S-72.311 ADVANCED ERROR CONTROL SCHEMES

Home assignment #3, submission date: March 27th 2001

1. Convolutional codes.

   I Consider the R=2/3 convolutional encoding matrix

   $$G(D) = \begin{bmatrix} 1+D & D & 1+D \\ 1 & 1 & D \end{bmatrix}$$

   Find the extended Transfer function T(D,L,I).

   II Find the distance profile and the free distance for the rate R=2/3 convolutional code with encoding matrix

   (a) $G_1(D) = \begin{bmatrix} 1+D & D & 1 \\ D^2 & 1 & 1+D+D^2 \end{bmatrix}$

   (b) $G_2(D) = \begin{bmatrix} 1+D & D & 1 \\ 1+D^2+D^3 & 1+D+D^2+D^3 & 0 \end{bmatrix}$

   (c) Show that $G_1(D)$ and $G_2(D)$ encode the same code.

   III Consider a BSC with error sequence **e**=01 01 10 01 00 00…. Decode the received sequence for the following cases. *Use a List decoder with L=3.*

   (a) Systematic convolutional encoding matrix
   $G(D)=(1 \quad 1+D+D^2+D^4)$, $d_{free}=5$.

   (b) Systematic convolutional encoding matrix
   $G(D)=(1 \quad 1+D+D^2+D^5+D^6+D^8+D^{10}+D^{11})$, $d_{free}=9$.

- **Note**: List decoding is a nonbacktracking breadth-first search of the code tree. At each depth, only the **L** most promising subpaths are extended, not all, as in the case with Viterbi decoding. These subpaths form a *list* of size **L** (needless to say, starting at the root, all subpaths are extended until we have obtained **L** or more subpaths). Since the search is breadth-first, all subpaths of the tree are of the same length; finding the **L** best extensions reduces to choosing the **L** extensions with the largest values of the Viterbi metric.
- **Algorithm-List Decoding**
- **LD1**.Load the list with the root and metric zero; set depth $t = 0$.
- **LD2**.Extend all stored subpaths to depth $t+1$ and place the **L** best (largest Viterbi metric) of their extensions on the list.

- **LD3**.If we have reached the end, then stop and choose as the decoded codeword a path to the terminating node with the largest Viterbi metric; otherwise increment *t* by *1* and go to **LD2**.

2. Punctured Convolutional Codes

Puncturing a given convolutional code is a method of constructing new convolutional codes with rates that are higher than the rate of the original code. The punctured codes are in general less powerful than nonpunctured codes of the same rate and memory, but they have two advantages.

From a given original low-rate convolutional code we can obtain a series of convolutional codes with successively higher rates. They can be decoded by the Viterbi algorithm with essentially the same structure as that for the original code.

A *puncturing sequence* is a binary sequence; 0 and 1 means that the corresponding code symbol is not transmitted and transmitted, respectively. This is illustrated in the following little example:

11	10	00	10	11	00	original code symbols
11	10	01	11	10	01	puncturing sequence
11	1x	x0	10	1x	x0	punctured code symbols –x-not transmitted

We have used the periodic sequence $[11\ 10\ 01]^{\infty}$ as puncturing sequence, where $[.....]^{\infty}$ denotes a semi-infinite sequence that starts at time 0 and that consists of an infinite repetition of the sub sequence between the square brackets.

Suppose that this puncturing sequence is used together with the encoding matrix $G(D) = (1 + D^2 \quad 1 + D + D^2)$ to communicate over a BSC with cross over probability ε.

(a) Find the rate of the punctured code.

(b) Use the Viterbi algorithm to decode **r** = 01010101. Consider a terminated trellis.

3. Trellis Coded Modulation(TCM)

(a) *Decoding:* Suppose that we operate with the 4-state , R=2/3 coded 8-PSK ungerboeck design. Assume that the encoder begins at state 0 and ends in state 0 after the transmission of 10 bits (5 trellis levels). We receive the complex number sequence

$\mathbf{r}$ = ($1.1e^{j80}$, $0.9e^{-j30}$, $1.2e^{j160}$, $0.7e^{j60}$, $1.1e^{j200}$). All angles are in degrees.

What is the MLS estimate?

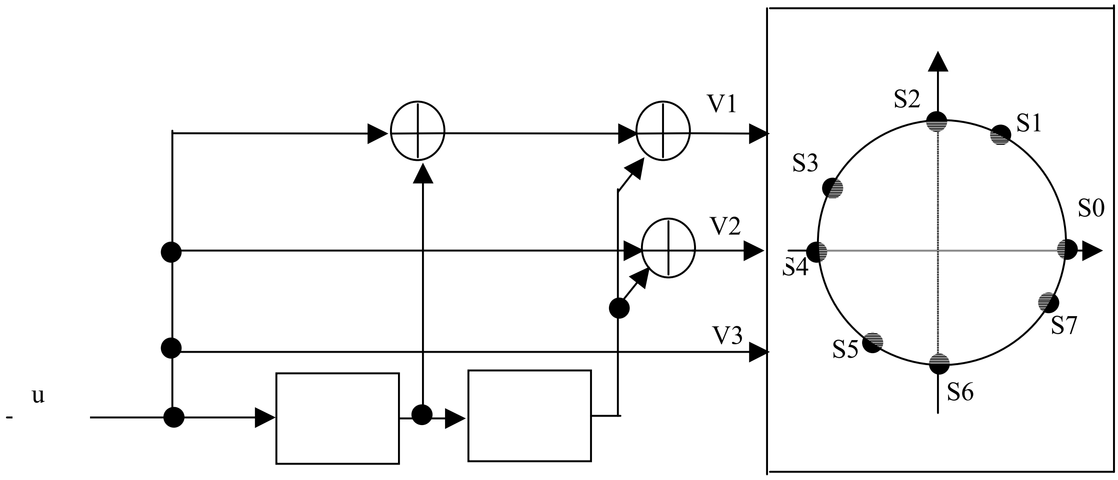
(b) Consider the set partitioning of the 32-point cross constellation. Show the partition sequence until subsets of size 4 are produced.

Suppose that we wish to transmit k=4 bits per modulator interval with a 4-state code. To provide the 16 branches per state, adopt two-sets-of –eight branching so that each parallel transition amounts to labeling with a subset of size 8. Use Ungerboeck's rules to label the 4-state trellis. Here there are single-step, three-step, and four-step, and so on, error events. Find the minimum distance of the single (parallel) error events and the smallest three-step error event. What is the free distance of the code.

(c) If we wish an 8-state code with the same parameters as in (b), what change in structure is required to increase the free distance?

4. Trellis Coded Modulation(TCM)

Consider the following TCM scheme.



The constellation (s_1,s_3,s_5,s_7) is rotated +60 degrees compared to the constellation (s_0,s_2,s_4,s_6).

(a) Which of the following three mappings {m_a,m_b,m_c} from v=(v_1 v_2 v_3) to s_i is best?
(b) Assume the best mapping in (a) is used. What is the asymptotic coding gain for the scheme?

	v_1 v_2 v_3		
m_a	**m_b**	**m_c**	s_i
111	**111**	**111**	**s_0**
010	**010**	**110**	**s_1**
101	**001**	**001**	**s_2**
110	**011**	**011**	**s_3**
000	**000**	**000**	**s_4**
001	**101**	**101**	**s_5**
011	**110**	**010**	**s_6**
100	**100**	**100**	**s_7**

5. MAP Decoding

Consider the following rate R=1/2 non systematic convolutional encoder

$$G(D) = (\ 1+D+D^2 \qquad 1+D^2\ ).$$

This is used for transmission over a binary input, 4-ary output DMC with transition probabilities shown below.

Use MAP algorithm to decode **r** = $0_1 1_1\ \ 1_2 0_1\ \ 1_1 0_2\ \ 0_1 1_1\ \ 1_2 1_1\ \ 1_1 1_2$ for the following three cases.

Assume that the trellis was terminated in the last bits.

The information symbols a *priori* are

(a) equiprobable.
(b) P(u=0)=2/3 and
(c) P(u=0)=1/3 respectively.