S-72.245 Transmission Methods in Telecommunication Systems
Tutorial 10 SOLUTION

<u>Objectives</u>

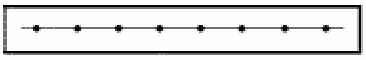
- To investigate coherent and incoherent binary systems and respective error rate analysis
- To identify the respective circuit block diagrams and signaling waveforms

<u>Quizzes</u>

<u>Q10.1</u>
Sketch signal constellations for a) 8-PAM b) 8-QAM c) 8-PSK

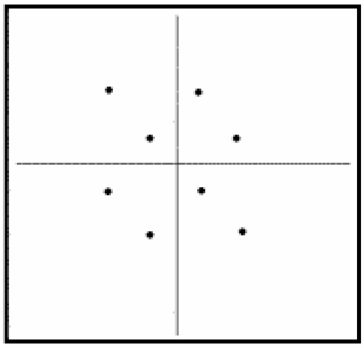
SOLUTIONS

a) 8-PAM is amplitude modulation, and has only one degree of freedom - the amplitude. The signal space is thus *one-dimensional*, and the constellation must be depicted on a line.



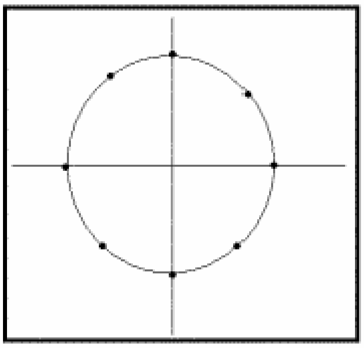
(a)

b) 8-QAM we know is transmitted using two mutually orthogonal channels. On both these channels the only freedom of choice is the transmitted amplitude. Thus we have one-dimensional chances on two mutually orthogonal channels, i.e., we have a two-dimensional signal-space (a plane). The traditional way to depict QAM signal constellations is on a plane whose x-axis represents the *in-phase amplitude* and the y-axis representing the *quadrature-phase amplitude*.



(b)

c) 8PSK means having different phases of a cosine. We thus have one degree of freedom, *the phase of a sinusoidal*. Therefore we could depict 8PSK on a line as in a), but traditionally MPSK signals are depicted on a circle in the iq-plane. Why? Because the angle can easily be depicted on a circle, and the carrier can also be detected using *orthogonal in- and quadrature -phase coherent receivers*.



(c)

Q10.2

Explain by your own words what is a signal constellation diagram?

SOLUTION

Signal constellations are graphical descriptions of the used signal space (set of signal vectors in terms of desired basis). The signal space may be 1, 2, 3 or even N -dimensional. In practice, there is always a combination of modulation errors that may be difficult to separate and identify, as such, it is recommended to evaluate the measured constellation diagrams using mathematical and statistically methods.

Q10.3

Symbol error ratio for 8-PSK in AWGN channel can be determined by

$$P(E) = 2Q\left(\sqrt{\frac{2E_{avg}}{N_0}}\sin\frac{\pi}{8}\right), E_{avg} = A_c^2 D/2 = E_b \log_2 M \ .$$

Decision signal is characterized by A_c=1 and noise rms-value of 97.7 dBμV. What is the respective symbol error rate?

SOLUTION

$$p_\varepsilon = 2Q\left[\frac{A}{\sigma}\sin\left(\frac{\pi}{M}\right)\right] = 2Q\left[\frac{1}{\left(10^{-6}\cdot 10^{\frac{97,7}{20}}\right)}\sin\left(\frac{\pi}{8}\right)\right] \approx 2Q(5) \approx 2\cdot\frac{1}{5\sqrt{2\pi}}e^{\frac{-5^2}{2}} \approx 6,35\cdot 10^{-7}$$

Matlab assignments

M10.1 Design a digital implementation of the transmitter and receiver filters $G_T(f)$ and $G_R(f)$ such that their product satisfies

$$G_T(f)G_R(f) = X_{rc}(f)$$

where $X_{rc}(f)$ is the raised-cosine frequency response characteristic enabling the ISI at the sampling time $t=nT$ to be zero. $G_R(f)$ is the matched filter to $G_T(f)$.

Hints:

1) Raised-cosine frequency response $X_{rc}(f)$:

$$X_{rc}(f)=\begin{cases} T, & 0\le |f|\le \frac{1-\alpha}{2T} \\ \frac{T}{2}\left[1+cos\frac{\pi T}{\alpha}\left(|f|-\frac{1-\alpha}{2T}\right)\right], & \frac{1-\alpha}{2T}<|f|\le\frac{1+\alpha}{2T} \\ 0, & |f|>\frac{1+\alpha}{2T} \end{cases}$$

where 1/T is the symbol rate.

2) The simplest way to design and implement the transmitter and receiver filters in digital form is to employ FIR filters with linear phase (symmetric impulse response).

SOLUTION:

The desired magnitude response is

$$|G_T(f)|=|G_R(f)|=\sqrt{X_{rc}(f)}$$

The frequency response is related to the impulse response of the digital filter by the equation:

$$G_T(f)=\sum_{n=-(N-1)/2}^{(N-1)/2} g_T(n)e^{-j2\pi fnT_s}$$

where T_s is the sampling interval and N is the length of the filter. Note that N is odd. Since $G_T(f)$ is bandlimited, we may select the sampling frequency F_s to be at least $2/T$. Our choice is $F_s=1/T_s=4/T$. Hence, the folding frequency is $F_s/2=2/T$. Since $G_T(f)=\sqrt{X_{rc}(f)}$, we may sample $X_{rc}(f)$ at equally spaced points in frequency, with frequency separation $\Delta f=F_s/N$. Thus, we have

$$\sqrt{X_{rc}(m\Delta f)}=\sqrt{X_{rc}\left(\frac{mF_s}{N}\right)}=\sum_{n=-(N-1)/2}^{(N-1)/2} g_T(n)e^{-j2\pi mn/N}$$

The inverse transform relation is

$$g_T(n)=\sum_{m=-(N-1)/2}^{(N-1)/2}\sqrt{X_{rc}\left(\frac{4m}{NT}\right)}e^{j2\pi mn/N}, \qquad n=0,\pm1,\ldots,\pm\frac{N-1}{2}$$

Since $g_T(n)$ is symmetric, the impulse response of the desired linear phase transmitter filter is obtained by delaying $g_T(n)$ by $(N-1)/2$ samples.

Figure 1(a) illustrates transmitter impulse response, receiver impulse response and their corresponding frequency response characteristics, where $n=0,1,\ldots,N-1$, for $\alpha=\frac{1}{4}$, N = 31 and T=1.

Figure 1(b) the impulse response of the cascade of the transmitter and receiver FIR filters is shown.

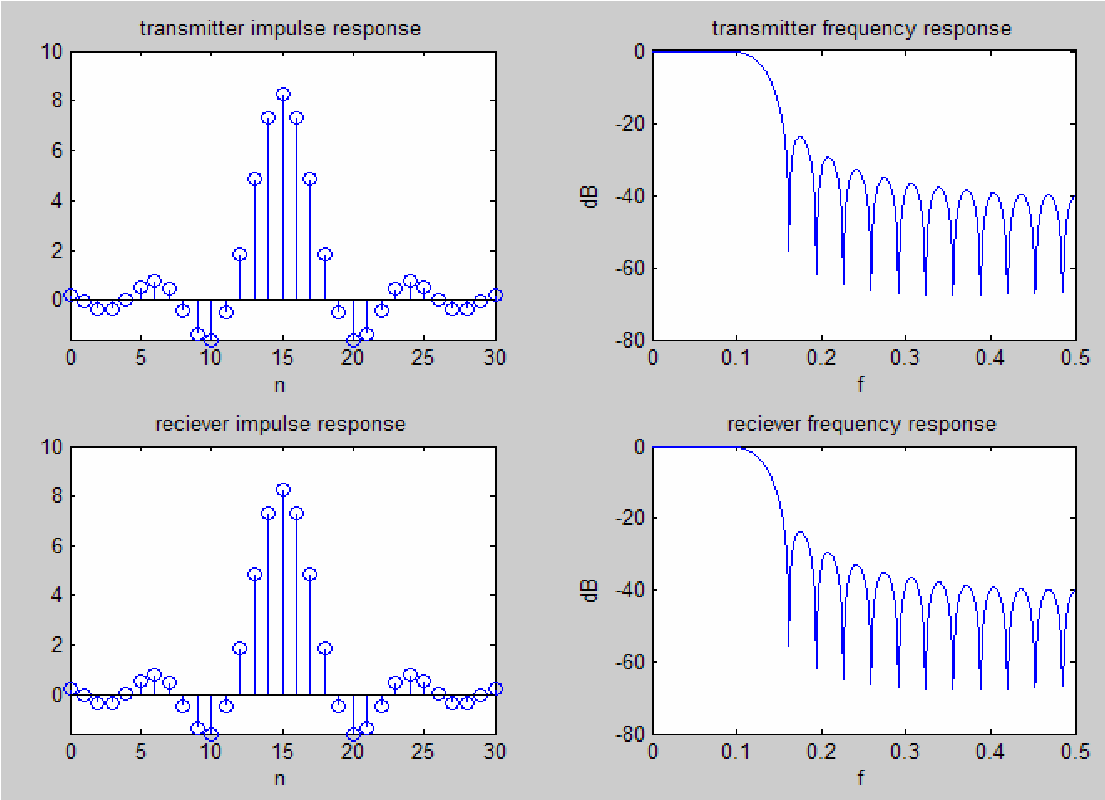


Figure 1(a)

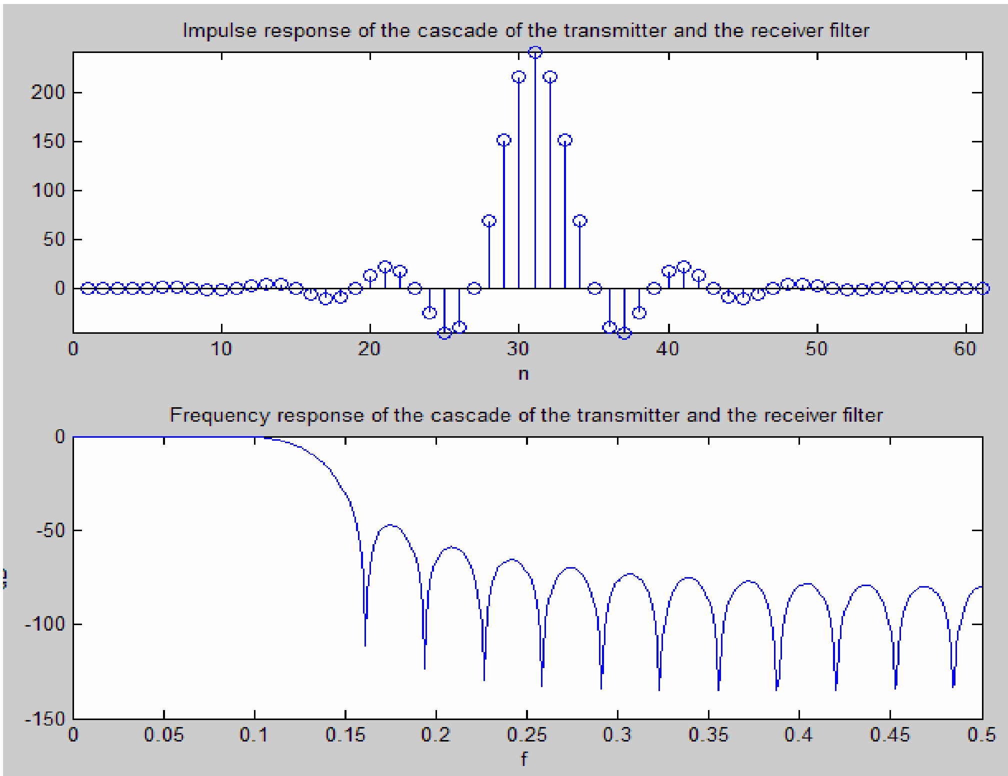


Figure 1(b)

M10.2 Consider a channel-distorted pulse $x(t$, at the input to the equalizer, given by the expression

$$x(t) = \frac{1}{1+(2t/T)^2}$$

where $1/T$ is the symbol rate. The pulse is sampled at the rate $2/T$ and is equalized by a zero-forcing equalizer. Determine the coefficients of a five-tap zero forcing equalizer.

SOLUTION

The zero-forcing equalizer must satisfy the equation:

$$p(mT) = \sum_{n=-2}^{2} c_n x\left(mT - \frac{nT}{2}\right) = \begin{cases} 1, & m=0 \\ 0, & m=\pm1,\pm2 \end{cases}$$

The matrix $\boldsymbol{X}$ with elements $x(mT-nT/2)$ is given as

$$X = \begin{bmatrix} \frac{1}{5} & \frac{1}{10} & \frac{1}{17} & \frac{1}{26} & \frac{1}{37} \\ 1 & \frac{1}{2} & \frac{1}{5} & \frac{1}{10} & \frac{1}{17} \\ \frac{1}{5} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{5} \\ \frac{1}{17} & \frac{1}{10} & \frac{1}{5} & \frac{1}{2} & 1 \\ \frac{1}{37} & \frac{1}{26} & \frac{1}{17} & \frac{1}{10} & \frac{1}{5} \end{bmatrix}$$

The coefficient vector $\boldsymbol{c}$ and the vector $\boldsymbol{p}$ are given as

$$c = \begin{bmatrix} c_{-2} \\ c_{-1} \\ c_0 \\ c_1 \\ c_2 \end{bmatrix} \qquad p = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Then the linear equations $\boldsymbol{Xc=p}$ can be solved by inverting the matrix $\boldsymbol{X}$. Thus, we obtain

$$c_{opt} = X^{-1}p = \begin{bmatrix} -2.2 \\ 4.9 \\ -3 \\ 4.9 \\ -2.2 \end{bmatrix}$$

Figure 2 illustrates the original pulse $x(t)$ and equalized pulse. Note the small amount of residual ISI in the equalized pulse.

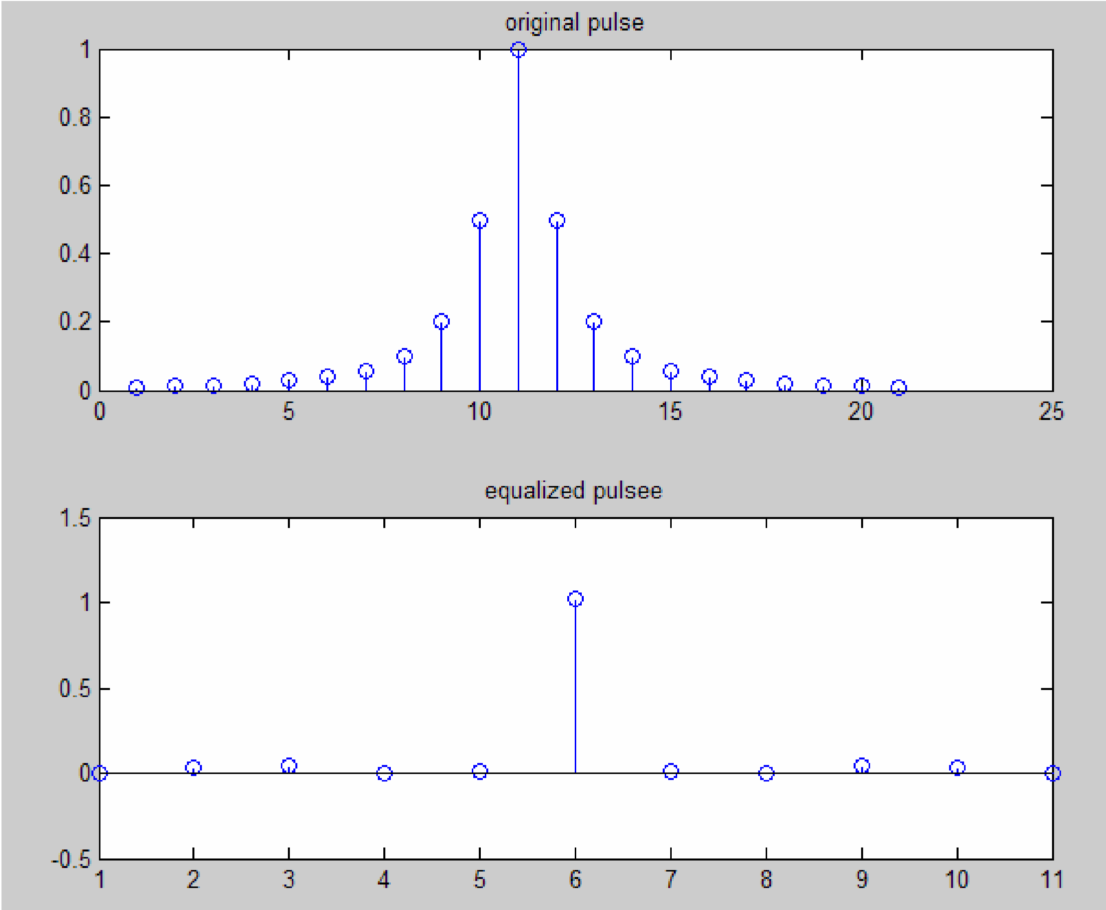

original pulse
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
equalized pulsee
1.5
1
0.5
0
-0.5
1
2
3
4
5
6
7
8
9
10
11

Figure 2