

## S-72.245 Transmission Methods in Telecommunication Systems

### Tutorial 11 Solutions

#### Objectives

- To understand and learn to apply channel coding
- Topics: Hamming distance, parity check and repetition coding, block coding principles and circuits

#### Quizzes

##### Q11.1

Show a table to encode 3-bit words by even parity. Give the expression for respective word error rate probability.

SOLUTION:

3-bit data word	Even parity code word
000	0000
001	0011
010	0101
011	0110
100	1001
101	1010
110	1100
111	1111

The word error rate probability in this 4-bit even-parity is:

$$P(i,4) \approx \binom{4}{i} \alpha^i \quad \alpha \ll 1$$

##### Q11.2

Explain by your own words a) Hamming distance, b) Hard decoding c) Soft decoding

SOLUTION

- a) Hamming distance  $d(X, Y)$  between two vectors  $X$  and  $Y$  is defined to equal the number of different elements.
- b) Hard decoding: calculates Hamming distance to all allowed code words and

- selects the code with the smallest distance.
- c) Soft decoding: calculates Euclidean distance to all allowed code words and selects the code with the smallest distance.

### Q11.3

A block code consists of the following codes: 10011, 11101, 01110, 00000. a) How many errors can be detected/corrected by this code? b) Is this a linear code?

### SOLUTION

a)

$$d_{\min} = 3$$

$$l = d_{\min} - 1 = 2$$

$$t = \lfloor l/2 \rfloor = 1$$

- b) This is a linear code, because two conditions are satisfied:  
It includes the all-zero vector  
The sum of any two code vectors produces another vector in the code

### Q11.4

What kind of channels interleaving should be used? Why?

### SOLUTION

Interleaving is used in fading multipath channels.

Burst errors occurred in fading channels can destroy several successive bits. Conventional error control methods are designed for errors that are isolated or statistically independent events. Interleaving can be used to make bits more independent and to distribute burst errors along data stream. When received, the bit sequence is reordered to its original form and FEC can correct the distributed errors.

### Q11.5

Create (5, 2) Hamming code, determine its code words and determine based on them its Hamming distance thus proving it is indeed a Hamming code.

### SOLUTION

Hamming codes are defined by:

$$n = 2^q - 1, q = 2, 3, \dots \quad k = n - q \quad d_{\min} = 3$$

Let  $q=3$ , so  $n = 2^q - 1 = 7, k = n - q = 4$

The generator matrix is

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$\underbrace{\hspace{2em}}_I \qquad \underbrace{\hspace{2em}}_P$

<b>M</b>	<b>C</b>	<b>d</b>
0 0	0 0 0	0
0 1	0 1 1	3
1 0	1 1 0	3
1 1	1 0 1	4

Table1. Encode for (5, 2) Hamming code

It is a Hamming code, because it includes the all-zero vector, it is a linear code and

$$d_{\min} = 3$$

Matlab assignments

M11.1 Investigate functions available in Matlab for GF(2) calculus and repeat problem Q.10.5 by using Matlab.

SOLUTION

Assume message polynomial  $m(x) = m_0 + m_1x$

code polynomial  $c(x) = m(x)g(x) = m_0g(x) + m_1xg(x) = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4$

where  $g(x) = 1 + x^2 + x^3$

Output of Matlab program:

ans =

message vector

m =

```

0    0
0    1
1    0
1    1

```

ans =

code vector

$c =$

0	0	0	0	0
0	1	0	1	1
1	0	1	1	0
1	1	1	0	1

M11.2 Design syndrome decoder for Q.10.5 by using Matlab.

SOLUTION

[P564-P566 in A.B.Carlson]

$$H = [P^T \ I] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$S = EH^T$$

Compute the syndromes and set up syndrome decoding table.

$S$	$E$
000	00000
110	10000
011	01000
100	00100
010	00010
001	00001

Table1. Syndromes decoding table for (5,2) Hamming code

$$Y = R+E$$

The following are all possible received vectors  $R$  and their corresponding decoder output vectors  $Y$ .

$R =$

$Y =$

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0
0	0	0	1	1	0	0	1	0	1	1
0	0	1	0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	1	0	1
0	0	1	1	0	1	1	0	1	1	0
0	0	1	1	1	0	0	0	1	1	1
0	1	0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	1	0	0	1	1
0	1	0	1	0	0	0	1	0	1	1

