S-72.245 Transmission Methods in Telecommunication Systems (4 cr)

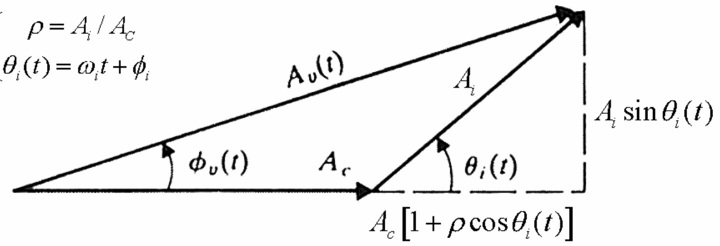
Review

Additive interference in unmodulated carrier

- Consider a general tone interference signal

$$v(t) = \underbrace{A_C \cos(\omega_C t)}_{\text{carrier}} + \underbrace{A_i \cos\left[(\omega_C + \omega_i)t + \phi_i\right]}_{\text{interference}}$$

$$\begin{cases} \rho = A_i / A_C \\ \theta_i(t) = \omega_i t + \phi_i \end{cases}$$



- interference produces both AM and FM:

$$A_v(t) = A_C\sqrt{1 + \rho^2 + 2\rho\cos\theta_i(t)}$$

$$\phi_v(t) = \arctan\frac{\rho\sin\theta_i(t)}{1 + \rho\cos\theta_i(t)}$$

Additive interference and demodulators

- Further simplification under weak interference: $A_i << A_c, \rho << 1$

$$A_v(t) = A_c\sqrt{1+\underbrace{\rho^2}_{\approx \rho^2\cos^2\theta_i(t)} + 2\rho\cos\theta_i(t)} \approx A_c\left[1+\rho\cos\theta_i(t)\right]$$

$$\phi_v(t) = \arctan\frac{\rho\sin\theta_i(t)}{1+\rho\cos\theta_i(t)} \approx \arctan\left[\rho\sin\theta_i(t)\right] \approx \rho\sin\theta_i(t)$$

- Demodulation functions:

$$y_D(t) \approx \begin{cases} K_{D,AM}A_v(t), \text{AM} \\ K_{D,PM}\phi(t), \text{PM} \\ K_{D,FM}d\phi(t)/dt, \text{FM} \end{cases}$$

- And therefore

$$y_D(t) \approx \begin{cases} K_{D,AM}A_C\left[1+\rho\cos\theta_i(t)\right], \text{AM} \\ K_{D,PM}\rho\sin\theta_i(t), \text{PM} \\ K_{D1,FM}\rho f_i\cos\theta_i(t), \text{FM} \end{cases}$$

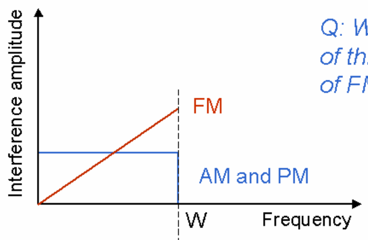
$$d\left[\rho\sin\theta_i(t)\right]/dt$$
$$= d\left[\rho\sin(\omega_i t+\phi_i)\right]/dt$$
$$= \rho\omega_i\cos(\omega_i t+\phi_i)$$

Implications for demodulator design

$$y_D(t) \approx \begin{cases} K_{D,AM}A_C\left[1+\rho\cos\theta_i(t)\right], \text{AM} \\ K_{D,PM}\rho\sin\theta_i(t), \text{PM} \\ K_{D,FM}\rho f_i\cos\theta_i(t), \text{FM} \end{cases}$$

$$\rho = A_i/A_C$$
$$\theta_i(t) = \omega_i t+\phi_i$$

- In AM and PM a tone interference produces a tone to reception whose amplitude is comparable to ρ and position comparable to $\theta_i(t) = \omega_i t+\phi_i$
- Interference in FM is more severe the more remote the interfering tone is from the carrier (but still at the reception band *W*):



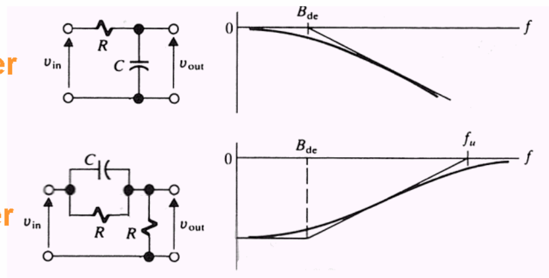
Q: What are the implications of this to noise sensitivity of FM bandwidth determination

FM preemphases and deemphases filters

- FM related noise emphases can be suppressed by *pre-distortion* and post detection filters (preemphases and deemphases filters):

receiver filter

transmitter filter



$H_{de}(f)$

$H_{pe}(f)$

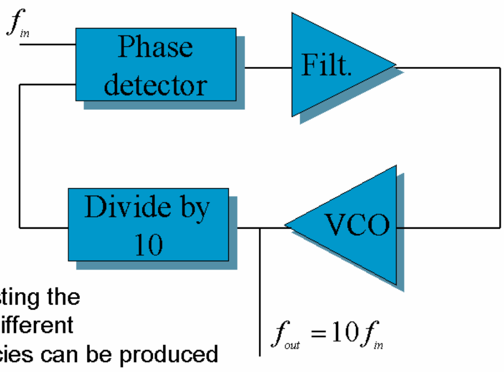
Q: What would happen if the filters would be reversed? (TX filter in receiver & vice versa)

$$H_{de}(f) = [1 + j(f / B_{de})]^{-1} \approx \begin{cases} 1, |f| << B_{de} \\ B_{de} / (jf), |f| >> B_{de} \end{cases}$$ LPF

$$H_{pe}(f) = [1 + j(f / B_{de})] \approx \begin{cases} 1, |f| << B_{de} \\ j(f / B_{de}), |f| >> B_{de} \end{cases}$$ HPF

PLL based frequency synthesizer

Reference signal f_{in} is locked for instance to the fundamental frequency of a crystal oscillator



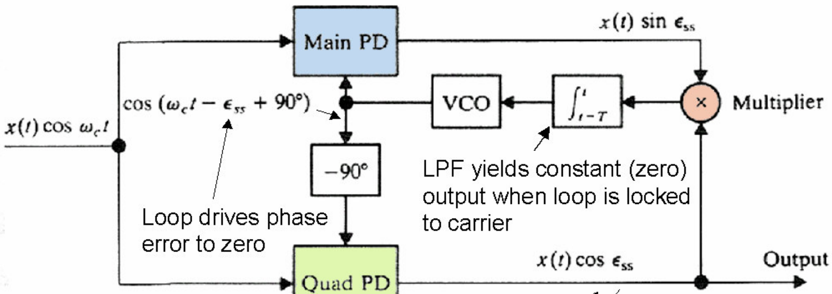
By adjusting the divider different frequencies can be produced whose phase is locked into f_{in}

Detecting DSB using PLL-principle

- An important application for PLLs is in **synchronization** of receiver local oscillator in synchronous detection
- In the **Costas PLL** (below) two phase discriminators are used to:
  - cancel out DSB modulation *x(t)* in the driving signal
  - synchronize the output frequency to the center frequency of the DSB spectra (the suppressed carrier)
  - to detect the DSB signal

Costas PLL detector for DSB

PD: phase detector (=multiply+LPF)



$$\sin\varepsilon_{ss}\cos\varepsilon_{ss} = \tfrac{1}{2}\sin 2\varepsilon_{ss} + \sin 0 \approx \varepsilon_{ss}$$

PAM Power Spectral Density (PSD)

- PSD for PAM can be determined by using a general expression

Amplitude autocorrelation

$$G_x(f) = \frac{1}{D}|P(f)|^2 \sum_{n=-\infty}^{\infty} R_a(n)\exp(-j2\pi nfD)$$

- For uncorrelated message bits

$$R_a(n) = \begin{cases} \sigma_a^2 + m_a^2, n = 0 & \text{Total power} \\ m_a^2, n \neq 0 & \text{DC power} \end{cases}$$

and therefore

$$\sum_{n=-\infty}^{\infty} R_a(n)\exp(-2\pi nfD) = \sigma_n^2 + m_a^2 \sum_{n=-\infty}^{\infty} \exp(-j2\pi nfD)$$

on the other hand $\sum_{n=-\infty}^{\infty} \exp(-j2\pi nfD) = \frac{1}{D}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{D}\right)$ and $r = 1/D$

$$G_x(f) = \sigma_a^2 r|P(f)|^2 + m_a^2 r^2 \sum_{n=-\infty}^{\infty} |P(nr)|^2 \delta(f - nr)$$

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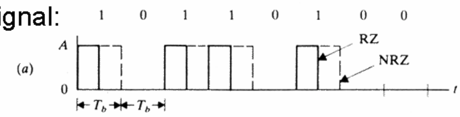
Example

- For unipolar binary RZ signal:

$$P(f) = \frac{1}{2r_b}\text{sinc}\frac{f}{2r_b}$$

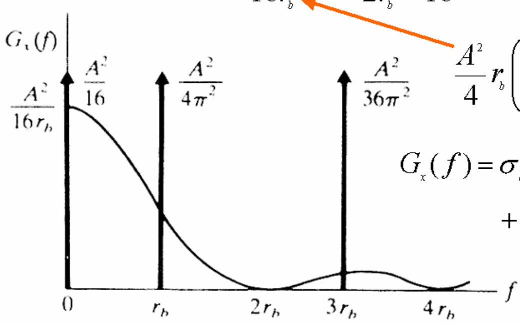


- Assume source bits are equally alike and independent, thus

$$\sigma_a^2 = (1/2T_b)\int_0^{T_b/2} A^2 dt = A^2/4, m_a^2 = \sigma_a^2$$

$$\Rightarrow G_x(f) = \frac{A^2}{16r_b}\text{sinc}^2\frac{f}{2r_b} + \frac{A^2}{16}\sum_{n=-\infty}^{\infty}\delta(f - nr_b)\text{sinc}^2\frac{n}{2}$$

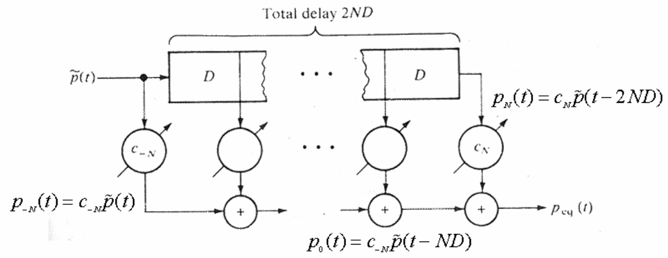
$$\frac{A^2}{4}r_b\left(\frac{1}{2r_b}\right)^2$$



$$G_x(f) = \sigma_a^2 r|P(f)|^2 + m_a^2 r^2 \sum_{n=-\infty}^{\infty}|P(nr)|^2\delta(f - nr)$$

Equalization: Removing Residual ISI

- Consider a tapped delay line equalizer with



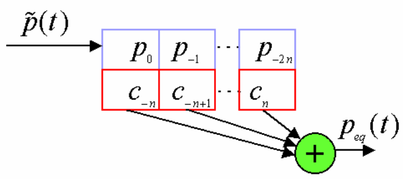
- Search for the tap gains c_N such that the output equals zero at sample intervals D except at the decision instant when it should be unity. The output is (think for instance paths c_{-N}, c_N or c_0)

$$p_{eq}(t) = \sum_{n=-N}^{N} c_n \tilde{p}(t - nD - ND)$$

that is sampled at $t_k = kD + ND$ yielding

$$p_{eq}(kD + ND) = \sum_{n=-N}^{N} c_n \tilde{p}(kD - nD) = \sum_{n=-N}^{N} c_n \tilde{p}\left[D(k-n)\right]$$

Tapped Delay Line: Matrix Representation



- At the instant of decision:

$$p_{eq}(t_k) = \sum_{n=-N}^{N} c_n \tilde{p}\left[D(k-n)\right] = \sum_{n=-N}^{N} c_n \tilde{p}_{k-n} = \begin{cases} 1, k = 0 \\ 0, k = \pm 1, \pm 2, \ldots, \pm N \end{cases}$$

- That leads into *(2N+1)x(2N+1)* matrix where *(2N+1)* tap coefficients can be solved:

$$\tilde{p}_0 c_{-n} + \tilde{p}_{-1} c_{-n+1} + \ldots + \tilde{p}_{-2n} c_n = 0$$
$$\tilde{p}_1 c_{-n} + \tilde{p}_0 c_{-n+1} + \ldots + \tilde{p}_{-2n+1} c_n = 0$$
$$\ldots$$
$$\tilde{p}_n c_{-n} + \tilde{p}_{n-1} c_{-n+1} + \ldots + \tilde{p}_{-n} c_n = 1$$
$$\ldots$$
$$\tilde{p}_{2n} c_{-n} + \tilde{p}_{2n-1} c_{-n+1} + \ldots + \tilde{p}_0 c_n = 0$$

$$\begin{bmatrix} \tilde{p}_0 & \cdots & \tilde{p}_{-2N} \\ \vdots & \cdots & \vdots \\ \tilde{p}_{N-1} & \cdots & \tilde{p}_{-N-1} \\ \tilde{p}_N & \cdots & \tilde{p}_{-N} \\ \tilde{p}_{N+1} & \cdots & \tilde{p}_{-N+1} \\ \vdots & & \vdots \\ \tilde{p}_{2N} & \cdots & \tilde{p}_0 \end{bmatrix} \begin{bmatrix} c_{-N} \\ \vdots \\ c_{-1} \\ c_0 \\ c_1 \\ \vdots \\ c_N \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

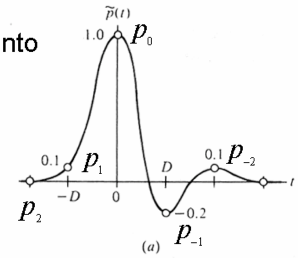
Example of Equalization

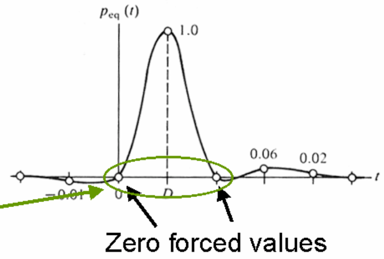
- Read the distorted pulse values into matrix from fig. (a)

$$\begin{bmatrix} 1.0 & 0.1 & 0.0 \\ -0.2 & 1.0 & 0.1 \\ 0.1 & -0.2 & 1.0 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

and the solution is

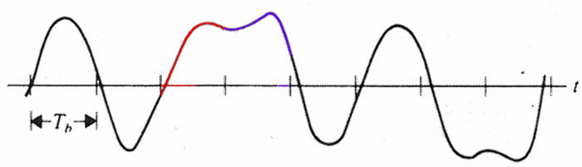
$$\begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} -0.096 \\ 0.96 \\ 0.2 \end{bmatrix}$$

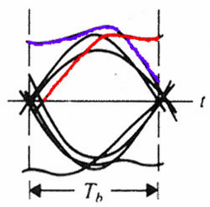




Question: what does these zeros help because they don't exist at the sampling instant?

Monitoring Transmission Quality by Eye Diagram





Required minimum bandwidth is

$$B_T \geq r/2$$

Nyqvist's sampling theorem:

> Given an ideal LPF with the bandwidth B it is possible to transmit independent symbols at the rate:
>
> $$B_T \geq r/2 = 1/(2T_b)$$