

## S-72.311 ADVANCED ERROR CONTROL SCHEMES

Home assignment #4, submission date: April 3<sup>rd</sup> 2001

### 1. TCM bounds (TCM text)

Derive the expression for the *Chernoff upperbound* of the pairwise error probability for a TCM scheme in a Rician flat fading channel *giving all the details*. The fading coefficients for different time intervals are assumed to be independent.

### 2. TCM Design

Consider 16-PSK 4-state trellis coded modulation scheme.

- a) Perform the set partitioning of the 16-PSK constellations.
- b) Construct the labeled trellis diagram for the above TCM scheme (assuming 4 parallel transitions in each branch).
- c) Determine the asymptotic coding gain of the TCM scheme relative to uncoded 8-PSK system.
- d) Find the Calderbank-Mazo realization of the system.
- e) Using the results obtain in section (d) find the Ungerboeck realization of the system.  
(For part b) Keep the Ungerboeck rules in mind and try to get a symmetrical trellis diagram.

### 3. MTCM Design

Consider 8-PSK constellation.

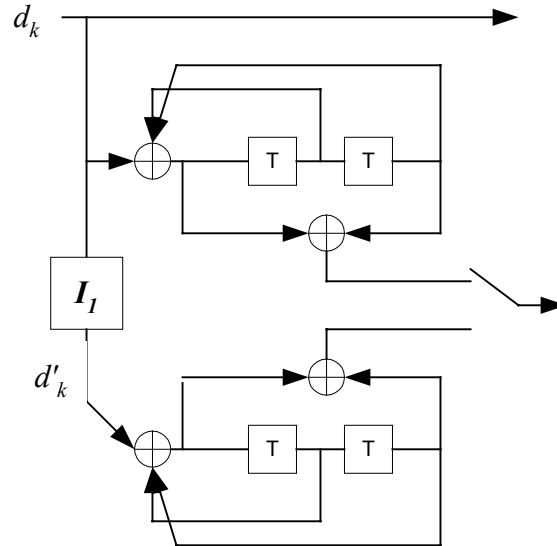
- a) Perform the set partitioning using Ungerboeck Rule.
- b) Construct the labeled trellis diagram for 8 PSK 4 state MTCM scheme for  $n=2$  and  $k=2$  (assume 8 transitions paths in each branch).
- c) Find the asymptotic coding gain.

### 4. Space time processing

- (a) Consider the design of 2-space time codes based on QPSK with 4 and 8 states using two transmit and one receive antenna.  
Prove that in each case the code achieves the required diversity advantage -clearly giving details for at least one case.
- (b) Use four transmit antennas and one receive antenna to design a QPSK STCM system. Give the output equations. What diversity advantage you'd expect to achieve? Justify.

## 5. Iterative Decoding- Max-Log-MAP algorithm

Consider the following encoder. Carry out iterative decoding using the information given for at least two iterations. Writing a MATLAB program may simplify things considerably.



**Figure B.1** Turbo encoder with RSC (7,5)

- Assume data sequence  $\{d_k\} = \{1,0,0,1,0,1,0,0,1\}$
- The last 2 bits are not the information bits, but the bits inserted in order to terminate the trellis of the RSC1.
- $I_1$  is a  $3 \times 3$  block interleaver.

So that the interleaved sequence  $\{d'_k\}$  is  $\{1,1,0,0,0,0,0,1,1\}$ .

*Verify that the punctured sequence is given by*

(Assuming that the puncturing function initially chooses  $c_k^1$ , the punctured sequence  $c_k^{2x}$ ,  $x = 1,2$ )

$$\{c_k^{2x}\} = \{1,0,1,1,0,0,1,0,1\}.$$

- Now, the output coded bit sequence consists of  $c_k^1, c_k^{2x}$  at time  $k$ . They will be modulated using BPSK into  $x_k^1, x_k^{2x}$  with the relation:

$$\{x_k^1, x_k^{2x}\} = \{2c_k^1 - 1, 2c_k^{2x} - 1\}.$$

Next, for a discrete memoryless Gaussian channel, noise  $n_k^1, n_k^{2x}$  are being added to the transmitted symbol  $x_k^1, x_k^{2x}$  at time  $k$ .  $n_k^1$  and  $n_k^{2x}$  are two independent noise with the same variance  $\sigma$ . Here, we specify  $\sigma^2 = \mathbf{0.89125}$  and hence:

$$\{n_k^1\} = \{1.186691, 0.992661, -1.96959, 1.09031, -1.24656, 0.163554, 2.33662, 2.83439, 0.43984\}$$

$$\{n_k^{2x}\} = \{0.0795, 0.463952, -0.712115, -1.10892, 2.2841, -2.373092, 0.752278, -0.224861, -0.977405\}$$

For each time instant  $k$ , the receiver input consists of  $R_k^1, R_k^{2x}$  which can be stated as:

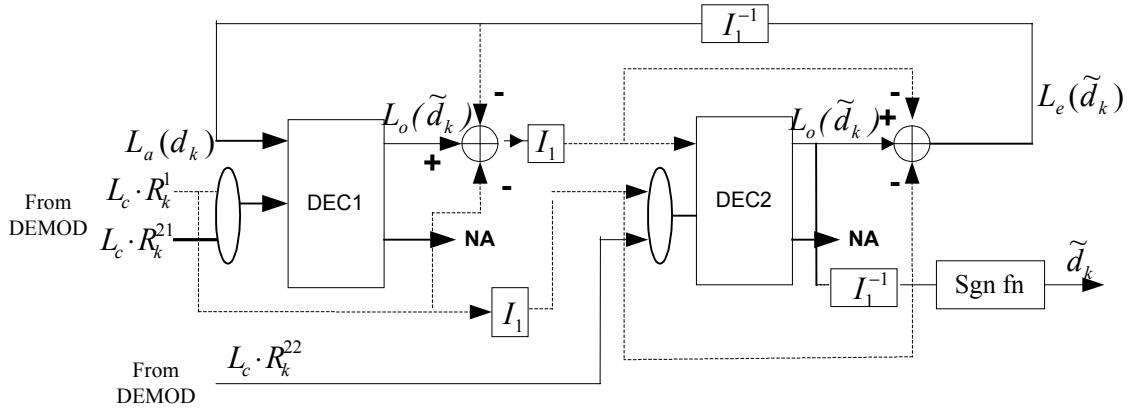
$$\begin{aligned} R_k^1 &= x_k^1 + n_k^1 \\ R_k^{2x} &= x_k^{2x} + n_k^{2x} \end{aligned}$$

*check that the values,  $R_k^1, R_k^{2x}$  can be obtained as:*

$$\{R_k^1\} = \{2.18669, -0.00733882, -2.44162, 2.09031, -2.24656, 1.16355, 1.33662, 1.83439, 1.43984\}$$

$$\{R_k^{2x}\} = \{2.0795, -0.536048, 0.287885, -0.108924, 1.2841, -3.37309, -1.22486, -1.4892, 0.0225949\}$$

You may use the following decoder configuration.



**Figure B.2** Iterative Decoding