Sampling and Pulse Coded Modulation

- Pulse amplitude modulation
- Sampling
  - Ideal sampling by impulses
  - Practical chopper sampler
- Line coding
- Quantization
  - Uniform
  - Non-uniform
  - \(\mu\)-law - compression
  - Quantization noise
- PCM and channel noise
- PCM multiplexing

TDM: Time Division Multiplexing
FDM: Frequency Division Multiplexing
PAM: Pulse Amplitude Modulation
PCM: Pulse Coded Modulation
Short history of pulse coded modulation

- A problem of PSTN analog techniques (e.g., SSB-FDM) was that transmitting multiple channels was difficult due to non-linearities resulting channel **cross-talk**
- **1937** Reeves and Delorane ITT labs. tested **TDM**-techniques by using electron-tubes
- **1948** PCM tested in Bell Labs: Using this method it is possible to represent a 4 kHz analog telephone signal as a 64 kbit/s **digital bit stream**
- TDM was taken into use in **1962** with a 24 channel PCM link
- The first **30-channel** PCM system installed in Finland **1969**
- Nowadays all exchanges in Finland use ISDN & PCM based cables, microwave or optical links

PCM coding is a form of waveform coding

- **Waveform coders** reply signal by quantized (discrete) values, - precise waveform replay but requires a lot of bandwidth
- **Parameterized coders** count on **system model** that reproduces the signal. Only model parameters are transmitted and updated. Very low rate can be obtained but this is paid by quality degradation
- **Hybrid coders** (as Δ-modulation) are a compromise solution
Pulse Coded Modulation (PCM)

- PCM is a method by which an analog message can be transformed into numerical format and then decoded at the receiver.

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<th>Standard no.</th>
<th>Name</th>
<th>Description</th>
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<tr>
<td>G.711</td>
<td>Pulse code modulation (PCM) of voice frequencies (64 kbit/s)</td>
<td>Adopted 1984</td>
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<tr>
<td>G.722, G.725</td>
<td>7 kHz audio-coding within 64 kbit/s</td>
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<td>G.726</td>
<td>16/24/3248 kbit/s adaptive differential pulse code modulation (ADPCM)</td>
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<td>G.728</td>
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<td>H.221</td>
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<td>MPEG1</td>
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<td>JPEG</td>
<td>Still-frame graphics for multimedia</td>
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MPEG = Motion picture expert group, JPEG = Joint photograph expert group

Helsinki University of Technology, Communications Laboratory, Timo O. Korhonen
**PCM principles: Ideal sampling**

- The rectangular pulse train
  \[ s(t) = \sum_{n=-\infty}^{\infty} \Pi \left( \frac{t - kT_s}{\tau} \right) \]

- The ideal sampling pulse train
  \[ s_\tau(t) = \sum_{k=-\infty}^{\infty} \lim_{\tau \to 0} \left( \frac{1}{\tau} s(t - kT_s) \right) = \sum_{n=-\infty}^{\infty} \delta(t - kT_s) \]

- The ideal sampled signal is a pulse train of weighted impulses
  \[ x_\tau(t) = x(t) \cdot s_\tau(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - kT_s) = \sum_{n=-\infty}^{\infty} x(kT_s) \delta(t - kT_s) \]

- Translation Fourier tables: \[ \mathcal{F} \left[ \sum_{n=-\infty}^{\infty} \delta(t - kT_s) \right] = \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \]

  the ideally sampled signal is then
  \[ X_\tau(f) = X(f) \otimes x_\tau(f) = X(f) \otimes \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \]

**Ideal sampling: reconstruction**

- Reconstruction is obtained by lowpass (LP) filtering. Assume the ideal LP filter with
  \[ H(f) = K \Pi \left( \frac{f}{2B} \right) \exp(-j\omega t_s) \]

- Due to the translations
  \[ \sin(2\pi f t) \leftrightarrow \frac{1}{2} \Pi \left( \frac{f}{2W} \right) \quad v(t - t_s) \leftrightarrow V(f) \exp(-j\omega t_s) \]

  the respective impulse response of the ideal LP is therefore
  \[ h(t) = 2BK \sin(2B(t - t_s)) \]

- In ideal sampling reconstruction weighted impulse train (representing the sampled signal) is applied to this filter and the output is
  \[ y(t) = h(t) \otimes x_\tau(t) = 2BK \sin(2B(t - t_s)) \otimes \sum_{n=-\infty}^{\infty} x(kT_s) \delta(t - kT_s) = 2BK \sum_{n=-\infty}^{\infty} x(kT_s) \sin(2B(t - t_s - kT_s)) \]
Reconstructed time-domain signal consists of interpolated sinc-functions

- At the sample instances all but one sinc functions are zero
- Therefore all band limited signals can be expressed by using the sinc-series:

\[ 2BK \sum_{k=-\infty}^{\infty} x(kT_s) \text{sinc} 2B(t - t_s - kT_s) \]

\[ y(t) = x(t) \]

Aliasing and sampling theorem

- Nyquist sampling theorem:
  
  If a signal contains no frequency components for \( |f| \geq W \) it is completely described by instantaneous uniformly spaced time samples having period \( T_s \geq 1/2W \). The signal can hence been reconstructed from its samples by an ideal LPF of bandwidth \( B \) such that \( W \leq B \leq f_s - W \)

- Note: If the signal contains higher frequencies than twice the sampling frequency they will also be present at the sampled signal! An application of this is the sampling oscilloscope (next slide)

- Also, it follows from the sampling theorem that

  Two pieces of independent information / second (independent samples) can be transmitted in 1 Hz wide channel

  because signal having bandwidth \( B \) can be constructed from rate \( 2B \) independent samples
1. Sampling wave pulses have finite duration and risetimes -> linear distortion
2. Reconstruction filters are not ideal lowpass filters -> spectral folding
3. Sampled messages are time limited and therefore their spectra is not frequency limited -> spectral folding
4. Samples digitized by finite length words -> quantization noise
The chopper sampler waveforms

- Sampling wave consists of a periodic pulse train whose duration is $\tau$ and period is $T_o$
  
  $c_s = \frac{1}{T_o} \int_{-\frac{T}{2}}^{\frac{T}{2}} v(t) \exp(-2\pi ft/T_o) dt$

  $c_s = \frac{A \tau}{T_o} \text{sinc}(nf_c \tau)$

  $s(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi nf_c t)$

  The Fourier series for real signals is

  $s(t) = c_s + \sum_{m=1}^{\infty} 2c_m \cos m\omega_s t$

Chopper sampler: sampled spectra

- Consider the sampled signal from the chopper sampler by term-by-term multiplication

  $x_s(t) = x(t) s(t)$

  $s(t)$: sampling wave

  $x(t)$: signal to be sampled

  $x_s(t) = x(t) c_s + \sum_{m=1}^{\infty} 2c_m x(t) \cos m\omega_s t + 2c_s x(t) \cos 2\omega_s t$

  Remember the modulation theorem:

  $v(t) \cos(\omega_t + \phi) \leftrightarrow \frac{1}{2}[V_f + V_f \exp(j\phi) + V_f \exp(-j\phi)]$

- Therefore the sampled signal is in frequency domain

  $X_s(f) = c_s X(f) + c_s [X(f-f_c) + X(f+f_c)]$

  $+ c_s [X(f-2f_c) + X(f+2f_c)]$

  $s(t) = c_s + \sum_{m=1}^{\infty} 2c_m \cos m\omega_s t$
Observations on chopper sampling

- Resulting spectra
  - has the envelope of the sampling waveform
  - has the sampled signal repeated at the integer multiples of the sampling frequency
- Therefore the sampled signal can be reconstructed by filtering provided that
  \[ X(f) = 0, |f| > W \]  Sampled signal is band limited
  \[ f_s \geq 2W \]  Sampling rate is high enough

If these conditions are not met, spectral folding (aliasing) results:

\[ X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) \]
Quantization

- Original signal has continuous amplitudes in its dynamic range
- PAM signal is a discrete constant period, pulse train having continuous amplitude values
- Quantized PAM signal has only the values that can be quantized by the words available (here by 3 bit words)

Uniform quantization: transmitter

- Transforming the continuous samples into discrete level samples is called quantization
- In uniform quantization quantization step size is constant

![Diagram of uniform quantization](image)

- M: number of quantization levels
- v: number of quantization bits
- q: number of quantization levels
- 2q = 7.8 \times 10^{-3}
- q = M^v = 256 (v = \log_2 q)
- ADC: Analog-to-Digital Converter
- r: output bitrate
Reconstruction from the quantized signal

Note that quantization error amplitude is limited to $|\varepsilon| \leq 1/q$

Quantization noise: uniform quantization

- Model the quantized signal by assuming ideal PAM sampling using the quantization error $\varepsilon_q$:
  \[ y(t) = \sum_k x(kT_s) \delta(t - kT_s) + \varepsilon_q(t-kT_s) \]

- Quantization error is the difference of the PAM and the Q-PAM signals
  \[ \varepsilon_q = x(kT_s) - x(kT_s)^+ \]

- The final output is obtained by using the ideal LPF:
  \[ y_q(t) = y(t) + \sum_k \varepsilon_q(t-k) \sin(\frac{f_s}{2} t) \]

- Assuming equal probable signal at all amplitude levels yields for quantization noise average power
  \[ \sigma^2 = \frac{q^2}{2} \int_{-q}^{q} \varepsilon^2 d\varepsilon = \frac{1}{3q^2} \]

  note: $2 \frac{q}{q/2} = 1$
PCM versus analog modulation

- PCM suppresses wideband noise
- A PCM system well above threshold
  \[ S_o / N_o = 3^{q^*} S_o \approx 3M^{15} S_o, \quad q = M^n \]
  transmission BW: \( B_s = b W \)

- Observations:
  - operate PCM near threshold!
- Note that for radio 75 dB dynamic range (12 bits without compression) transmission bandwidth \( B_s \) is
  \[ W = 15 \text{kHz}, \quad n = 12 \text{bit} \]
  \[ B_s \approx \frac{n f_s}{2} = 12 \times 35 / 2 = 210 \text{kHz} \]
  (This is about the same as in FM)
- Why to use PCM then??
  - regenerative digital repeaters
  - digital multiplexing
  - etc!

Line coding

- Line codes are used to enable baseband transmission in
  - Fiber optic systems
  - Cable transmission
  - Data access and storage
- In PCM-links line coding is used to alleviate clock synchronization at the receiver (F-transform of the pulse train should contain spikes that the receiver clock can be synchronized)
- Line codes should
  - be immune to long strings of zeros that can lead to missing receiver clock synchronization
  - contain zero long term averaged DC-component
  - have minimum bandwidth
- Line codes can also be used for error detection
Line coding (cont.)

Unipolar \([0, A]\) RZ and NRZ

Polar \([-A/2, A/2]\) RZ and NRZ

Bipolar \([-A/2, 0, A/2]\) AMI

Split-Phase Manchester

Code rate reduced by \(n\)

Split-Polar quaternary NRZ

\[ M = 2^n \]
\[ r_n = r / n = r / \log_2 M \]

Line coding methods ... (cont.)

Bipolar-AMI
- 0 = no line signal
- 1 = positive or negative level, alternating for successive ones

Pseudoternary
- 0 = positive or negative level, alternating for successive zeros
- 1 = no line signal

Manchester
- 0 = transition from high to low in middle of interval
- 1 = transition from low to high in middle of interval

Differential Manchester
- Always a transition in middle of interval
- 0 = transition at beginning of interval
- 1 = no transition at beginning of interval

BZS
- Same as bipolar AMI, except that any string of eight zeros is replaced by a string with two code violations

HDB3
- Same as bipolar AMI, except that any string of four zeros is replaced by a string with one code violation

AMI

HDB-3

Emitted balance pulse due to following 1
Repeaters

- At the transmission path regenerative repeaters are often used.
- At the receiver signal is transformed back to analog form by lowpass filtering removing harmonics produced by sampling.
- Repeaters are categorized as:
  - analog repeater: gain equal to the line attenuation between repeaters.
  - digital repeater: regenerates bits by decoding and encoding.

Error rates for polar, baseband, \( m \)-stage repeater chains:

\[
S_e / N_0 \approx \frac{(S/N)_0}{m} \text{ analog repeaters}^* \\
\Rightarrow P = Q\left(\sqrt{(S/N)_0 / m}\right) \text{ digital repeaters}
\]

\[
P_e \approx mQ\left[\sqrt{(S/N)_0}\right] \text{ digital repeaters}
\]

* Formula follows from the reverse inspection than coherent averaging.

Digital Repeater Chain Error Rate

- Assume each repeater has the bit error rate

\[
\alpha = Q\left(\sqrt{(S/N)_0}\right)
\]

- There are \( m \) repeaters that introduce errors that are produced for odd* number of error digits and the probability of \( i \) errors in the chain is

\[
P(i, m) = \binom{m}{i} \alpha^i (1-\alpha)^{m-i}, i \text{ odd}
\]

\[
P_e = \binom{m}{1} \alpha(1-\alpha)^{-1} + \binom{m}{3} \alpha^3 (1-\alpha)^{-3} \ldots
\]

\[
\approx m\alpha (\alpha << 1, m \text{ moderate})
\]

- And therefore

\[
P_e \approx mQ\left[\sqrt{(S/N)_0}\right] \left[\frac{m}{1} \alpha(1-\alpha)^{-1} = \frac{m!}{(m-1)!} \alpha = m\alpha \right.
\]

\[
\left. \frac{m}{3} \alpha(1-\alpha)^{-1} = \frac{m!}{3!(m-3)!} \alpha^3 << m\alpha \right]
\]

\[
\approx m\alpha (\alpha << 1, m \text{ moderate})
\]

*Even number of errors cancel out each other!
PCM encoding and decoding circuits \((n=3)\)

\[
x(kT_s) = \text{sign-bit} \cdot (x(kT_s) \mod 2^n)\]

\[
\text{ADC}
\]

\[
\text{DAC}
\]

\[
\text{weighted-resistor decoder}
\]

\[
\text{direct-conversion encoder}
\]

PCM Systems and Digital Time Division Multiplexing (TDM)

- In digital multiplexing several messages are transmitted via same physical channel. For multiplexing 64 kbit/s channels in digital exchanges following three methods are available:
  - **PDH** (plesiochronous digital hierarchy) (the dominant method today, E1 & T1) ('50-'60, G.702)
  - **SONET** (synchronous optical network) ('85)
  - **SDH** (synchronous digital hierarchy) (CCITT '88)

\[
\begin{array}{c}
\text{European PCM frame} \\
\text{125 \(\mu\)s} \\
32 \text{time slots} \times 8 \text{bits} \times 8000 \text{Hz} = 2048 \text{kbit/s} \\
\end{array}
\]

PDH E-1 frame
**E1 and T1 First Order Frames Compared**

- **T1** is byte-interleaved: blocks of eight bits from the same channel are inserted to the multiplexed flow.
- **USA & Japan**
  - **T1**: 24 time-slots and a framing (F) bit serves 24 channels.
  - Frame length: 1 + 8x24 = 193 bits.
  - Rate 193x8000 bits/second = 1544 kb/s.
- **E1**
  - Frame has 32 time-slots, TS 0 holds a synchronization pattern and TS 16 holds signaling information.
  - Rate 8000x256 = 2048 kb/s.

**NOTE**: In T1 one bit in each time slot in every sixth frame is replaced by signaling information yielding 56 kb/s only.

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**T1 and E1 Summarized**

- In PSTN two PCM systems dominate:
  - T1, developed by Bell Laboratories, used in USA & Japan.
  - E1, developed by CEPT* used in most of the other countries.
- In both data streams divided in frames of 8000 frames/sec.
- In T1:
  - 24 time-slots and a framing (F) bit serves 24 channels.
  - Frame length: 1 + 8x24 = 193 bits.
  - Rate 193x8000 bits/second = 1544 kb/s.
- In E1:
  - Frame has 32 time-slots, TS 0 holds a synchronization pattern and TS 16 holds signaling information.
  - An E1 frame has 32x8 = 256 bits and its rate is 8000x256 = 2048 kb/s.

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*European Conference of Postal and Telecommunications Administration*
If one wishes to disassemble a tributary from the main flow the main flow must be demultiplexed step by step to the desired main flow level in PDH.

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**PCM-method summarized**

- Analog speech signal is applied into a LP-filter **restricting its bandwidth** into 3.4 kHz
- Sampling circuit forms a PAM pulse train having rate of 8 kHz
- Samples are **quantized** into 256 levels that requires a 8 bit-word for each sample ($2^8=256$).
- Thus a telephone signal requires $8 \times 8 \text{ kHz} = 64 \text{ kHz bandwidth}$
- The samples are **line coded** by using the HDB-3 scheme to enable synchronization and channel adaptation
- Usually one transmits several channels simultaneously following a **digital hierarchy** (as SDH or PDH)
- Transmission link can be an **optical fiber, radio link or an electrical cable**
- At the receiver the PAM signal is reconstructed where after it is **lowpass filtered** to yield the original-kind, analog signal