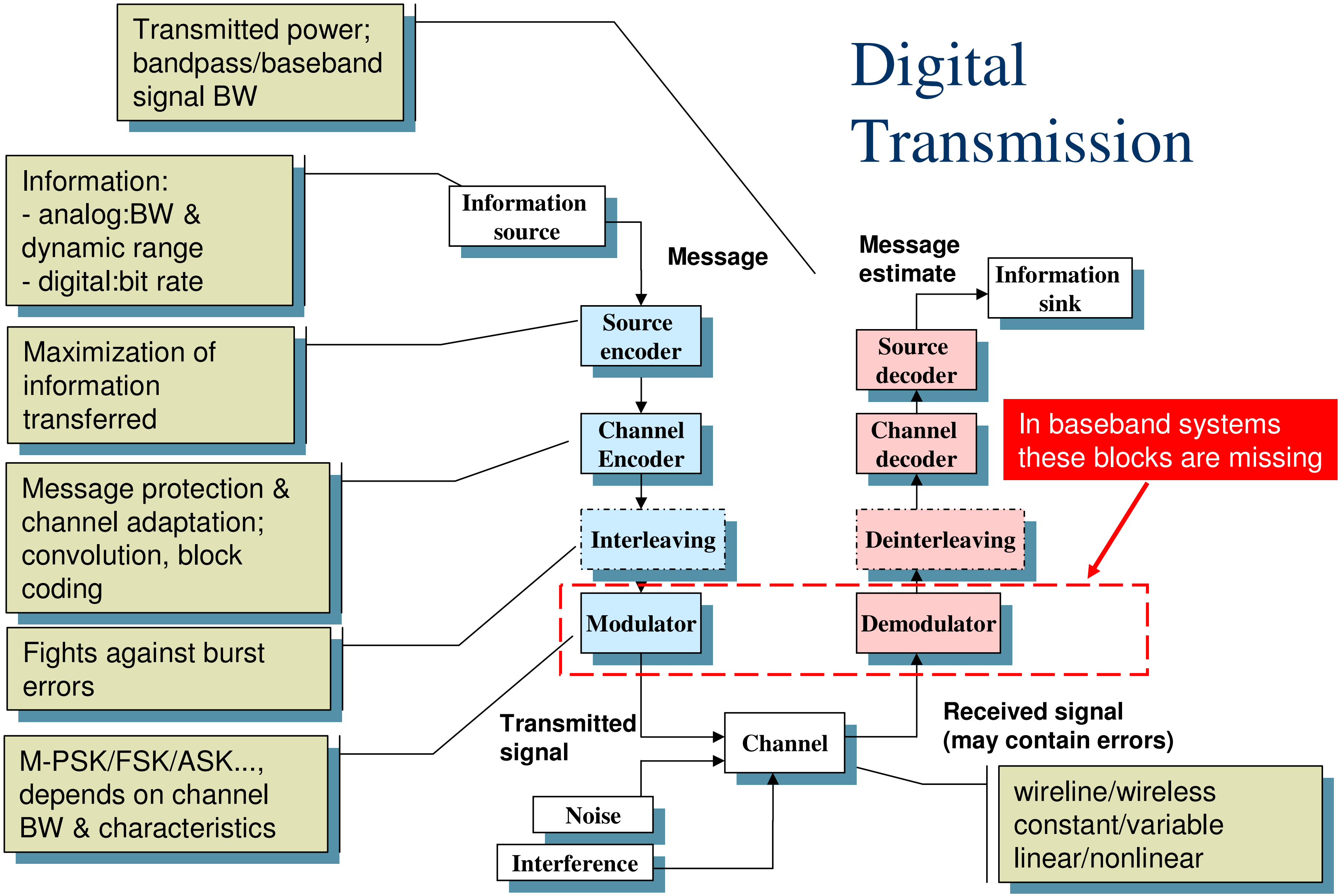
This means that at the instant of decision

$$s(t) = \sum_k a_k p(t - kD) = a_k$$

- n : number of bits
- M : number of levels
- D : Symbol duration
- T_b : Bit duaration

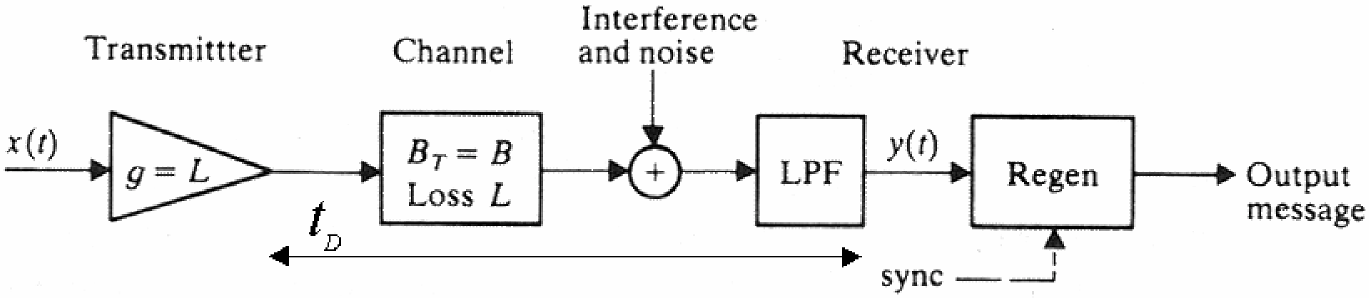
*Pulse Amplitude Modulation

Digital Transmission

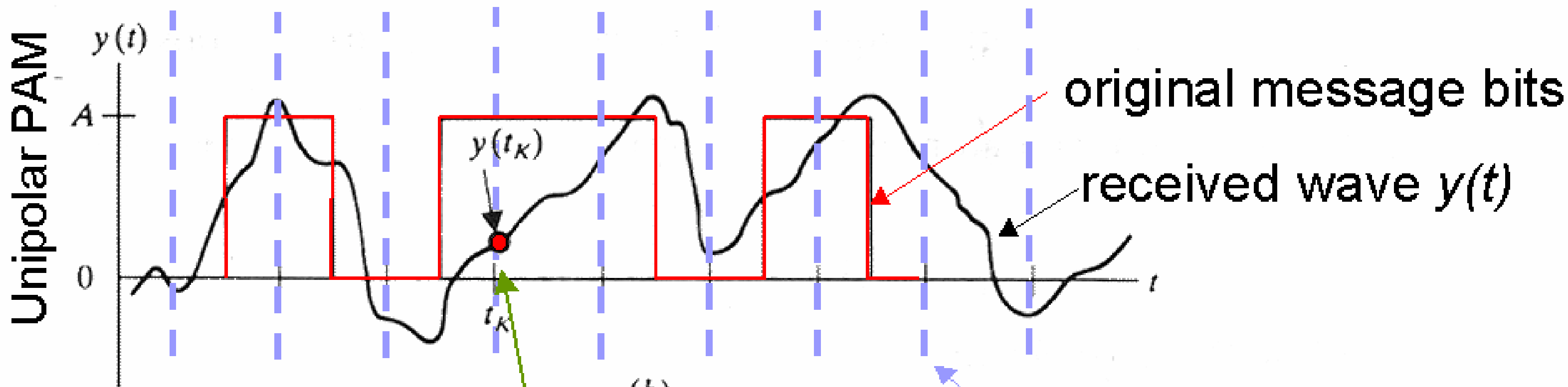


- ‘Baseband’ means that no carrier wave modulation is used for transmission

Baseband Digital Transmission Link



(a)



(b)

decision instances

$$y(t) = \sum_k a_k p(t - t_d - kD) + n(t)$$

message reconstruction at $t_K = KD + t_d$ yields

$$y(t_K) = a_k + \sum_{k \neq K} a_k \tilde{p}(KD - kD) + n(t)$$

message ISI Gaussian bandpass noise

Baseband Unipolar Binary Error Probability

Assume binary & unipolar *x(t)*

The sample-and-hold circuit yields:

$$\text{r.v. } Y : y(t_k) = a_k + n(t_k)$$

Establish H_0 and H_1 hypothesis:

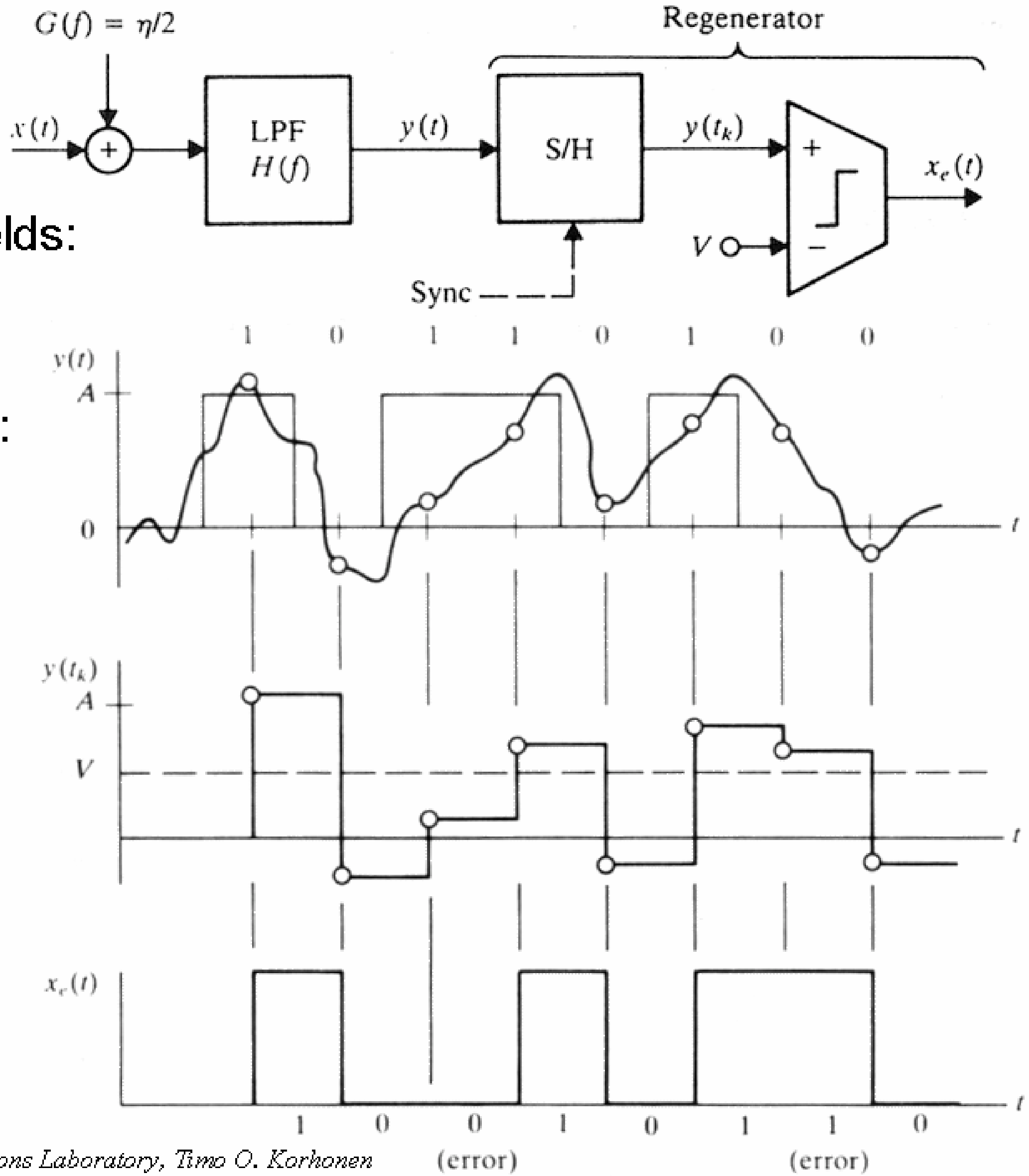
$$H_0 : a_k = 0, Y = n$$

$$p_Y(y \mid H_0) = p_N(y)$$

and

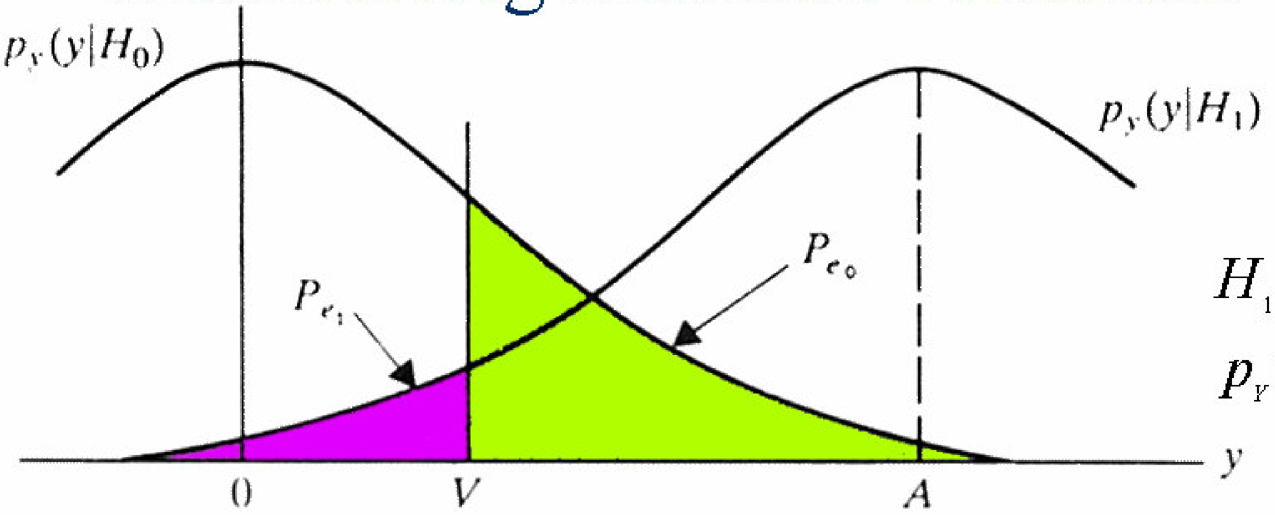
$$H_1 : a_k = 1, Y = A + n$$

$$p_Y(y \mid H_1) = p_N(y - A)$$



$p_N(y)$: Noise spectral density

Determining Decision Threshold



$H_0 : a_k = 0, Y = n$

$p_Y(y \mid H_0) = p_N(y)$

$H_1 : a_k = 1, Y = A + n$

$p_Y(y \mid H_1) = p_N(y - A)$

The comparator implements decision rule:

Choose Ho *(a_k=0)* if *Y<V*
Choose H1 *(a_k=1)* if *Y>V*

$$p_{e1} \equiv P(Y < V \mid H_1) = \int_{-\infty}^{V} p_Y(y \mid H_1)dy$$

$$p_{eo} \equiv P(Y > V \mid H_0) = \int_{V}^{\infty} p_Y(y \mid H_0)dy$$

Transmitted ‘0’ but detected as ‘1’

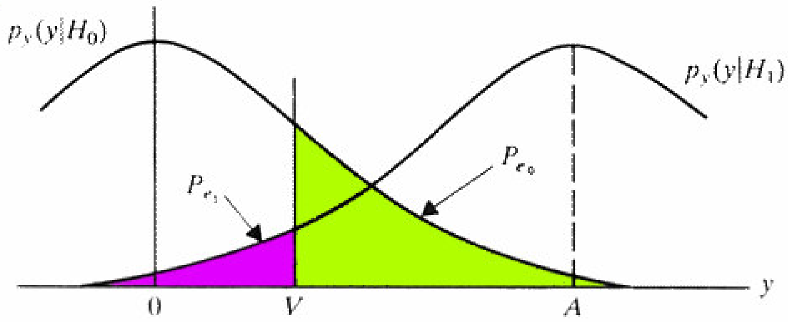
Average error error probability: $P_e = P_0 P_{e0} + P_1 P_{e1}$

$$P_0 = P_1 = 1/2 \Rightarrow P_e = ½(P_{e0} + P_{e1})$$

Assume Gaussian noise:

$$p_N(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

Determining Error Rate



$$p_{e0} = \int_V^\infty p_N(y)dy$$

$$p_{e0} = \frac{1}{\sigma\sqrt{2\pi}} \int_V^\infty \exp\left(-\frac{x^2}{2\sigma^2}\right)dx$$

that can be expressed by using the Q-function, defined by

$$Q(k) = \frac{1}{\sqrt{2\pi}} \int_k^\infty \exp\left(-\frac{\lambda^2}{2}\right)d\lambda \Rightarrow$$

$$\sigma p_{e0} = \frac{1}{\sqrt{2\pi}} \int_V^\infty \exp\left(-\frac{x^2}{2\sigma^2}\right)dx = \sigma Q\left(\frac{V}{\sigma}\right)$$

and therefore

$$p_{e0} = Q\left(\frac{V}{\sigma}\right)$$ and also $$P_{e1} = \int_{-\infty}^V p_N(y-A)dy = Q\left(\frac{A-V}{\sigma}\right)$$

Baseband Binary Error Rate in Terms of Pulse Shape and γ

setting *V=A/2* yields then

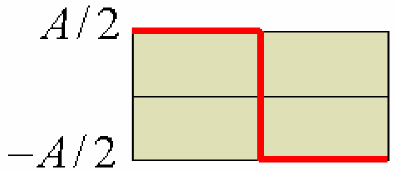
$$p_e = \tfrac{1}{2}(p_{e0} + p_{e1}) = p_{e0} = p_{e1} \Rightarrow \boxed{p_e = Q\left(\frac{A}{2\sigma}\right)}$$

for unipolar, **rectangular** NRZ [0,A] bits

$$S_R = \overline{x_{DC}^2} + \sigma^2 = A^2/2$$

for polar, **rectangular** NRZ [-A/2,A/2] bits

$$S_R = \underbrace{\overline{x_{DC}^2}}_{=0} + \sigma^2 = A^2/4$$



$$\overline{a_k^2} = \frac{2}{T}\int_0^{T/2}\left(\frac{A}{2}\right)^2 = \frac{2T}{2T}\left(\frac{A}{2}\right)^2 = \frac{A^2}{4}$$

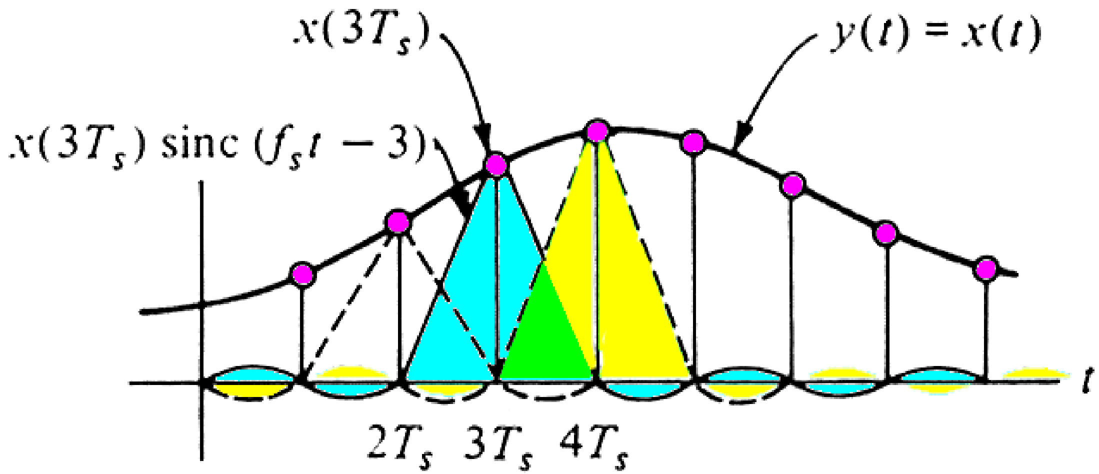
and hence

$$\left(\frac{A}{2\sigma}\right)^2 = \frac{A^2}{4N_R} = \begin{cases} S_R/(2N_R), \text{unipolar} \\ S_R/N_R, \text{polar} \end{cases}$$

$$\begin{cases} \gamma_b = E_b/N_0 = S_R/N_0 r_b \\ N_R = N_0 r_b/2 \end{cases} \Rightarrow \begin{cases} 2\gamma_b N_0 r_b/(2N_0 r_b) = \gamma_b, \text{unipolar} \\ 2\gamma_b N_0 r_b/N_0 r_b = 2\gamma_b, \text{polar} \end{cases}$$

Note that $N_R = N_0 B_N \geq N_0 r_b/2$ (lower limit with sinc-pulses (see later))

Pulse Shaping and Band-limited Transmission



- In digital transmission <u>signaling pulse shape</u> is chosen to satisfy the following requirements:
  - yields **maximum SNR at the time instance of decision** (matched filtering)
  - accommodates signal **to channel bandwidth:**
    - rapid **decrease of pulse energy** outside the main lobe in frequency domain alleviates filter design
    - **lowers cross-talk** in multiplexed systems

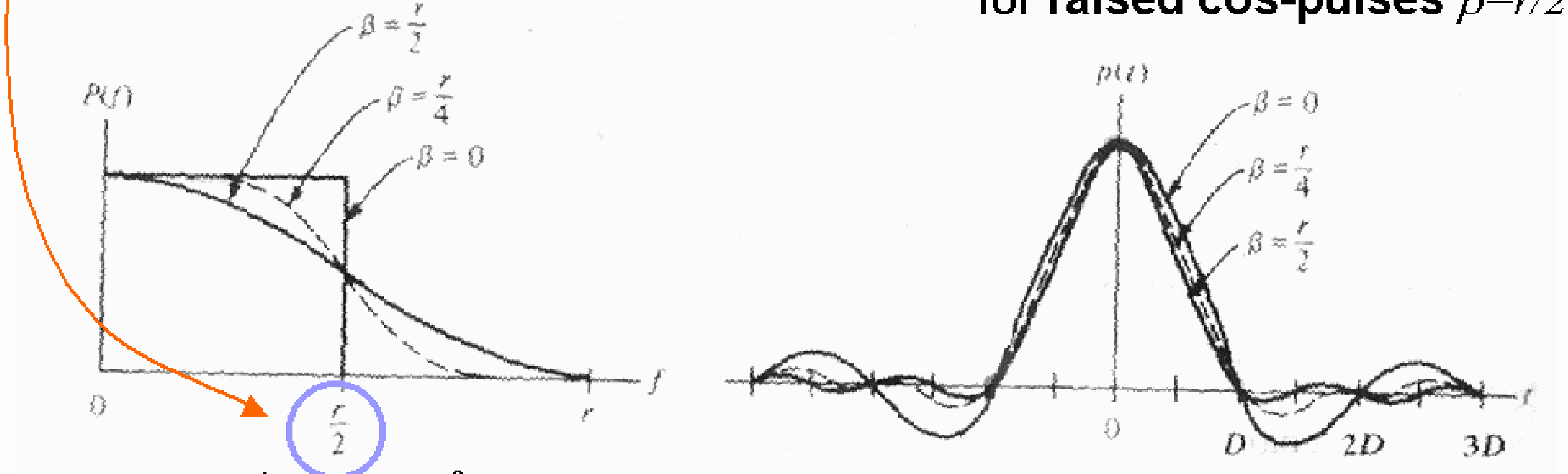
Signaling With Cosine Roll-off Signals

- **Maximum transmission rate** can be obtained with sinc-pulses

$$\begin{cases} p(t) = \text{sinc}(rt) = \text{sinc}(t/D) \\ P(f) = F[p(t)] = \dfrac{1}{r}\Pi\left(\dfrac{f}{r}\right) \end{cases}$$

- However, they are not time-limited. A more practical choice is the **cosine roll-off signaling**:

for **raised cos-pulses** $\beta = r/2$

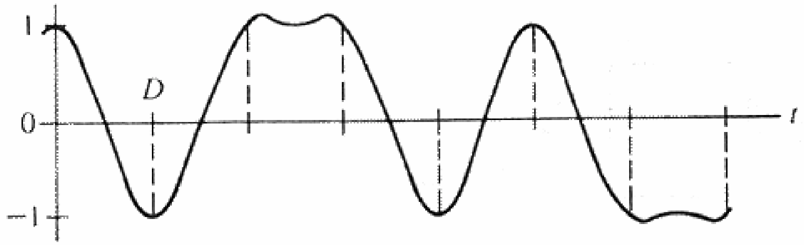


$$P(f)\Big|_{\beta=r/2} = \frac{1}{r}\cos^2\frac{\pi f}{2r}\Pi(f/2r)$$

$$p(t) = \frac{\cos 2\pi\beta t}{1-(4\beta t)^2}\,\text{sinc}\, rt$$

Example

- By using $\beta = r/2$ and polar signaling, the following waveform is obtained:



- Note that the zero crossing are spaced by *D* at

$$t = \pm 0.5D, \pm 1.5D, \pm 2.5D, ...$$

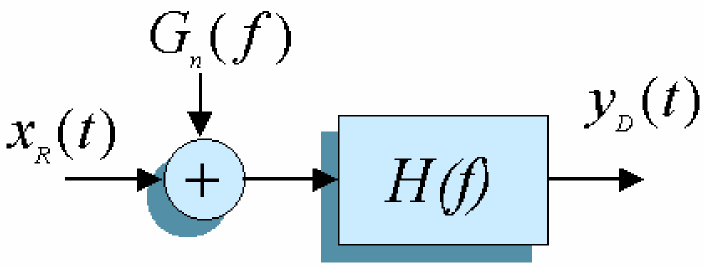
(this could be seen easily also in eye-diagram)

- The zero crossing are easy to detect for clock recovery. Note that unipolar baseband signaling involves performance penalty of 3 dB compared to polar signaling:

$$p_e = \begin{cases} Q(\sqrt{\gamma_b}), \text{ unipolar } [0/1] \\ Q(\sqrt{2\gamma_b}), \text{ polar}[\pm 1] \end{cases}$$

Matched Filtering

$$\begin{cases} x_R(t) = A_R\, p(t - t_0) \\ X_R(f) = A_R P(f)\exp(-j\omega t_0) \end{cases}$$



$$E_R = \int_{-\infty}^{\infty} |X_R(f)|^2\, df = A_R^2 \int_{-\infty}^{\infty} |P(f)|^2\, df$$

$$A = F^{-1}[H(f)X_R(f)]\big|_{t=t_0+t_d}$$

$$= A_R \int_{-\infty}^{\infty} H(f)P(f)\exp\left(j\omega t_d\right)df$$ Peak amplitude to be maximized

$$\sigma^2 = \int_{-\infty}^{\infty} |H(f)|^2 G_n(f)df = \frac{\eta}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df$$ Post filter noise

$$\left(\frac{A}{\sigma}\right)^2 = A_R^2 \frac{\left|\int_{-\infty}^{\infty} H(f)P(f)\exp\left(j\omega t_d\right)df\right|^2}{\frac{\eta}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df}$$ Should be maximized

Matched Filtering SNR and Transfer Function

$$\frac{\left|\int_{-\infty}^{\infty} V(f)W^*(f)df\right|^2}{\int_{-\infty}^{\infty} |V(f)|^2 df} \le \int_{-\infty}^{\infty} |W(f)|^2 df$$

Schwartz's inequality applies when

$$V(f) = KW^*(f)$$

$$\left(\frac{A}{\sigma}\right)^2 = A_R^2 \frac{\left|\int_{-\infty}^{\infty} H(f)P(f)\exp(j\omega t_d)df\right|^2}{\frac{\eta}{2}\int_{-\infty}^{\infty} |H(f)|^2 df}$$

SNR at the moment of sampling

$$\left.\begin{aligned} V(f) &= H(f) \\ W^*(f) &= P(f)\exp(j\omega t_d) \end{aligned}\right\}$$

$$\Rightarrow H(f) = KP(f)\exp(j\omega t_d)$$

impulse response is: $\Rightarrow h(t) = Kp(t_d - t)$

Considering right hand side yields max SNR

$$\left.\left(\frac{A}{\sigma}\right)^2\right|_{MAX} = \frac{2A_R^2}{\eta}\int_{-\infty}^{\infty} |W(f)|^2 df = \frac{2A_R^2 \int_{-\infty}^{\infty} |P(f)|^2 df}{\eta}$$

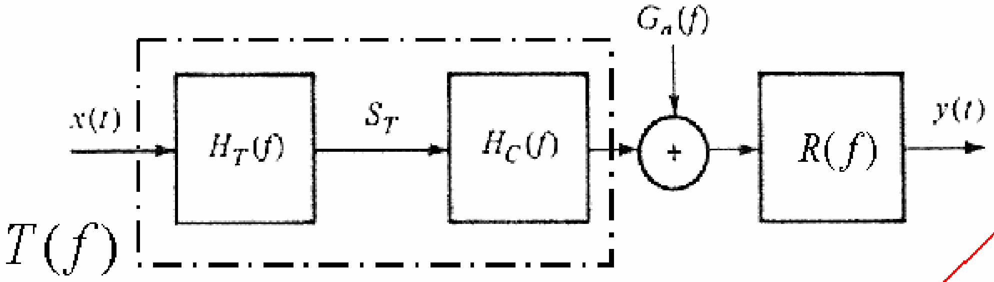
pulse energy

Optimum terminal filters

- Assume
  - arbitrary TX pulse shape *x(t)*
  - arbitrary channel response $h_c(t)$
  - multilevel PAM transmission

$$\begin{cases} P_x : \text{transmitting waveform} \\ H_T : \text{transmitter shaping filter} \\ H_C : \text{channel transfer function} \\ R : \text{receiver filter} \end{cases}$$

- What kind of filters are required for TX and RX to obtain matched filtered, non-ISI transmission?



Nyqvist shaped pulse in *y(t)*

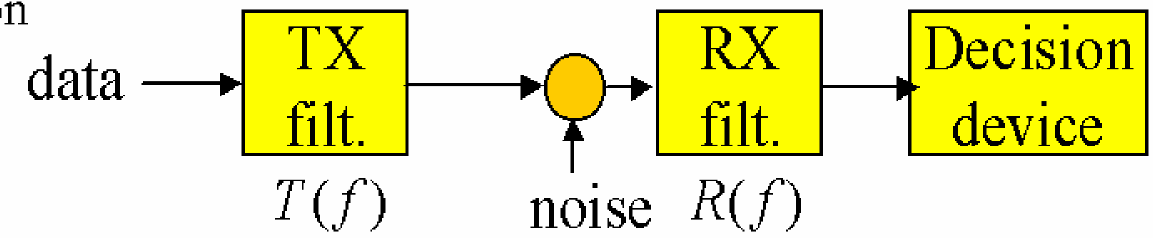
- The following condition must be fulfilled:

$$P_x(f)H_T(f)H_C(f)R(f) = P(f)\exp(-j\omega t_d)$$

that means that undistorted transmission is obtained

Avoiding ISI and enabling band-limiting in radio systems

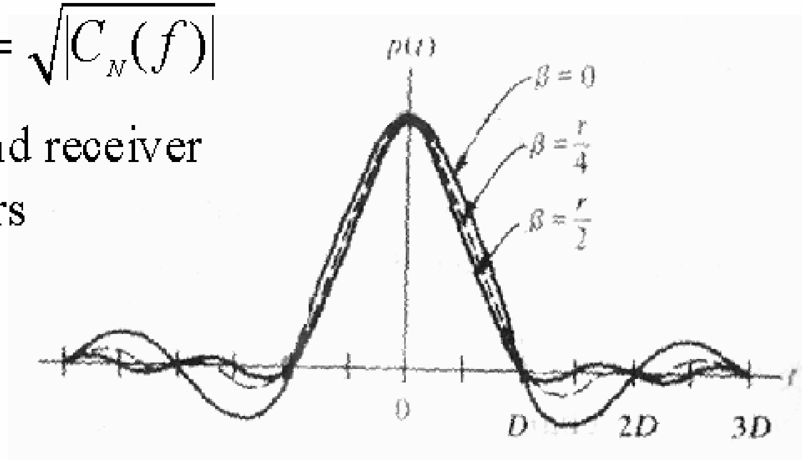
- Two goals to achieve: band limited transmission & matched filter reception



$$\begin{cases} T(f)R(f) \equiv C_N(f), & \text{raised-cos shaping} \\ T(f) = R^*(f), & \text{matched filtering} \end{cases}$$

$$\Rightarrow |R(f)| = |T(f)| = \sqrt{|C_N(f)|}$$

- Hence at the transmitter and receiver alike **root-raised** cos-filters must be applied



raised cos-spectra $C_N(f)$

Determining Transmission Bandwidth for an Arbitrary Baseband Signaling Waveform

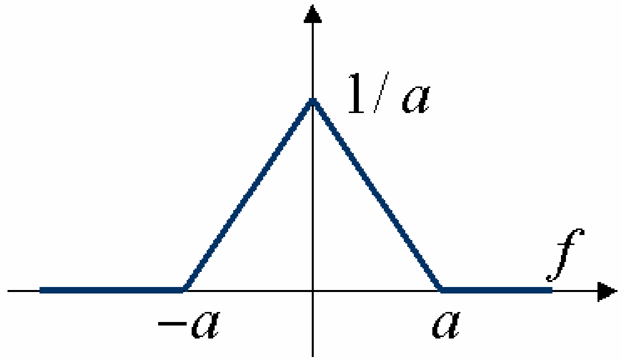
- Determine the relation between *r* and *B* when *p(t)=sinc² at*
- First note from time domain that

$$\text{sinc}^2\, at = \begin{cases} 1, t = 0 \\ 0, t = \pm 1/a, \pm 2/a \ldots \end{cases} \Rightarrow r = a$$

hence this waveform is suitable for signaling

- There exists a Fourier transform pair

$$\text{sinc}^2\, at \leftrightarrow \frac{1}{a}\Lambda\left(\frac{f}{a}\right)$$



- From the spectra we note that $B_T \geq a$ and hence it must be that for baseband

$$\Rightarrow B_T \geq r$$

PAM Power Spectral Density (PSD)

- PSD for PAM can be determined by using a general expression

Amplitude autocorrelation

$$G_x(f) = \frac{1}{D}\left|P(f)\right|^2 \sum_{n=-\infty}^{\infty} R_a(n)\exp(-j2\pi nfD)$$

- For uncorrelated message bits

$$R_a(n) = \begin{cases} \sigma_a^2 + m_a^2, n = 0 & \text{Total power} \\ m_a^2, n \neq 0 & \text{DC power} \end{cases}$$

and therefore

$$\sum_{n=-\infty}^{\infty} R_a(n)\exp(-2\pi nfD) = \sigma_n^2 + m_a^2 \sum_{n=-\infty}^{\infty} \exp(-j2\pi nfD)$$

on the other hand $\sum_{n=-\infty}^{\infty} \exp(-j2\pi nfD) = \frac{1}{D}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{D}\right)$ and $r = 1/D$

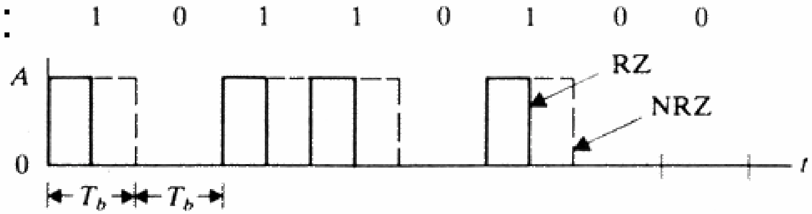
$$G_x(f) = \sigma_a^2 r\left|P(f)\right|^2 + m_a^2 r^2 \sum_{n=-\infty}^{\infty} \left|P(nr)\right|^2 \delta(f - nr)$$

Example

- For unipolar binary RZ signal:

$$P(f) = \frac{1}{2r_b}\,\mathrm{sinc}\,\frac{f}{2r_b}$$

(a)

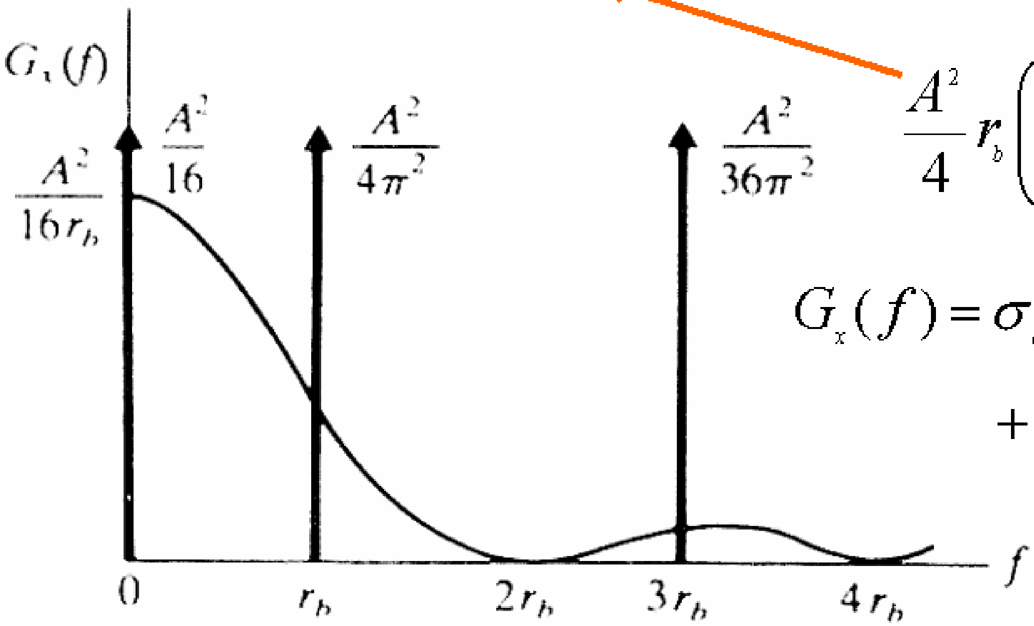


- Assume source bits are equally alike and independent, thus

$$\sigma_a^2 = (1/2T_b)\int_0^{T_b/2} A^2 dt = A^2/4, m_a^2 = \sigma_a^2$$

$$\Rightarrow G_x(f) = \frac{A^2}{16r_b}\mathrm{sinc}^2\frac{f}{2r_b} + \frac{A^2}{16}\sum_{n=-\infty}^{\infty}\delta(f - nr_b)\,\mathrm{sinc}^2\frac{n}{2}$$

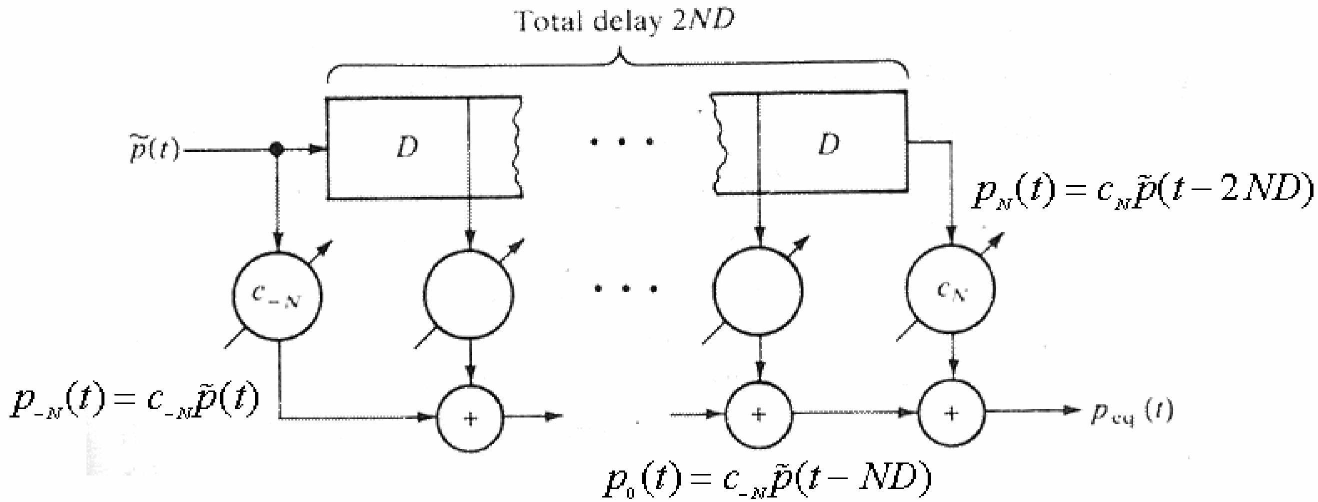
$$\frac{A^2}{4} r_b \left(\frac{1}{2r_b}\right)^2$$



$$G_x(f) = \sigma_a^2 r|P(f)|^2 + m_a^2 r^2 \sum_{n=-\infty}^{\infty}|P(nr)|^2\,\delta(f - nr)$$

Equalization: Removing Residual ISI

- Consider a tapped delay line equalizer with



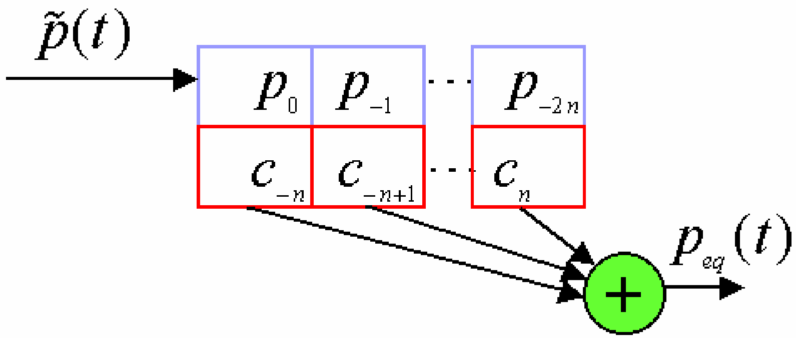
- Search for the tap gains c_N such that the output equals zero at sample intervals *D* except at the decision instant when it should be unity. The output is (think for instance paths c_{-N}, c_N or c_0)

$$p_{eq}(t) = \sum_{n=-N}^{N} c_n \tilde{p}(t - nD - ND)$$

that is sampled at $t_k = kD + ND$ yielding

$$p_{eq}(kD + ND) = \sum_{n=-N}^{N} c_n \tilde{p}(kD - nD) = \sum_{n=-N}^{N} c_n \tilde{p}\left[D(k-n)\right]$$

Tapped Delay Line: Matrix Representation



- At the instant of decision:

$$p_{eq}(t_k) = \sum_{n=-N}^{N} c_n \tilde{p}\left[D(k-n)\right] = \sum_{n=-N}^{N} c_n \tilde{p}_{k-n} = \begin{cases} 1, k=0 \\ 0, k = \pm 1, \pm 2, \ldots, \pm N \end{cases}$$

- That leads into *(2N+1)x(2N+1)* matrix where *(2N+1)* tap coefficients can be solved:

$$\begin{aligned}
&\tilde{p}_0 c_{-n} + \tilde{p}_{-1} c_{-n+1} + \ldots + \tilde{p}_{-2n} c_n = 0 \\
&\tilde{p}_1 c_{-n} + \tilde{p}_0 c_{-n+1} + \ldots + \tilde{p}_{-2n+1} c_n = 0 \\
&\qquad \ldots \\
&\tilde{p}_n c_{-n} + \tilde{p}_{n-1} c_{-n+1} + \ldots + \tilde{p}_{-n} c_n = 1 \\
&\qquad \ldots \\
&\tilde{p}_{2n} c_{-n} + \tilde{p}_{2n-1} c_{-n+1} + \ldots + \tilde{p}_0 c_n = 0
\end{aligned}$$

$$\begin{bmatrix} \tilde{p}_0 & \cdots & \tilde{p}_{-2N} \\ \vdots & \cdots & \vdots \\ \tilde{p}_{N-1} & \cdots & \tilde{p}_{-N-1} \\ \tilde{p}_N & \cdots & \tilde{p}_{-N} \\ \tilde{p}_{N+1} & \cdots & \tilde{p}_{-N+1} \\ \vdots & & \vdots \\ \tilde{p}_{2N} & \cdots & \tilde{p}_0 \end{bmatrix} \begin{bmatrix} c_{-N} \\ \vdots \\ c_{-1} \\ c_0 \\ c_1 \\ \vdots \\ c_N \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

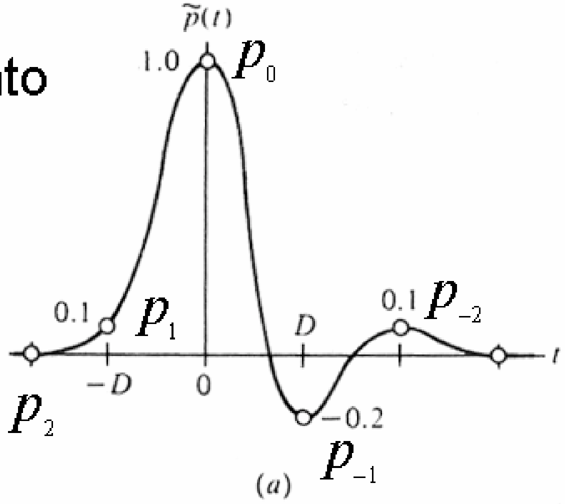
Example of Equalization

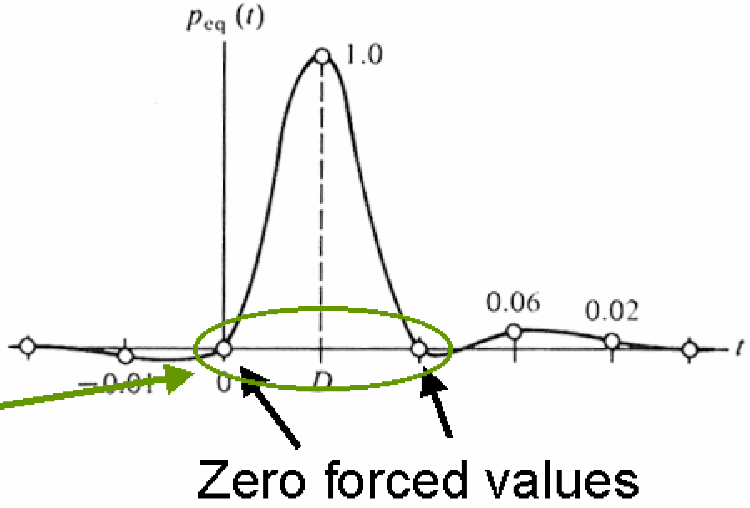
- Read the distorted pulse values into matrix from fig. (a)

$$\begin{bmatrix} 1.0 & 0.1 & 0.0 \\ -0.2 & 1.0 & 0.1 \\ 0.1 & -0.2 & 1.0 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

and the solution is

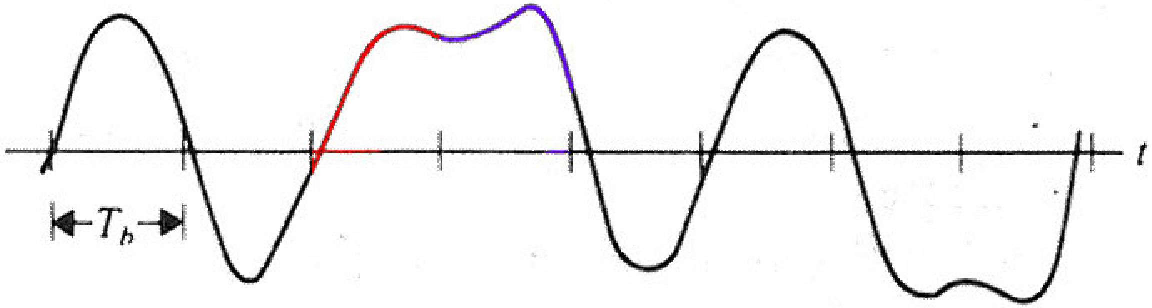
$$\begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} -0.096 \\ 0.96 \\ 0.2 \end{bmatrix}$$

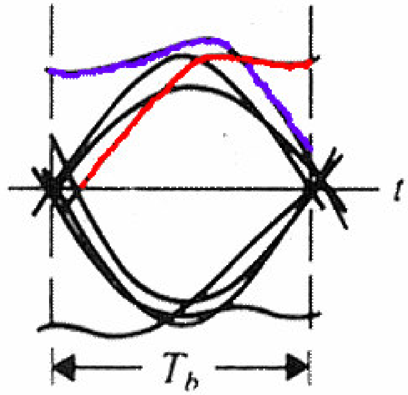




Question: what does these zeros help because they don't exist at the sampling instant?

Monitoring Transmission Quality by Eye Diagram



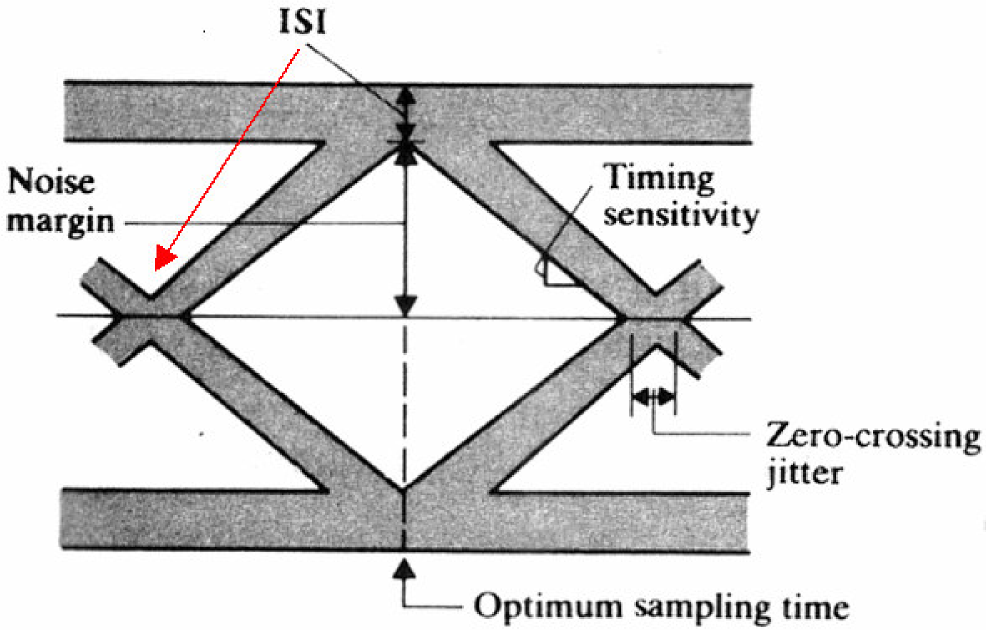


Required minimum bandwidth is

$$B_T \geq r/2$$

Nyqvist's sampling theorem:

> Given an ideal LPF with the bandwidth B it is possible to transmit independent symbols at the rate:
>
> $$B_T \geq r/2 = 1/(2T_b)$$



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