S-72.245 Transmission Methods in Telecommunication Systems (4 cr)

Sampling and Pulse Coded Modulation

Sampling and Pulse Coded Modulation

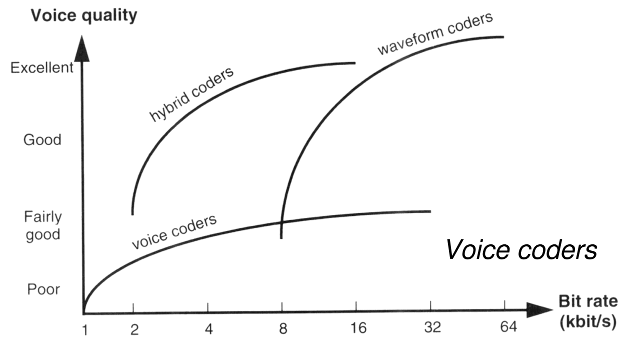
- Pulse amplitude modulation
- Sampling
  - Ideal sampling by impulses
  - practical chopper sampler
- Line coding
- Quantization
  - Uniform
  - Non-uniform
  - - μ - law - compression
  - quantization noise
- PCM and channel noise
- PCM multiplexing

TDM: Time Division Multiplexing
FDM: Frequency Division Multiplexing
PAM: Pulse Amplitude Modulation
PCM: Pulse Coded Modulation

Short history of pulse coded modulation

- A problem of PSTN analog techniques (eg SSB-FDM) was that transmitting multiple channels was difficult due to non-linearities resulting channel **cross-talk**
- **1937** Reeves and Delorane ITT labs. tested **TDM**-techniques by using electron-tubes
- **1948 PCM** tested in Bell Labs: Using this method it is possible to represent a 4 kHz analog telephone signal as a **64 kbit/s digital bit stream**
- TDM was taken into use in **1962** with a **24 channel PCM link**
- The first **30-channel** PCM system installed **in Finland 1969**
- Nowadays all exchanges in Finland use ISDN & PCM based cables, microwave or optical links

PCM coding is a form of waveform coding



- **Waveform coders** reply signal by quantized (discrete) values, - precise waveform replay but requires a lot of bandwidth
- **Parameterized coders** count on **system model** that reproduces the signal. Only model parameters are transmitted and updated. Very low rate can be obtained but this is paid by quality degradation
- **Hybrid coders** (as Δ- modulation) are a compromise solution

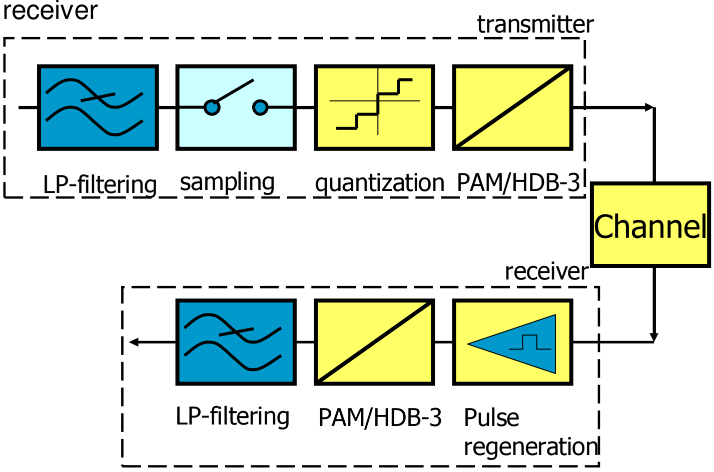
Some important ITU-T speech/video coding standards

Standard no. Name	Description	Current status
G.711	Pulse code modulation (PCM) of voice frequencies (64 kbit/s)	Adopted 1984
G.722, G.725	7 kHz audio-coding within 64 kbit/s	Adopted 1988
G.726	16/24/32/46 kbit/s adaptive differential pulse code modulation (ADPCM)	Adopted 1990
G.728	16 kbit/s speech coding with excited linear prediction	Adopted 1992
G.729	8 kbit/s speech coding	Adopted 1996
H.221	Frame structure for a 64 to 1920 kbit/s channel in audiovisual teleservices	Adopted 1990
H.230	Control and indication signals for audiovisual systems	Adopted 1990
H.231, H243	Multipoint videoconferencing	Adopted 1993
H.233	Encryption / Privacy systems	Adopted 1993
H.261	Video codec for audiovisual teleservices at p x 64 kbit/s	Adopted 1993
H.263	Video coding for low bit rate communication	Adopted 1996
MPEG1	Stored motion video stored at <2 Mbit/s	Adopted 1993
MPEG2	Stored/live motion video at 5–60 Mbit/s	Adopted 1994
MPEG4	Low bit rate (<64 kbit/s) coding of motion video	
JPEG	Still-frame graphics for multimedia	Adopted 1991

MPEG = Motion picture expert group. JPEG = Joint photograph expert group

Pulse Coded Modulation (PCM)

- PCM is a method by which an analog message can be transformed into numerical format and then decoded at the receiver



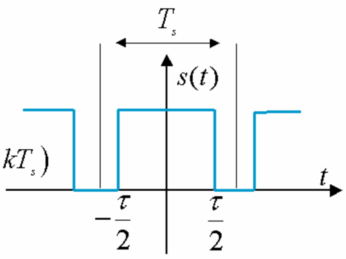
PCM principles: Ideal sampling

- The rectangular pulse train

$$s(t)=\sum_{k=-\infty}^{\infty}\Pi\left(\frac{t-kT_s}{\tau}\right)$$

- The ideal sampling pulse train

$$s_\delta(t)=\sum_{k=-\infty}^{\infty}\lim_{\tau\to 0}\left[\frac{1}{\tau}s(t-kT_s)\right]=\sum_{k=-\infty}^{\infty}\delta(t-kT_s)$$



- The ideal sampled signal is a pulse train of weighted impulses

$$x_\delta(t)=x(t)s_\delta(t)=x(t)\sum_{k=-\infty}^{\infty}\delta(t-kT_s)$$
$$=\sum_{k=-\infty}^{\infty}x(kT_s)\delta(t-kT_s)$$

- Translation Fourier tables: $\mathcal{F}[\sum_{k=-\infty}^{\infty}\delta(t-kT_s)]=f_s\sum_{n=-\infty}^{\infty}\delta(f-nf_s)$

the ideally sampled signal is then $X_\delta(f)=X(f)\otimes s_\delta(f)$

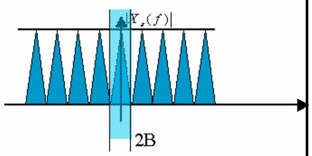
$$=X(f)\otimes f_s\sum_{n=-\infty}^{\infty}\delta(f-nf_s)$$

$$X_\delta(f)=f_s\sum_{n=-\infty}^{\infty}X(f-nf_s)$$

Ideal sampling: reconstruction

- Reconstruction is obtained by lowpass (LP) filtering. Assume the ideal LP filter with

$$H(f)=K\Pi\left(\frac{f}{2B}\right)\exp(-j\omega t_d)$$



- Due to the translations

$$\text{sinc}\,2Wt\leftrightarrow\frac{1}{2W}\Pi\left(\frac{f}{2W}\right)\qquad v(t-t_d)\leftrightarrow V(f)\exp(-j\omega t_d)$$

the respective impulse response of the ideal LP is therefore

$$h(t)=2BK\,\text{sinc}\,2B(t-t_d)$$

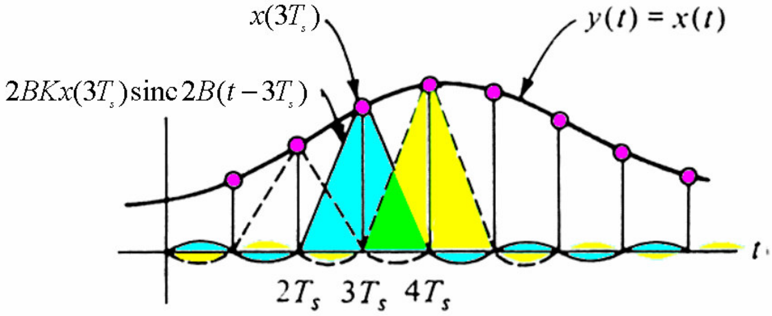
- In ideal sampling reconstruction weighted impulse train (representing the sampled signal) is applied to this filter and the output is $y(t)=h(t)\otimes x_\delta(t)$

$$=2BK\,\text{sinc}\,2B(t-t_d)\otimes\sum_{k=-\infty}^{\infty}x(kT_s)\delta(t-kT_s)$$
$$=2BK\sum_{k=-\infty}^{\infty}x(kT_s)\text{sinc}\,2B(t-t_d-kT_s)$$

Reconstructed time-domain signal consists of interpolated sinc-functions

- At the sample instances all but one sinc functions are zero
- Therefore all *band limited* signals can be expressed by using the sinc-series:

$$2BK\sum_{k=-\infty}^{\infty} x(kT_s)\,\text{sinc}\,2B(t-t_d-kT_s)$$



Aliasing and sampling theorem

- Nyqvist sampling theorem:

> If a signal contains no frequency components for $|f| \geq W$ it is **completely** described by instantaneous uniformly spaced time samples having period $T_s \leq 1/2W$. The signal can hence been reconstructed from its samples by an ideal LPF of bandwidth *B* such that $W \leq B \leq f_s - W$.

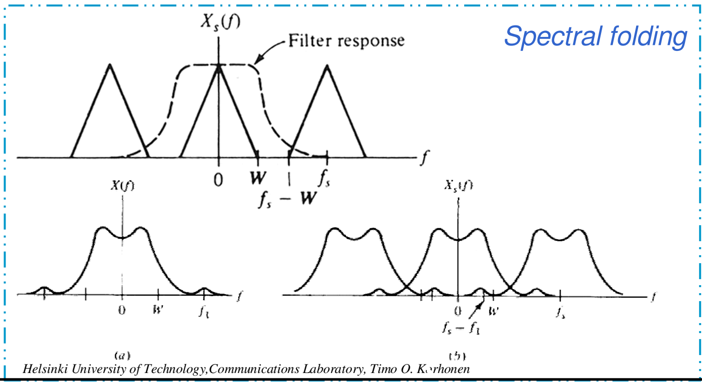
- Note: If the signal contains higher frequencies than twice the sampling frequency they **will also be present** at the sampled signal! An application of this is the sampling oscilloscope (next slide)
- Also, it follows from the sampling theorem that

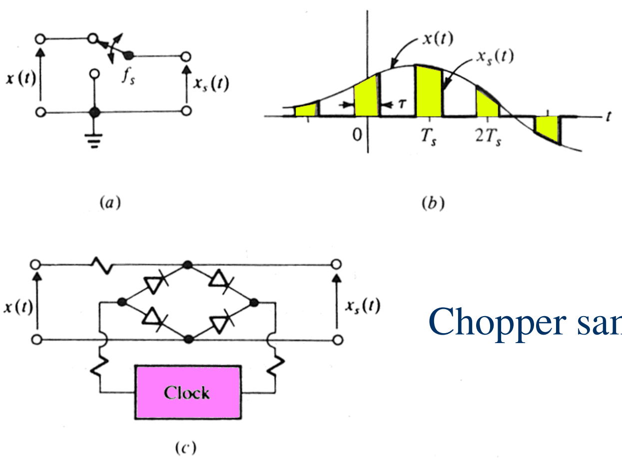
> *Two pieces of <u>independent</u> information / second (independent samples) can be transmitted in 1 Hz wide channel*

because signal having bandwidth B can be constructed from rate 2B independent samples

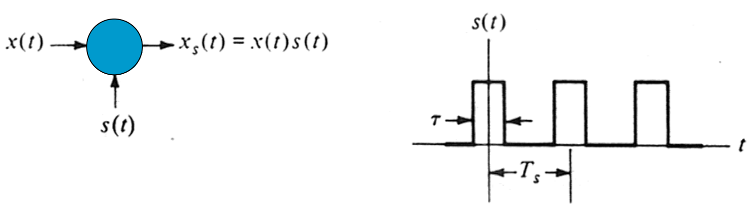
Unperfect reconstruction

1. Sampling wave pulses have finite duration and risetimes
   -> linear distortion
2. Reconstruction filters are not ideal lowpass filters
   -> spectral folding
3. Sampled messages are time limited and therefore their spectra is not frequency limited -> spectral folding
4. Samples digitized by finite length words -> quantization noise





Chopper sampling



The chopper sampler waveforms

- Sampling wave consists of a periodic pulse train whose duration is τ and period is T_o

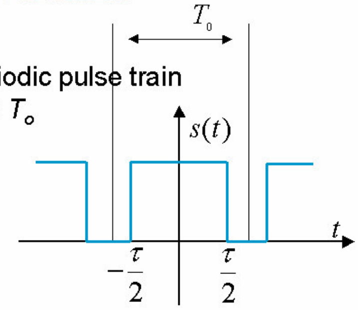
$$c_n = \frac{1}{T_0}\int_{T_0} v(t)\exp(-2\pi t / T_0)dt$$

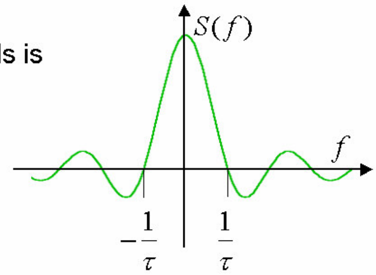
$$c_n = \frac{A\tau}{T_o}\mathrm{sinc}(nf_0\tau)$$

$$s(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_0 t)$$

The Fourier series for real signals is

$$s(t) = c_0 + \sum_{n=1}^{\infty} 2c_n \cos n\omega_s t$$





Chopper sampler: sampled spectra

- Consider the sampled signal from the chopper sampler by term-by-term multiplication

s(t): sampling wave
x(t): signal to be sampled

$$x_s(t) = x(t)s(t)$$
$$= x(t)\left[c_0 + \sum_{n=1}^{\infty} 2c_n \cos n\omega_s t\right]$$

$$x_s(t) = x(t)s(t)$$
$$= x(t)c_0 + 2c_1 x(t)\cos\omega_s t + 2c_2 x(t)\cos 2\omega_s t \ldots$$

Remember the modulation theorem:

$$v(t)\cos(\omega_c t + \phi) \leftrightarrow ½[V(f - f_c)\exp(j\phi) + V(f + f_c)\exp(-j\phi)]$$

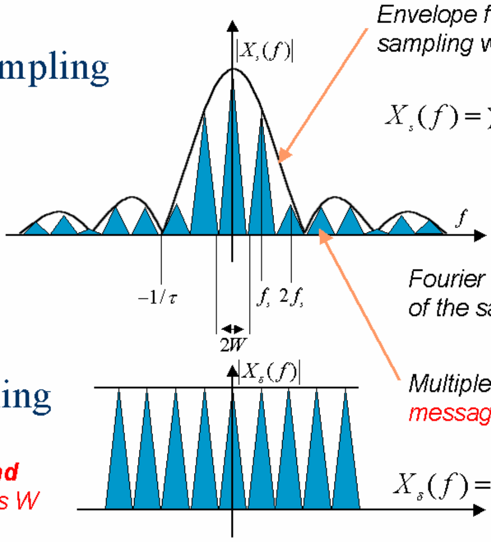
- Therefore the sampled signal is in frequency domain

$$X_s(f) = c_0 X(f) + c_1[X(f - f_s) + X(f + f_s)]$$
$$+ c_2[X(f - 2f_s) + X(f + 2f_s)]\ldots$$

$$s(t) = c_0 + \sum_{n=1}^{\infty} 2c_n \cos n\omega_s t$$

Spectra of ideal sampling and chopper sampling compared

Chopper sampling



Envelope follows sampling waveform spectra

$$X_s(f) = \sum_{n=-\infty}^{\infty} c_n X(f - nf_s)$$

Fourier coefficients of the sampling waveform

Ideal sampling

Multiple copies of the message spectra

Message's ***baseband*** *spectral width equals W*

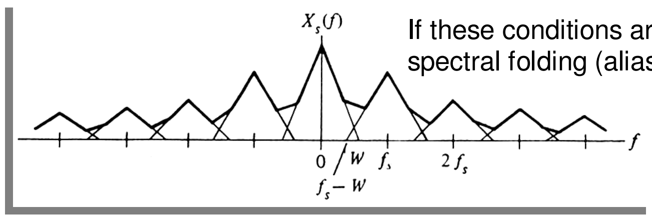
$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

Observations on chopper sampling

- Resulting spectra
  - has the envelope of the sampling waveform
  - has the sampled signal repeated at the integer multiples of the sampling frequency
- Therefore the sampled signal can be reconstructed by filtering provided that

  $X(f) = 0, |f| > W$ Sampled signal is band limited

  $f_s \geq 2W$ Sampling rate is high enough



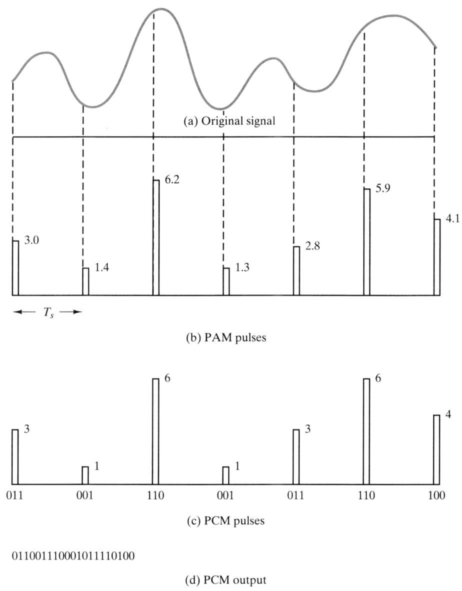
If these conditions are not met, spectral folding (aliasing) results:

Quantization

Analog signal to be transmitted

Continuous PAM-pulse train

Quantized PAM-pulse train

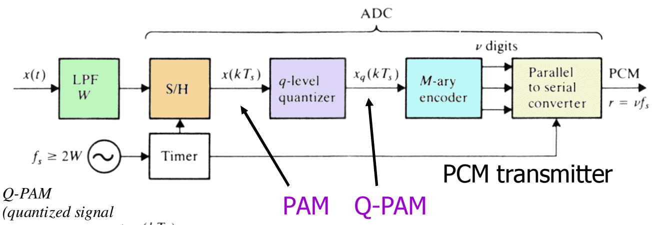


- **Original signal** has continuous amplitudes in its dynamic range
- **PAM - signal** is a discrete constant period, pulse train having continuous amplitude values
- **Quantized PAM** signal has only the values that can be quantized by the words available (here by 3 bit words)

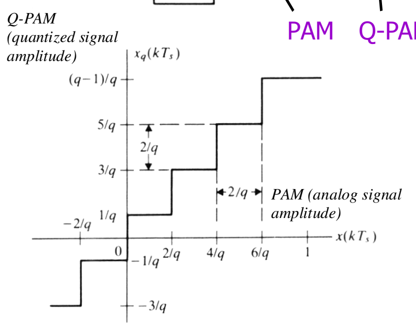
Helsinki University of Technology,Communications Laboratory, Timo O. Korhonen

Uniform quantization: transmitter

- Transforming the continuos samples into discrete level samples is called quantization
- In uniform quantization quantization step size is constant



PCM transmitter



$M = 2, \nu = 8$

$q = M^{\nu} = 256 \ (\nu = \log_M q)$

$2/q \approx 7.8 \cdot 10^{-3}$

q: number of quantization levels

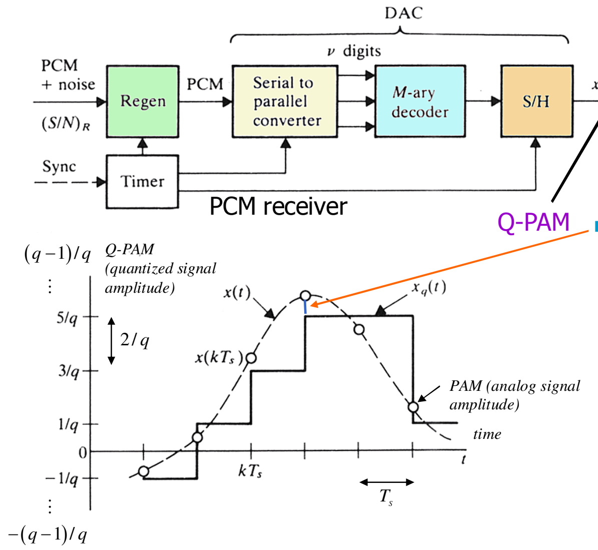
ν: number of quantization bits

ADC: Analog-to-Digital Converter

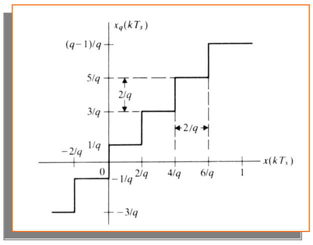
r: output bitrate

18

Reconstruction from the quantized signal



- Note that quantization error amplitude is limited to $|\varepsilon_k| \leq 1/q$



Quantization noise: uniform quantization

- Model the quantized signal by assuming ideal PAM sampling using the quantization error ε_k:

$$y(t) = \sum_k \left[x(kT_s) + \varepsilon_k \right] \delta(t - kT_s)$$

- Quantization error is the difference of the PAM and the Q-PAM signals

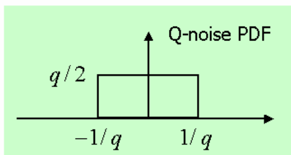
$$\varepsilon_k = x_q(kT_s) - x(kT_s)$$

- The final output is obtained by using the ideal LPF:

$$y_D(t) = x(t) + \sum_k \varepsilon_k \operatorname{sinc}(f_s t - k)$$

- Assuming equal probable signal at all amplitude levels yields for quantization noise average power

$$\overline{\sigma_k^2} = \frac{q}{2} \int_{-1/q}^{1/q} \varepsilon^2 d\varepsilon = \frac{1}{3q^2}$$



note: $\frac{2}{q}\frac{q}{2} = 1$

PCM versus analog modulation

- PCM suppresses wideband noise
- A PCM system well above threshold

$$S_D / N_D = 3q^2 S_x \approx 3M^{2b} S_x, q = M^n$$

$$\text{transmission BW: } B_T = bW$$

$$(S_D / N_D)_{FM} = 3D^2 S_x \gamma$$

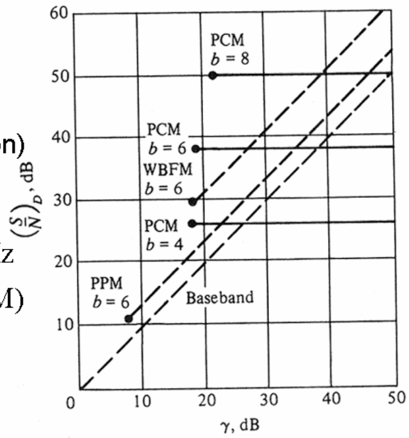
$$\left(\frac{S_D}{N_D}\right)_{PCM} = \frac{3q^2}{1+4q^2 P_e} S_x$$

- Observations:
  - operate PCM near threshold!
- Note that for radio 75 dB dynamic range (12 bits without compression) transmission bandwidth B_T is

$$W = 15\,\text{kHz}, n = 12\,\text{bit}$$

$$B_T \approx nf_s / 2 = 12 \times 35 / 2 = 210\,\text{kHz}$$

(This is about the same as in FM)



- Why to use PCM then??
  - regenerative digital repeaters
  - digital multiplexing
  - etc!

Line coding

- Line codes are used to **enable baseband transmission** in
  - Fiber optic systems
  - Cable transmission
  - Data access and storage
- In PCM-links line coding is used to alleviate clock **synchronization** at the receiver (F-transform of the pulse train should contain spikes that the receiver clock can be synchronized)
- Line codes should
  - be immune to **long strings of zeros** that can lead to missing receiver clock synchronization
  - contain zero long term averaged **DC-component**
  - have minimum **bandwidth**
- Line codes can also be used for **error detection**

Line coding (cont.)

Unipolar [0,A] RZ and NRZ

Polar [-A/2,A/2] RZ and NRZ

Bipolar [-A/2,0,A/2] AMI

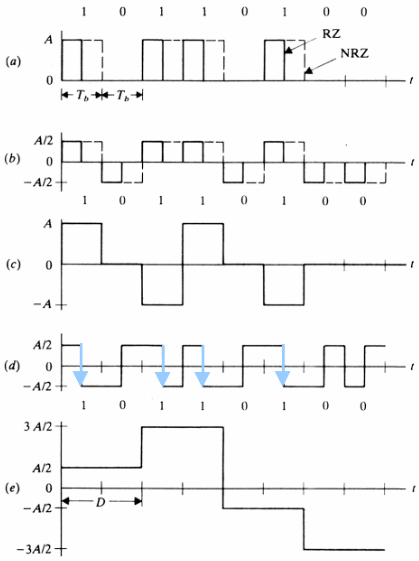
Split-Phase Manchester

Split-Polar quaternary NRZ

Code rate reduced by n

$$M = 2^n$$

$$r_{be} = r/n = r/\log_2 M$$



Line coding methods … (cont.)

Bipolar-AMI
- 0 = no line signal
- 1 = positive or negative level, alternating for successive ones

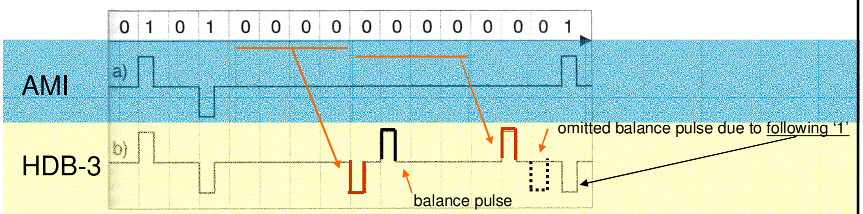
Pseudoternary
- 0 = positive or negative level, alternating for successive zeros
- 1 = no line signal

Manchester
- 0 = transition from high to low in middle of interval
- 1 = transition from low to high in middle of interval

Differential Manchester
- Always a transition in middle of interval
- 0 = transition at beginning of interval
- 1 = no transition at beginning of interval

B8ZS
- Same as bipolar AMI, except that any string of eight zeros is replaced by a string with two code violations

HDB3
- Same as bipolar AMI, except that any string of four zeros is replaced by a string with one code violation



Repeaters

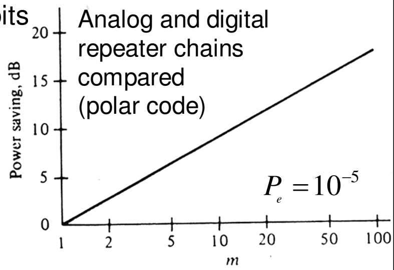
- At the transmission path regenerative repeaters are often used
- At the receiver signal is transformed back to analog form by lowpass filtering removing harmonics produced by sampling
- Repeaters are categorized as:
  - analog repeater: gain equal to the line attenuation between repeaters
  - digital repeater: regenerates bits by decoding and encoding

Error rates for polar, baseband, *m*-stage repeater chains:

$$S_D/N_D \approx \frac{(S/N)_1}{m} \quad \text{analog repeaters*}$$

$$\Rightarrow P_e = Q\left[\sqrt{(S/N)_1/m}\right]$$

$$P_e \approx mQ\left[\sqrt{(S/N)_1}\right] \quad \text{digital repeaters}$$



* Formula follows from the reverse inspection than coherent averaging

Digital Repeater Chain Error Rate

- Assume each repeater has the bit error rate

$$\alpha = Q\left[\sqrt{(S/N)_1}\right]$$

- There are *m* repeaters that introduce errors that are produced for *odd* number of error digits* and the probability of *i* errors in the chain is

$$P(i,m) = \binom{m}{i}\alpha^i(1-\alpha)^{m-i}, i \text{ odd}$$

$$P_e = \binom{m}{1}\alpha(1-\alpha)^{m-1} + \binom{m}{3}\alpha^3(1-\alpha)^{m-3}...$$

$$\approx m\alpha \; (\alpha << 1,\; m \text{ moderate})$$

- And therefore

$$P_e \approx mQ\left[\sqrt{(S/N)_1}\right]$$

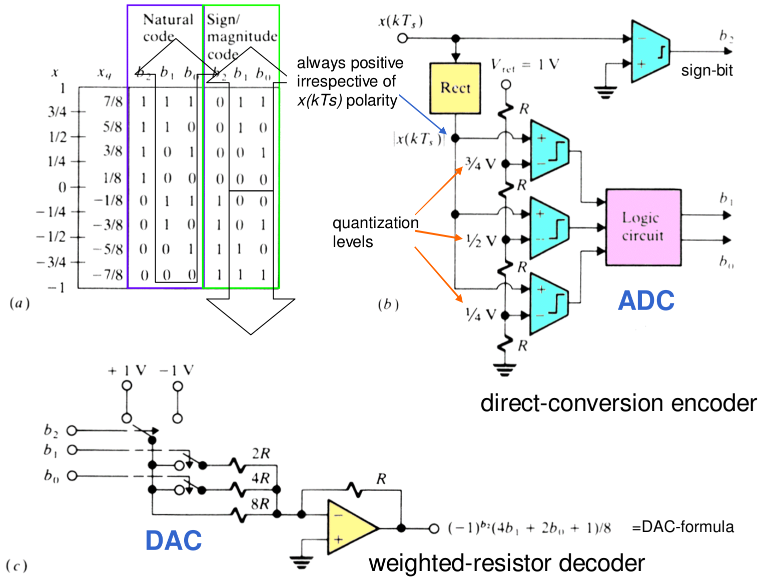
$$\binom{m}{1}\alpha(1-\alpha)^{m-1} \approx \frac{m!}{(m-1)!}\alpha = m\alpha$$

$$\binom{m}{3}\alpha^3(1-\alpha)^{m-3} \approx \frac{m!}{3!(m-3)!}\alpha^3 << m\alpha$$

$$\approx m\alpha \; (\alpha << 1,\; m \text{ moderate})$$

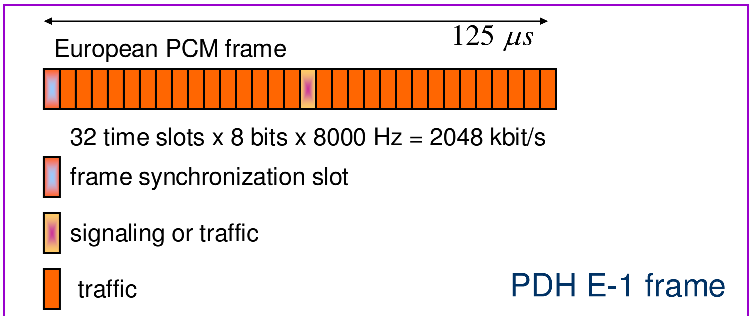
*Even number of errors cancel out each other!

PCM encoding and decoding circuits (n=3)



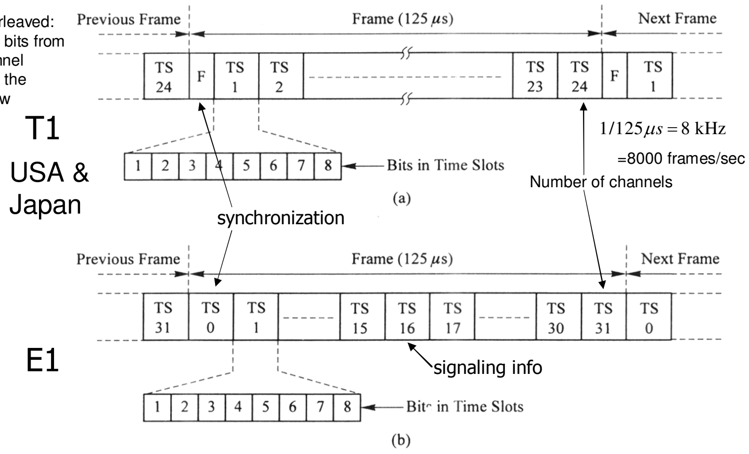
PCM Systems and Digital Time Division Multiplexing (TDM)

- In digital multiplexing several messages are transmitted via same physical channel. For multiplexing 64 kbit/s channels in digital exchanges following three methods are available:
  - **PDH** (plesiochronous digital hierarchy) (the dominant method today, E1 & T1) ('50-'60, G.702)
  - **SONET** (synchronous optical network) ('85)
  - **SDH** (synchronous digital hierarchy) (CCITT '88)



E1 and T1 First Order Frames Compared*

T1 is byte-interleaved: blocks of eight bits from the same channel are inserted to the multiplexed flow



NOTE: In T1 one bit in each time slot in every sixth frame is replaced by signaling information yielding 56 kb/s only

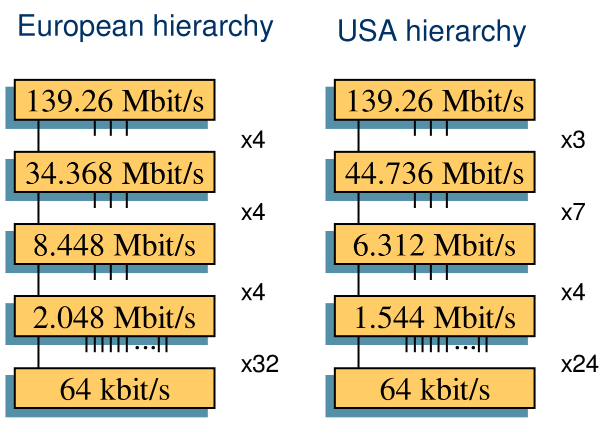
John G. van Bosse: Signaling in Telecommunication Networks

T1 and E1 Summarized

- In PSTN two PCM systems dominate:
  - T1, developed by Bell Laboratories, used in USA & Japan
  - E1, developed by CEPT* used in most of the other countries
- In both data streams divided in frames of 8000 frames/sec
- In T1
  - 24 time-slots and a framing (F) bit serves 24 channels
  - Frame length: 1+ 8x24=193 bits
  - Rate 193x8000 bits/second=**1544 kb/s**
- In E1
  - frame has 32 time-slots, TS 0 holds a synchronization pattern and TS 16 holds signaling information
  - An E1 frame has 32x8=256 bits and its rate us 8000x256=**2048 kb/s**

**European Conference of Postal and Telecommunications Administration*

PCM Hierarchy in PDH



If one wishes to disassemble a tributary from the main flow the main flow must be demultiplexed step by step to the desired main flow level in PDH.

PCM-method summarized

- Analog speech signal is applied into a LP-filter restricting its bandwidth into 3.4 kHz
- Sampling circuit forms a PAM pulse train having rate of 8 kHz
- Samples are quantized into 256 levels that requires a 8 bit-word for each sample (2^8=256).
- Thus a telephone signal requires 8x8 kHz = 64 kHz bandwidth
- The samples are line coded by using the HDB-3 scheme to enable synchronization and channel adaptation
- Usually one transmits several channels simultaneously following a digital hierarchy (as SDH or PDH)
- Transmission link can be an optical fiber, radio link or an electrical cable
- At the receiver the PAM signal is reconstructed where after it is lowpass filtered to yield the original-kind, analog signal