# Turbo Codes with High Spectral Efficiency

- In the previous lectures, the basic concepts of turbo codes were introduced and the performance of turbo codes was investigated.

- However, these discussions and observations were only for binary modulation over AWGN channel.

- With the remarkable performance of turbo codes, it is worthwhile to combine turbo codes with multilevel modulation in order to obtain large coding gains and high bandwidth efficiency over both AWGN and fading channels.

- The first attempt in combining turbo codes with multilevel modulation was described in (**Goff et al.,** 1994) and is called "*pragmatic*" approach to TCM.

- In this approach a Gray mapper is used after binary turbo encoder for multilevel modulation. *The coding and modulation are separate processes and hence it is actually not a coded-modulation scheme*. Decoding relies on binary turbo decoder, hence the term "pragmatic".

- In the second approach (**Robertson and Woerz,** 1995) binary RSC component codes in binary turbo code are replaced by Ungerboeck TCM codes to retain the advantages of both classical turbo code and TCM code.

- In order to do that certain special conditions need to be met at the encoder and the iterative decoder needs to be adapted to the decoding of the component Ungerboeck codes. *This approach is known as turbo TCM (T-TCM).*

- Another approach was proposed by (**Benedetto et al.,** 1995). Their approach is slightly different from the second one and is termed *parallel concatenated trellis coded-modulation (PCTCM).* They also proposed another scheme based on serial concatenation termed as *serial concatenated trellis coded-modulation (SCTCM).*

- In the subsequent sections, the first two approaches for combining turbo code with multilevel modulation are discussed, then simulation results will be given and the comparison of different approaches will be made.

## Pragmatic Approach

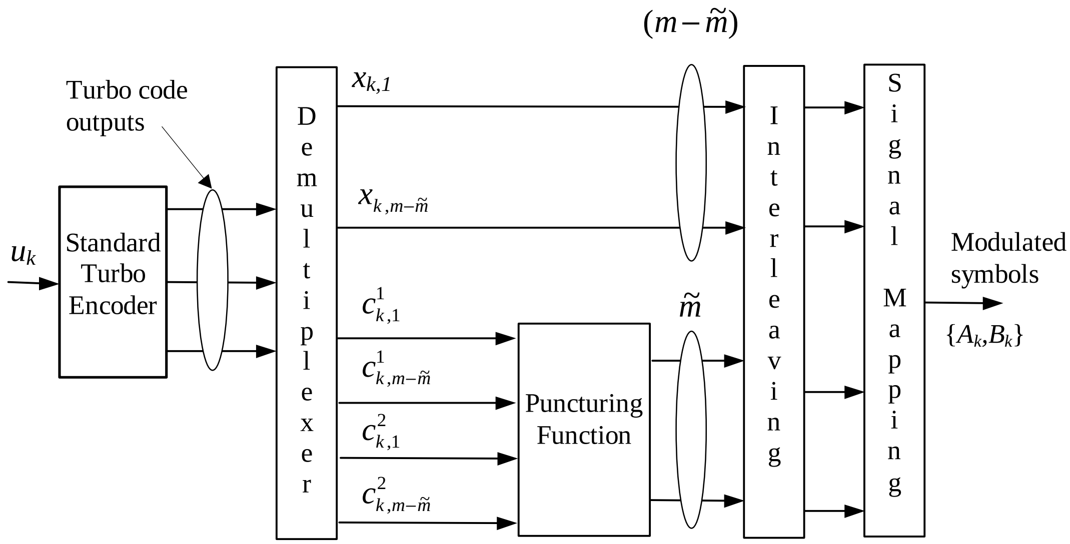


**Fig. 1.** Association of turbo codes with multilevel modulations (**Goff et al.,** 1994)

- Figure 1 shows the association of a binary turbo code with *M*-level modulation (MPSK, *M*-level QAM). The standard turbo code uses two rate-1/2 RSC codes as constituent codes.

- The parity check bits at the output of constituent codes are denoted as $c^1$ and $c^2$ respectively. The puncturing function is inserted at the output of the standard turbo code and thus it is possible to obtain a large code family with various rates $R = (m - \tilde{m})/m$.

- In addition to the interleaver $I_1$ inside the turbo encoder, another interleaver $I_2$ is inserted between the puncturing function and the modulator in order to obtain symbols affected with independent noise at the turbo decoder input. It was mentioned in (**Goff et al.,** 1994) that this second interleaver $I_2$ is only necessary when the symbols are transmitted over fading channel.

- At any time $k$ a set $\{u_{k,i}\}$ ($i$=1, ..., $m$) of $m$ bits is *Gray* mapped into a complex signal symbol $s_k^j$ ($j$=1, ..., $M$) to be transmitted over the channel. Each symbol

$s_k^j$ is represented by couple of real-valued symbols $\{A_k^j, B_k^j\}$. The redundant bits provided by the turbo encoder are always associated to the highest protected bits $u_{k,i}$.

- The receiver for the above association scheme is shown in Fig. 2.

- The correlation demodulator (or matched filter demodulator) produces the output of noisy symbol $r_k$ . Each noisy symbol $r_k$ consists of in-phase and quadrature components $X_k$ and $Y_k$ respectively and contains all the sufficient information in the received signal waveform.

- Based on the observation of $r_k$ the log-likelihood value associated with each bit $u_{k,i}$, $i$=1, ..., $m$ can be determined and then used as a relevant soft information by the binary turbo decoder:

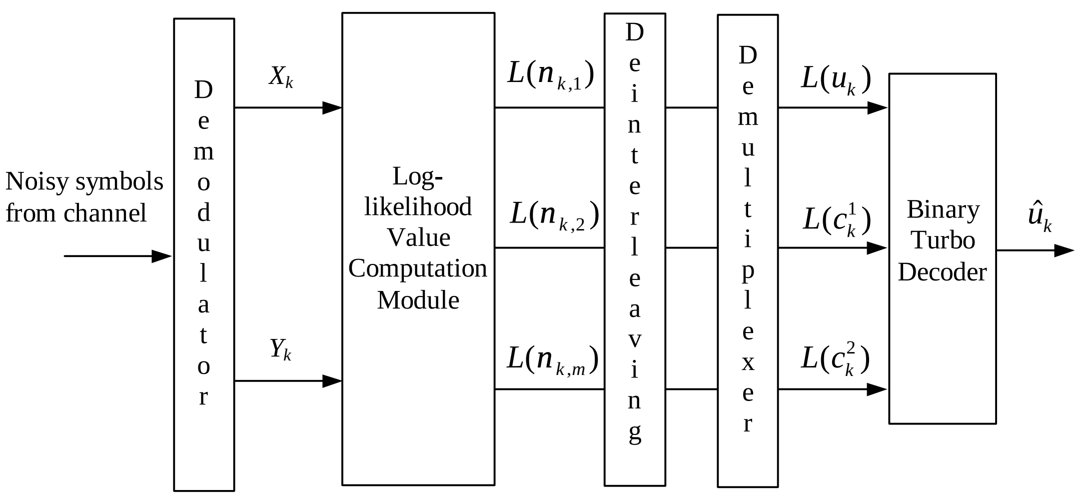


**Fig. 2** Decoder structure in pragmatic approach (**Goff et al.,** 1994)

$$L(u_{k,i}) = Log \frac{P(u_{k,i} = 1 \mid r_k)}{P(u_{k,i} = 0 \mid r_k)} \tag{1}$$

$$= Log \frac{P(u_{k,i} = 1 \mid X_k, Y_k)}{P(u_{k,i} = 0 \mid X_k, Y_k)} \quad i = 1, \ldots, m$$

- The computation of $m$ values $L(u_{k,i})$ for the case of square $M$-QAM constellation with $M=2^m$ and $m$ even, was described in (**Goff et al.,** 1994). Also a good approximation of these values was proposed.

- Here we will describe a general method to calculate the log-likelihood values $L(u_{k,i})$ from the observations $X_k$ and $Y_k$ and use that calculation in simulating the combination of turbo codes with MPSK modulation over AWGN channel.

- Define the *a posteriori* probability of transmitted symbol at every time instant $k$ as,

$$P(s_k^j \mid r_k) = P(\text{ signal was transmitted} \mid r_k), \; j=1,2,\ldots, M \tag{2}$$

where $s_k^j$ denotes one of $M$ possible transmitted symbols at time instant $k$. Using Bayes' rule, the *a posteriori* probability may be expressed as,

$$P(s_k^j \mid r_k) = \frac{p(r_k \mid s_k^j) P(s_k^j)}{p(r_k)} \tag{3}$$

where $p(r_k \mid s_k^j)$ is the conditional pdf of the observed signal given $s_k^j$, and $P(s_k^j)$ is the *a priori probability* of the $j$th signal being transmitted. The denominator of (3) can be expressed as,

$$p(r_k) = \sum_{j=1}^{M} p(r_k \mid s_k^j) P(s_k^j) \qquad (4)$$

- From (3) and (4) it can be seen that the computation of the posteriori probability $P(s_k^j \mid r_k)$ requires knowledge of the *a priori* probability $P(s_k^j)$ and the conditional pdfs $p(r_k \mid s_k^j)$ for $j$=1, ..., $M$.

- *When the M signals are equally likely* $P(s_k^j)$=1/$M$ for all $M$. The log-likelihood values $L(u_{k,i})$ now can be calculated from the *a posteriori probability* of transmitted symbol as,

$$L(u_{k,i}) = Log\,\frac{P(u_{k,i}=1 \mid r_k)}{P(u_{k,i}=0 \mid r_k)} \qquad (5)$$

$$= Log\,\frac{\sum_{u_{k,i}=1} P(s_k^j \mid r_k)}{\sum_{u_{k,i}=0} P(s_k^j \mid r_k)} \qquad \begin{cases} i = 1,\ldots,m \\ j = 1,\ldots,M \end{cases}$$

- where the summation in the numerator and denominator is taken over all $m$ symbol $s_k$ that has the $i^{th}$ bit equal to 1 and 0 respectively. Note that the denominator in (3) when calculating the posteriori probability of the transmitted symbol is independent of which signal is transmitted.

- Furthermore when assuming equally probable transmitted symbol (5) can be rewritten as,

$$L(u_{k,i}) = Log \frac{\sum_{u_{k,i}=1} p(r_k | s_k^j)}{\sum_{u_{k,i}=0} p(r_k | s_k^j)} \qquad (6)$$

$$= Log\left( \sum_{u_{k,i}=1} p(r_k | s_k^j) \right) - Log\left( \sum_{u_{k,i}=0} p(r_k | s_k^j) \right),$$

- The conditional pdf $p(r_k | s_k^j)$, usually called the *likelihood function* for signal transmitted over AWGN channel is calculated as,

$$p(r_k | s_k^j) = p(X_k, Y_k | A_k^j, B_k^j)$$

$$= \frac{1}{\sqrt{2p}\, s_N} \exp\left[ -\frac{\left| r_k - s_k^j \right|^2}{2 s_N^2} \right] \qquad (7)$$

$$= \frac{1}{\sqrt{2p}\, s_N} \exp\left[ -\frac{\left(X_k - A_k^j\right)^2 + \left(Y_k - B_k^j\right)^2}{2 s_N^2} \right]$$

- where $s_N^2$ is the noise variance and |.| denotes the magnitude calculation operation.

- Then the *log-likelihood function* is

$$Logp(r_k \left| s_k^j \right.) = -\frac{1}{2} Log(2ps_N^2) - \left[ \frac{\left(X_k - A_k^j\right)^2 + \left(Y_k - B_k^j\right)^2}{2s_N^2} \right] \tag{8}$$

For high SNR, we can use the following expression when evaluating (6)

$$Log(e^{d_1} + e^{d_2} + \ldots + e^{d_n}) \approx \max_{i \in \{1,\ldots,n\}} d_i \tag{9}$$

Then $L(u_{k,i})$ in (6) can be approximated as,

$$\begin{aligned} L(u_{k,i}) &= \max_{u_{k,i}=1} \left( Logp(r_k \left| s_k^j \right.) \right) - \max_{u_{k,i}=0} \left( Logp(r_k \left| s_k^j \right.) \right) \\ &= \max_{u_{k,i}=1} \left[ \frac{\left(X_k - A_k^j\right)^2 + \left(Y_k - B_k^j\right)^2}{2s_N^2} \right] \\ &- \max_{u_{k,i}=0} \left[ \frac{\left(X_k - A_k^j\right)^2 + \left(Y_k - B_k^j\right)^2}{2s_N^2} \right], \quad \begin{cases} i = 1,\ldots,m \\ j = 1,\ldots,M \end{cases} \end{aligned} \tag{10}$$

- In the following, simulation results when combining turbo codes with 8-PSK modulation scheme are presented. Calculation of log-likelihood value for each coded bit from modulated symbol follows the derivation above.

- *Figures 3 to 5 plot the error performance of different turbo codes combined with 8-PSK modulation scheme over AWGN channel.*

- Turbo codes are constructed using a *random* interleaver and different RSC codes with constraint $K$=3, 4, 5 and generator matrix ($G_1$=7, $G_2$=5), ($G_1$=15, $G_2$=17), ($G_1$=37, $G_2$=21) respectively.

- Simulations were carried out with 8 *iterations* and error probabilities were calculated after each iteration. The block length of bit sequence is 2400 (hence the symbol length is 1200) in all simulations.

- From these figures one can draw very similar conclusions with the performance of binary codes: a large coding gain is obtained after the first two iterations, coding gain then decreases when the number of iterations increases. After about 6 iterations the increment of coding gain becomes marginal.

- Figure 6 compares the performance of conventional 8-state 8-PSK TCM scheme with pragmatic approach based 8-PSK schemes.

- Both schemes have 2bits/sec/Hz spectral efficiency, and performance of pragmatic 8-PSK scheme is evaluated after 6 iterations. Comparison then shows the superiority of pragmatic approach over conventional TCM. At BER level of $10^{-3}$ coding gain from 0.6 to 0.9 dB is achieved for different turbo codes.

- More coding gain is possible at a lower level of BER. However the price paid for the improvement in performance of pragmatic approach over conventional TCM is the significantly more complexity of iterative decoding of turbo codes compared to that of Viterbi (or MAP) decoding of TCM. More details in complexity comparison of these schemes can be found in (**Jung,** 1996; **Koorapaty et al.,** 1997).

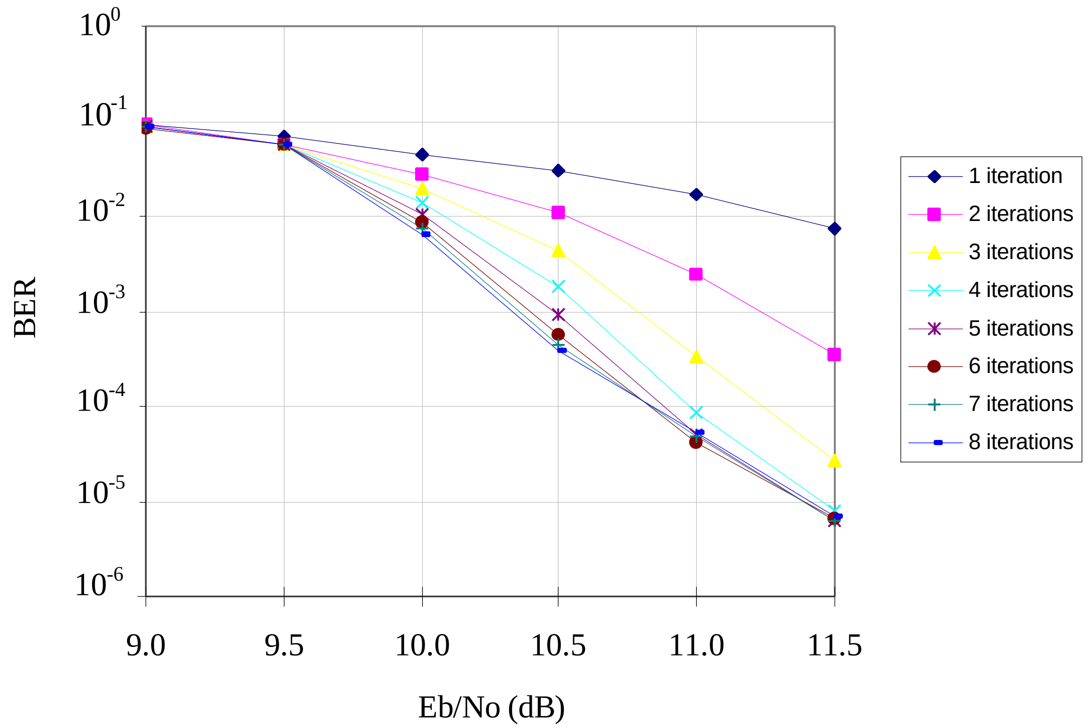


**Fig. 3.** Performance of TC with 8-PSK modulation
(TC is built from RSC component codes ($K$=3, $G_1$=7, $G_2$=5) and a random interleaver)

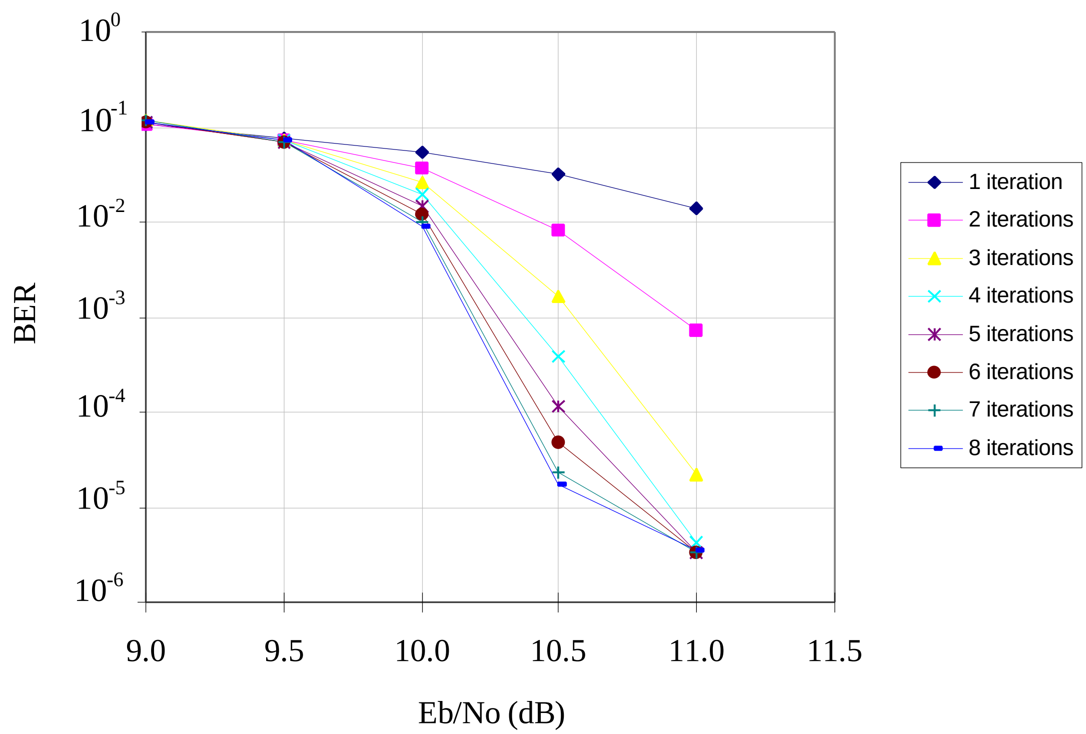


**Fig. 4.** Performance of TC with 8-PSK modulation
(TC is built from RSC component codes ($K$=4, $G_1$=15, $G_2$=17) and a random interleaver)

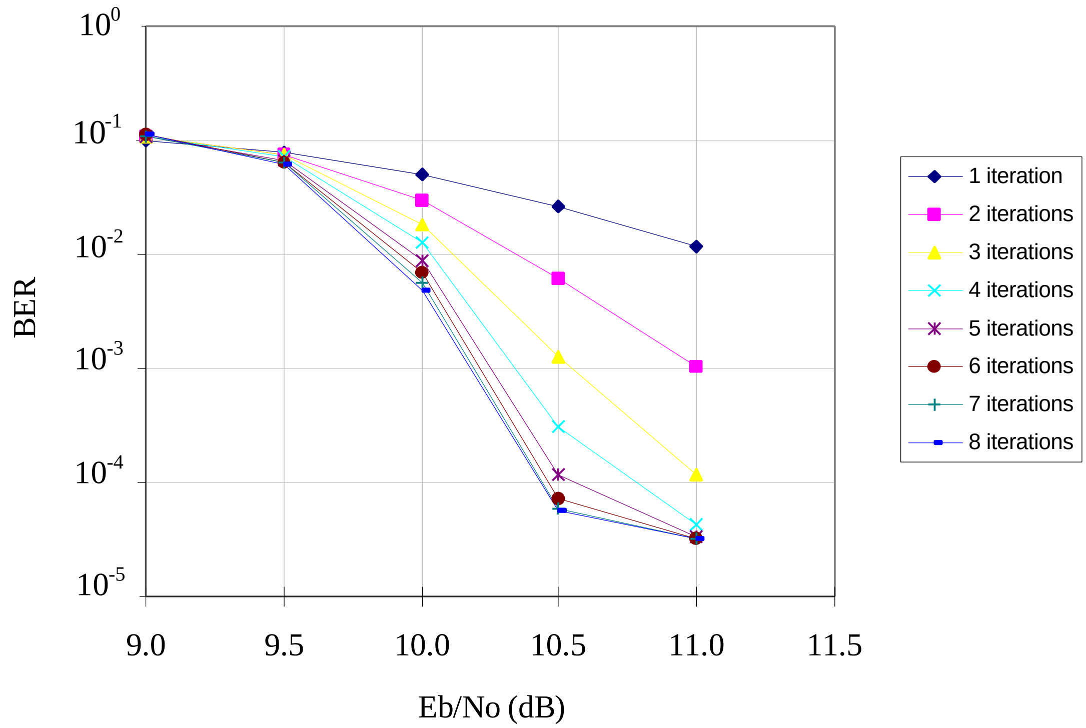


**Fig. 5.** Performance of TC with 8-PSK modulation
(TC is built from RSC component codes ($K$=5, $G_1$=37, $G_2$=21) and a random interleaver)

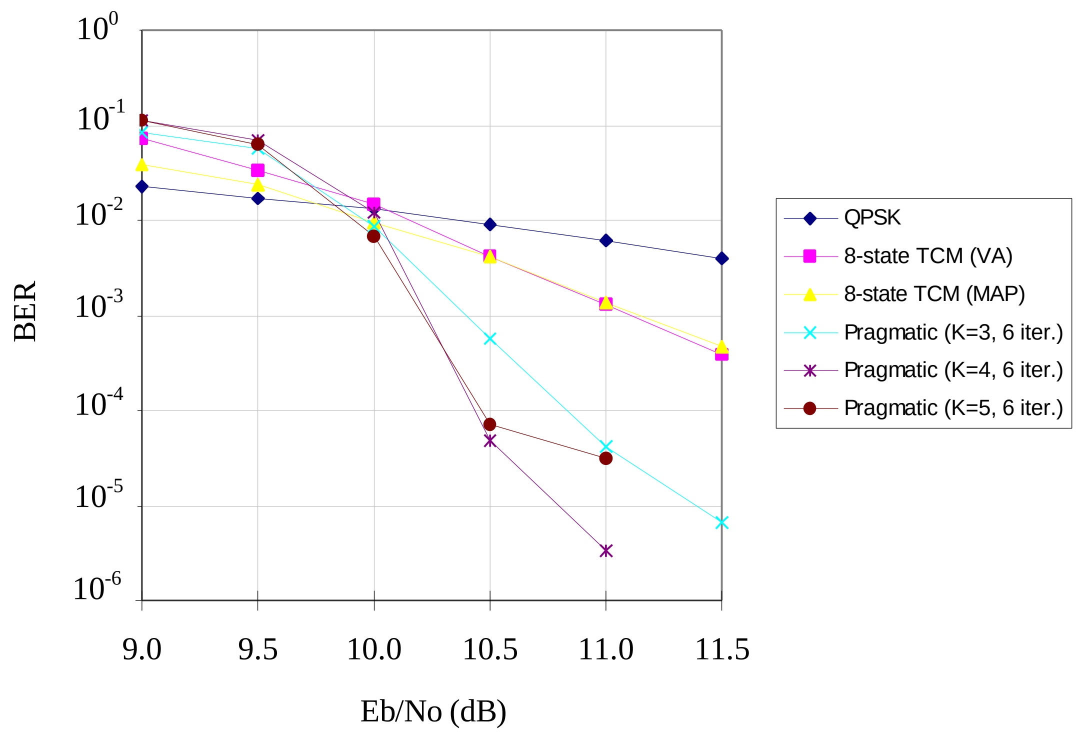


**Fig. 6** Comparison of pragmatic approach with TCM over AWGN channel

## Turbo Trellis Coded-Modulation (T-TCM)

- In this second approach, two *Ungerboeck TCM codes* in their recursive systematic feedback form are used as *component codes* in a similar way as in binary turbo codes.

- The recursive systematic TCM code is essentially a recursive systematic convolutional (RSC) encoder followed by symbol mapper.

- Usually the rate of RSC code is ($m$-1)/$m$. It means that one redundant bit is created for every ($m$-1) information bits and then $m$ bits are mapped into one of $M$=$2^m$ symbols following Ungerboeck's mapping by set partitioning rule.

- The general structure of RSC code that constructs recursive systematic TCM code is illustrated in Fig. 7.

- There are $n$ memories in Fig. 7, hence $2^n$ states of the trellis diagram. Out of $m$ input bits, there are ($m$-$k$) *uncoded* bits corresponding to *parallel transitions* in the trellis diagram of TCM;

- $k$ information bits are encoded by a rate-$k$/($k$+1) RSC code. The RSC encoder in Fig. 7 is completely defined by the following generator matrix.

$$G_{(k+1)\times n} = \begin{bmatrix} g_0^{(0)} & g_1^{(0)} & g_2^{(0)} & \cdots & g_{n-1}^{(0)} \\ g_0^{(1)} & g_1^{(1)} & g_2^{(1)} & \cdots & g_{n-1}^{(1)} \\ . & & & & . \\ . & & & & . \\ . & & & & . \\ g_0^{(k)} & g_1^{(k)} & g_2^{(k)} & \cdots & g_{n-1}^{(k)} \end{bmatrix} \qquad (11)$$

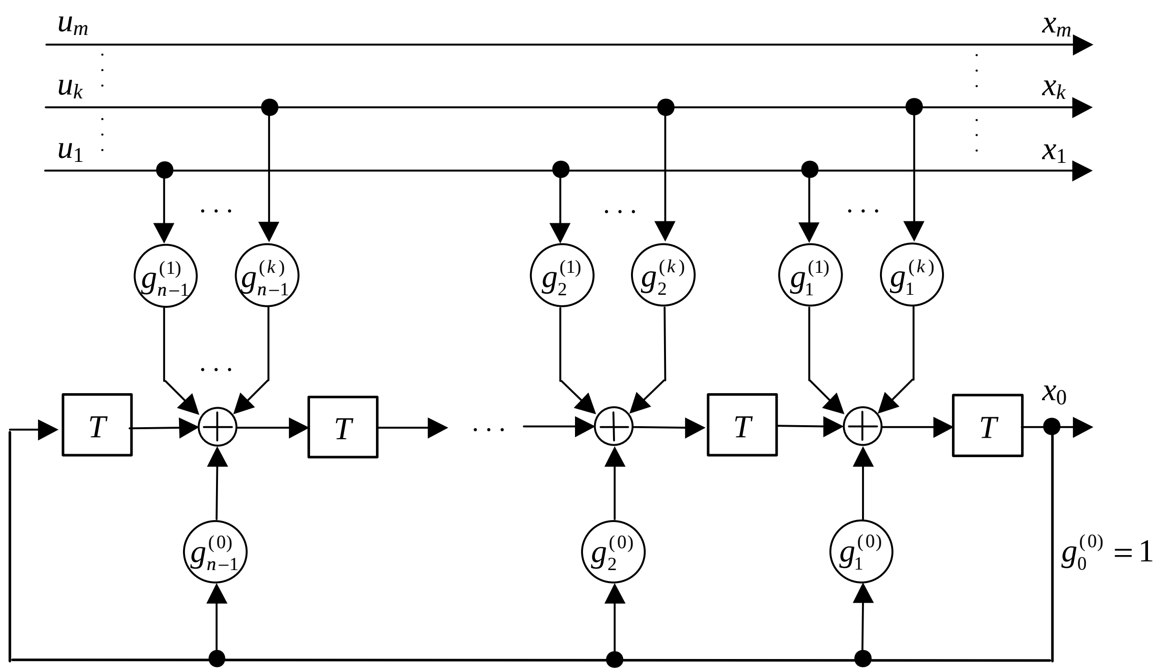


**Fig. 7.** Systematic canonical convolutional encoder with feedback

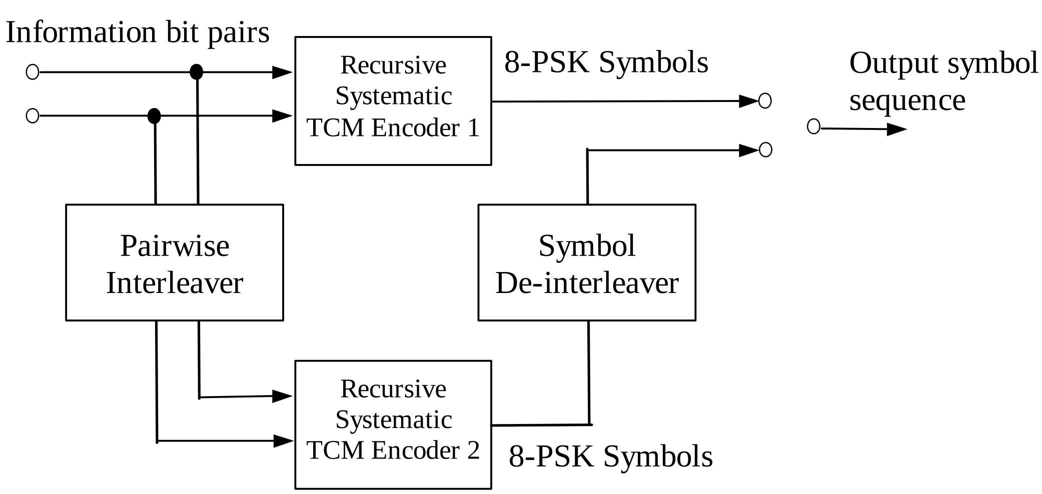


**Fig. 8.** T-TCM encoder

- Fig. 8 shows the encoder structure of T-TCM when the component recursive systematic TCM is 8-PSK TCM. *The following are the major differences when comparing the above encoder with binary turbo encoder (**Robertson and Woerz** 1995, 1996).*

  - Interleaving operates on group of ($m$-1) bits instead of single bits.

  - In order to achieve a systematic overall code, the *pairwise* interleaver must map even positions to even positions and odd positions to odd positions (or even-odd, odd-even).

  - For the component code, the corresponding trellis diagram should have no parallel transitions and the information bits at time $k$ do not affect the parity bits at time $k$.

- In Fig. 8 the sequence of information bit pairs is TCM encoded to yield the 8-PSK symbol sequence. The information bits are then interleaved on a pairwise basis and TCM encoded again into the second 8-PSK symbol sequence.

- The second 8-PSK symbol sequence is deinterleaved to ensure that the ordering of the two information bits partly defining each symbol corresponds to that of the first encoder.

- Finally the transmitted symbol sequence is alternatively selected from two output symbol sequences. *In this way, each information bit pair is contained in only one 8-PSK symbol and the parity bit is alternately chosen from the first and second encoder.*

- Fig. 9 illustrates structure of the complete T-TCM decoder.

- Basically iterative decoding in T-TCM is similar to that in binary turbo code.

- However, there is a difference in the nature of the information passed between component decoders and in the first decoding step.

- In the binary turbo code, *as shown before*, the output of component decoder can be split into three additive parts for each information bit $k$ in the logarithmic domain: the *systematic* component, the *a priori* component and the *extrinsic* component and only the last component can be passed to the next decoder.

- In T-TCM each decoder alternatively sees its corresponding encoder's noisy output symbol and then the other encoder's noisy output symbol. The noisy output symbol corresponding to the other encoder are referred to "punctured" symbols and denoted by "∗" in Fig. 9.

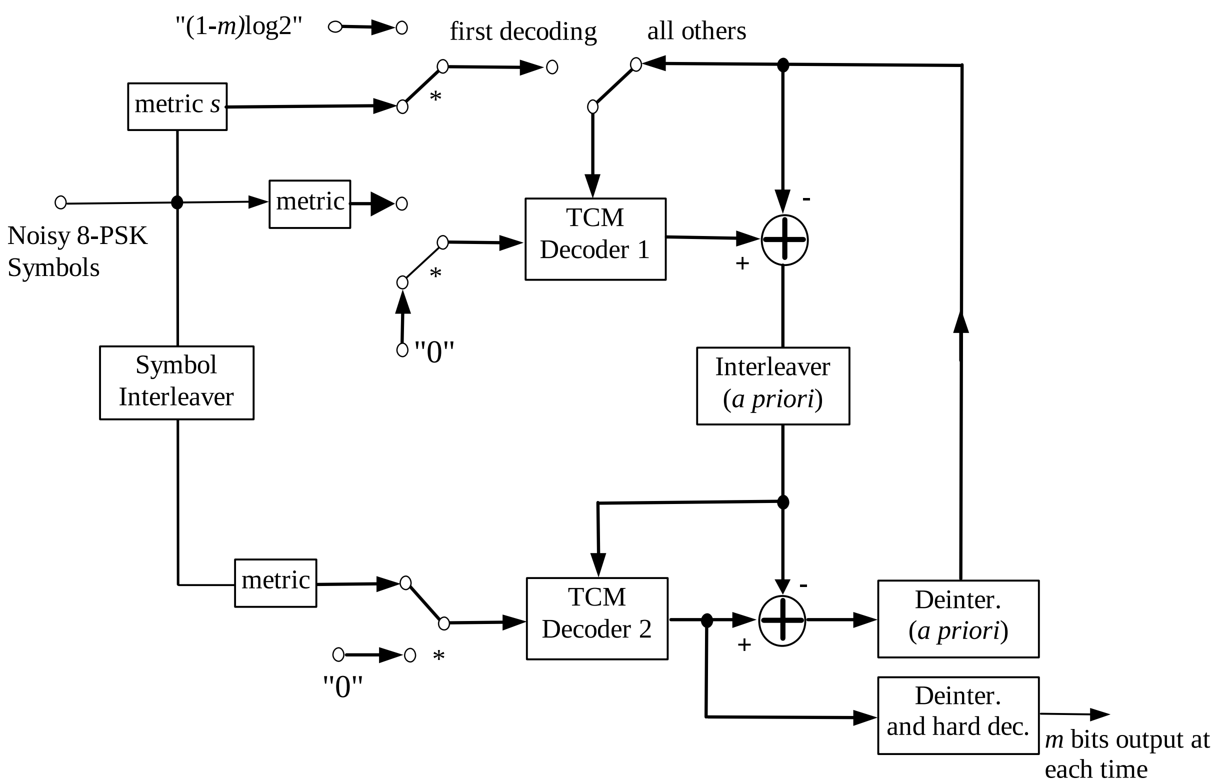


**Fig. 9.** T-TCM decoder

- In T-TCM, since the systematic bits are transmitted together with parity bits in the same symbol, if the noise affects the parity component it will also affect the systematic component, hence one ***cannot separate*** systematic component from the extrinsic information.

- In this case, the output of the component decoder can only be separated into two different components: *a priori* and *extrinsic&systematic*.

- Each decoder now will deliver to the next decoder the second component. In order to do this, the component decoder should employ a *symbol-by-symbol* MAP algorithm. In Fig. 9 the thin paths are channel outputs while the thick paths represent a group of $2^{m-1}$ values of logarithms of probabilities.

- Appendix describes in more detail the symbol-by-symbol MAP algorithm and how to handle the first decoding step following (**Roberson and Woerz** 1996).

• The simulation results for two different 8-PSK T-TCM schemes are presented below. The generator matrices of RSC codes used to construct component TCM codes in these T-TCM schemes are given in (12) and (13) respectively.

- In these RSC codes a parity check bit is appended to every two information bits and the resulting triple bits are used to select one of 8 PSK symbols according to Ungerboeck's rules.

- There is no uncoded bit in these RSC codes and hence no paralell transitions as required for component TCM codes in T-TCM.

$$G_{3\times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{12}$$

$$G_{3\times 4} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{13}$$

- Figures 10 and 11 plot the performance of two T-TCM schemes corresponding to the RSC codes in (12) and (13) respectively.

- Iterative decoding is carried out with 8 iterations and BER is calculated after each iteration. The information block length of 2400 bits (hence 1200 8-PSK symbols) is chosen in all simulations.

- From these two figures we can see the improvement in performance due to iterative decoding procedure. As in the case of binary turbo codes as well as pragmatic approach, a very large coding gain is obtained after the first iteration.

- The weak asymptotic performance of the component TCM codes (corresponding to the first decoding) seems not to affect the performance of T-TCM. Trellis termination for TCM component codes is done in the same way as before, i.e., the first TCM component code is terminated, whereas the second one is left "open".

- Figure 12 then compares the performance of different spectral efficiency modulation schemes over AWGN channel including 8-state 8-PSK Ungerboeck's TCM, pragmatic 8-PSK scheme presented in the earlier Section and T-TCM schemes of this section.

- All of these schemes have the same spectral efficiency of 2bits/sec/Hz. It can be seen that T-TCM scheme is better than pragmatic approach for the same complexity (i.e., the same constraint length for component codes).

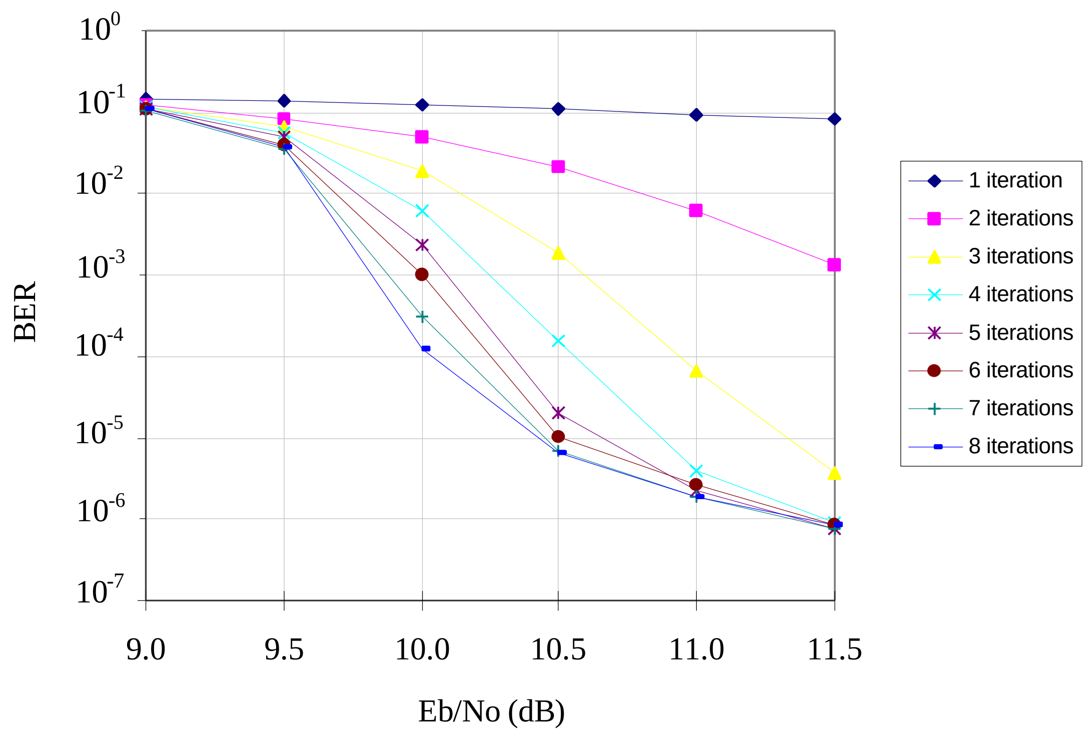


**Fig. 10** Performance of 8-PSK T-TCM scheme over AWGN channel
(The component Ungerboeck TCM encoder is constructed from 8-state RSC code)

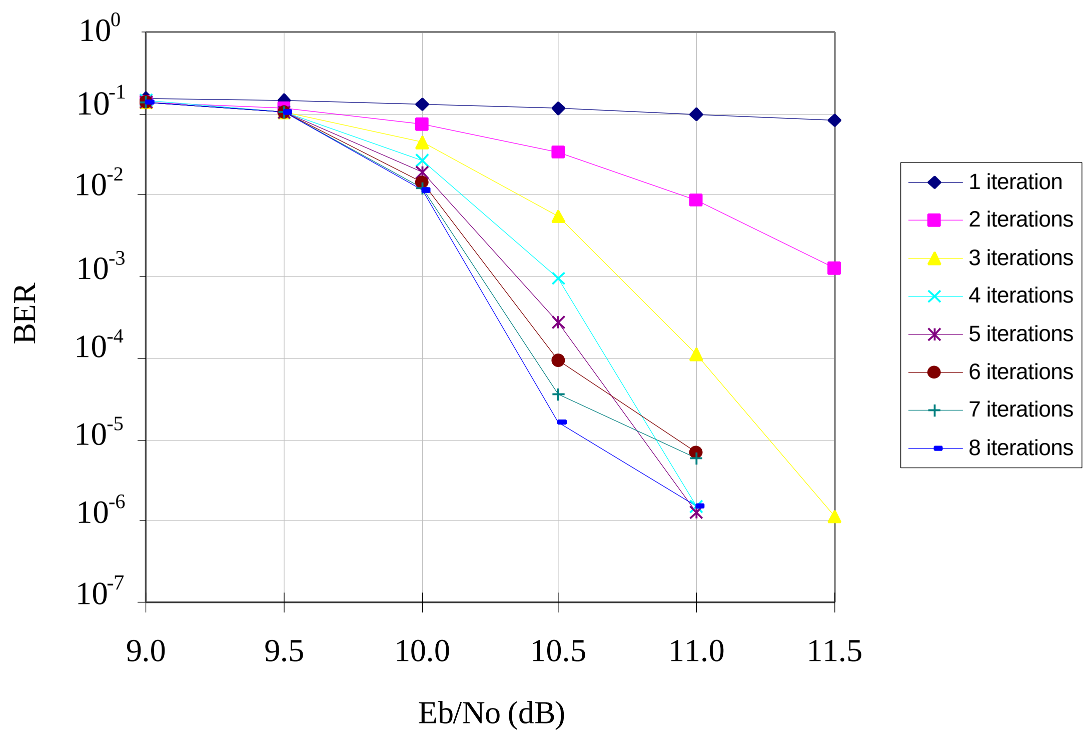


**Fig. 11.** Performance of 8-PSK T-TCM scheme over AWGN channel
(The component Ungerboeck TCM encoder is constructed from 16-state RSC code)

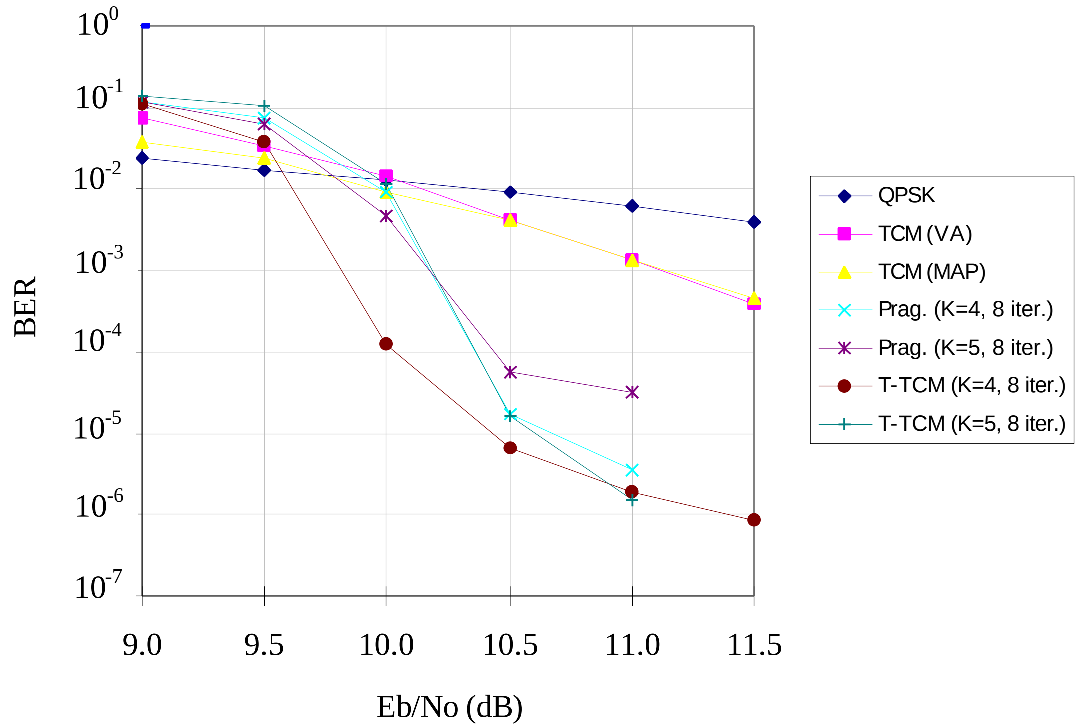


**Fig. 12** Comparison between various spectral efficiency modulation schemes over AWGN channel.

## Turbo Trellis Coded-Modulation (T-TCM) over Frequency-selective Rayleigh Fading Channels

- As mentioned earlier, there have been a lot of research on turbo codes since its first invention by (**Berrou et al.,** 1993). However, all of the work is related to AWGN and flat fading channel.

- Not much work is there on the application of turbo codes combined with multilevel modulation for frequency-selective fading channel.

- Based on the work of (**Robertson and Woerz** 1995, 1996) in successfully applying T-TCM for AWGN channel, the application of T-TCM over frequency-selective fading channel is desired and will be considered in this section.

- The section begins with the review of some of the previous work which employed turbo codes for fading channel. The system model is then described and the simulation results will be presented. The section closes with the conclusions for the new application of turbo codes.

- **Summary of Previous Work**

- The first attempt in applying turbo codes for fading channel was by (**Goff et al.,** 1994) where the *pragmatic* approach is considered. The coded sequence from turbo encoder is Gray mapped into multilevel symbol sequence and then this symbol sequence is transmitted over fading channel.

- The channel was Rayleigh *flat* fading with the fading coefficients are assumed to be known at the turbo decoder. Then by modifying the formulas to calculate the log-likelihood ratio of each encoded bit, it was shown that the turbo decoder optimized for a Gaussian channel is also optimal for a Rayleigh flat fading channel.

- The principle of iterative decoding binary turbo codes over flat fading channel was analyzed in (**Hagenauer,** 1996) and it is essentially the same as in AWGN channel.

- In (**Jung,** 1996) the performance of binary turbo codes over Rayleigh flat fading channel was investigated intensively for short frame length and for different algorithms of decoding component codes. Another similar research was done by (**Koorapaty et al.,** 1997).

- Recently in (**Ogiwara et al.,** 1997) a coded-modulation scheme similar to T-TCM was investigated for both AWGN channel and Rayleigh flat fading channel. The modulation scheme was QPSK and it was shown that the proposed codes can be iteratively decoded and hence achieve a very good performance after a small number of iterations.

- (**Raphaeli and Zarai,** 1997) investigated the performance of turbo codes for intersymbol interference channel. A new iterative decoder structure was constructed which combined the channel equalization and turbo decoding.

- At each iteration the extrinsic information from the channel detector is fed into the turbo decoder and then its extrinsic information is fed back to the channel detector.

- Simulation was carried out for turbo code with binary modulation over *static* ISI channel. Their performance results showed that at BER level of $10^{-5}$ the perfrormnce is about 0.9dB from the ISI channel capacity.

**System Model for T-TCM over Frequecy-Selective Fading Channel**

- Basically, the system model for the application of T-TCM over frequency-selective fading channel presented in Fig. 13 is similar to the one for the application of TCM over that type of channel.

- In essence, the interleaver and deinterleaver are still included to mitigate the effect of the fading. Equalizing multipath fading channel and channel decoding are done separately as before.

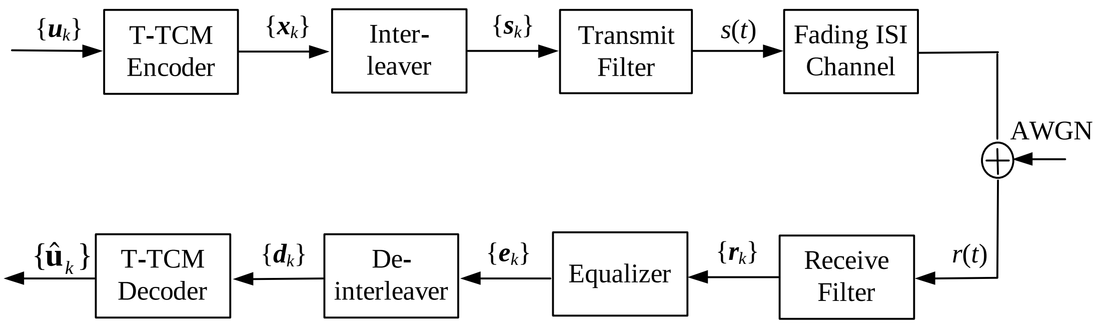


**Fig. 13.** System model for T-TCM over frequecy-selective fading channel

- In Fig. 13 the information sequence {$\boldsymbol{u}_k$} is T-TCM encoded as described earlier to produce the symbol sequence {$\boldsymbol{x}_k$}. The symbols at the output of T-TCM encoder take one of $M$ possible symbols depending on which type of constellation used in component TCM.

- This symbol sequence is then block interleaved before transmitting over frequency-selective fading channel. *The overall channel model consisting of transmit filter, receive filter and fading channel can be again modeled as a FIR filter having (L+1) taps.*

- In the system model of Fig. 13 we employ the same T-TCM decoder as the one used for decoding T-TCM over AWGN channel.

- However, in case of AWGN channel the input of T-TCM decoder is the transmitted symbol sequence corrupted by uncorrelated additive noise. When applying T-TCM for frequency-selective fading channel the input to T-TCM decoder is the output of equalizer.

- In order to successfully decode T-TCM, the equalizer has to be designed carefully so that sufficient information is passed to T-TCM decoder.

- As discussed earlier, the iterative decoding of T-TCM over AWGN channel relies on symbol-by-symbol MAP decoding of component TCM. The output of such a component decoder can be split into two parts and one of the two parts is passed to the other component decoder.

- The input to the component decoder is composed from direct channel symbols and *a priori* information from the other decoder. Whenever the symbols received directly from AWGN channel correspond to the underlying decoder, the statistical value $Logp(\mathbf{y}_k | u_k = i, s_k, s_{k-1})$ is calculated and used in symbol-by-symbol MAP algorithm as shown in the Appendix.

- If the received symbols do not correspond to the underlying component decoder, they are ignored and the above statistical values are set to 0. The very first decoding step of iterative decoding is also described in Appendix C.

- Keeping in mind the basic idea of iterative decoding T-TCM over AWGN channel above, the same T-TCM decoder is expected to perform well in the system model of Fig. 13 if the equalizer can provide the T-TCM decoder with the statistical values similar to $Logp(\mathbf{y}_k | u_k = i, s_k, s_{k-1})$ in AWGN channel.

- Since $Logp(\mathbf{y}_k | u_k = i, s_k, s_{k-1})$ is actually the logarithm of probability of received symbol given a transmitted symbol over AWGN channel, it is essential for the equalizer of Fig. 13 to use the Max-Log-MAP algorithm.

- In that case the outputs of equalizer are used directly to calculate the transition probabilities in the symbol-by-symbol MAP algorithm provided that the current symbol correspond to the underlying decoder, otherwise the logarithm of probabilities are set to 0. The first decoding step in this case is slightly different from the one for AWGN channel. We set the *a prirori* probabilities as $P(u_k = i) = \dfrac{1}{2^{m-1}}$ for all symbols.

## Simulation Results

- In this section our simulation results in applying T-TCM toghether with the Max-Log-MAP equalizer are presented.
- The T-TCM schemes considered in Fig. 14 and Fig. 15 are the same as those investigated for AWGN channel before, they are T-TCM constructed from different 8-PSK recursive systematic TCM codes given in (12) and (13).
- Random interleaver is used inside T-TCM encoder, whereas 10×120 block interleaver and deinterleaver are utilized to reduce the effect of fading as in the case of TCM schemes before.

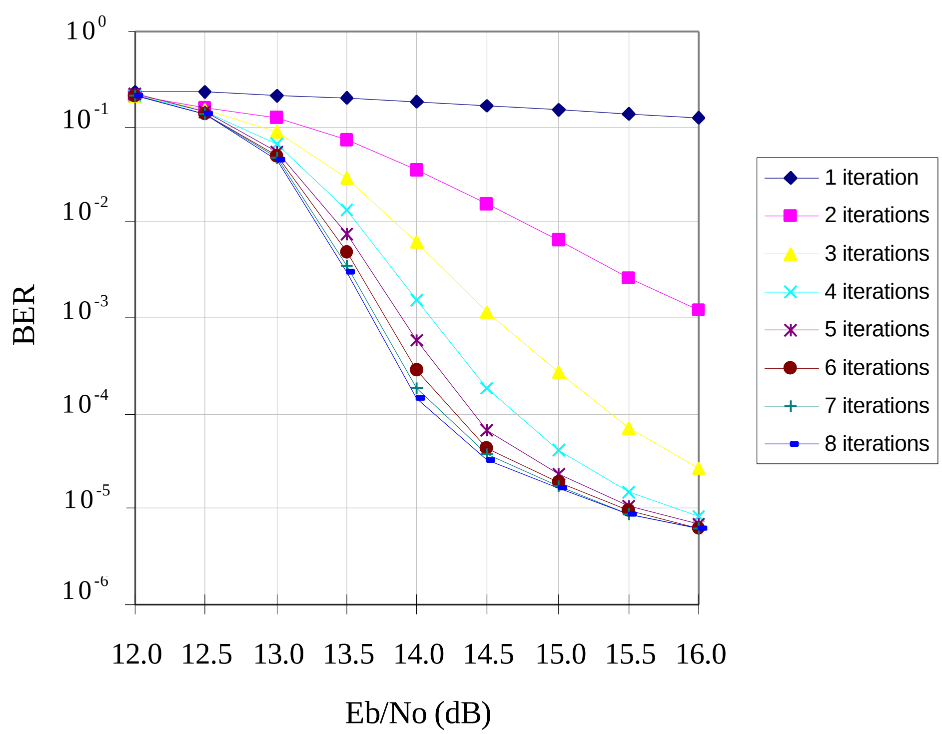


**Figure 14** Performance of 8-PSK T-TCM scheme over frequency-selective Rayleigh fading channel (The component Ungerboeck TCM encoder is constructed from 8-state RSC code)

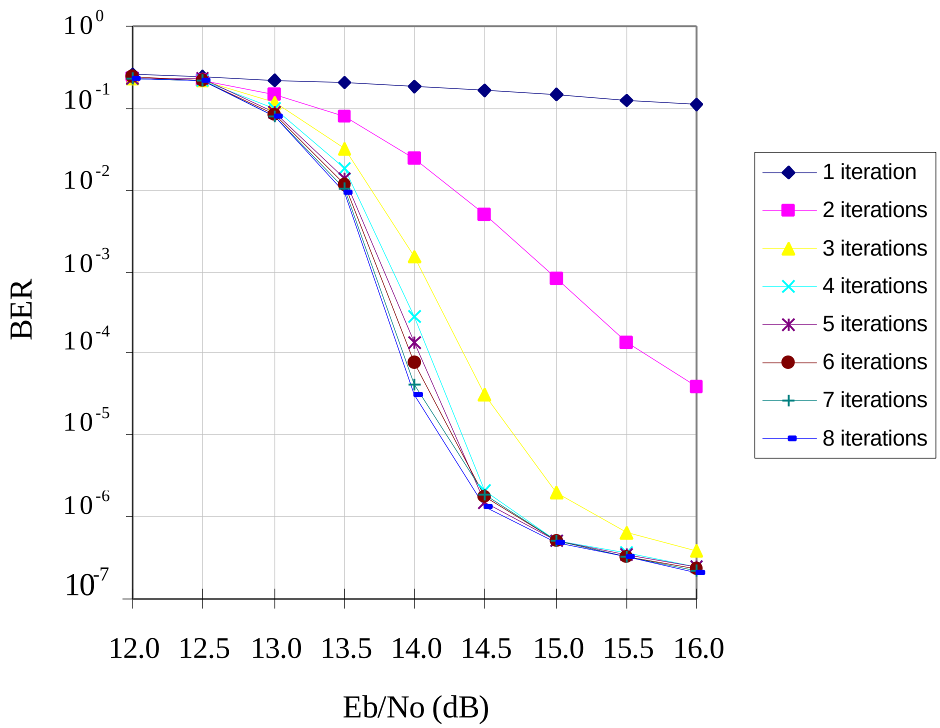


**Figure 15** Performance of 8-PSK T-TCM scheme over frequency-selective Rayleigh fading channel (The component Ungerboeck TCM encoder is constructed from 16-state RSC code)

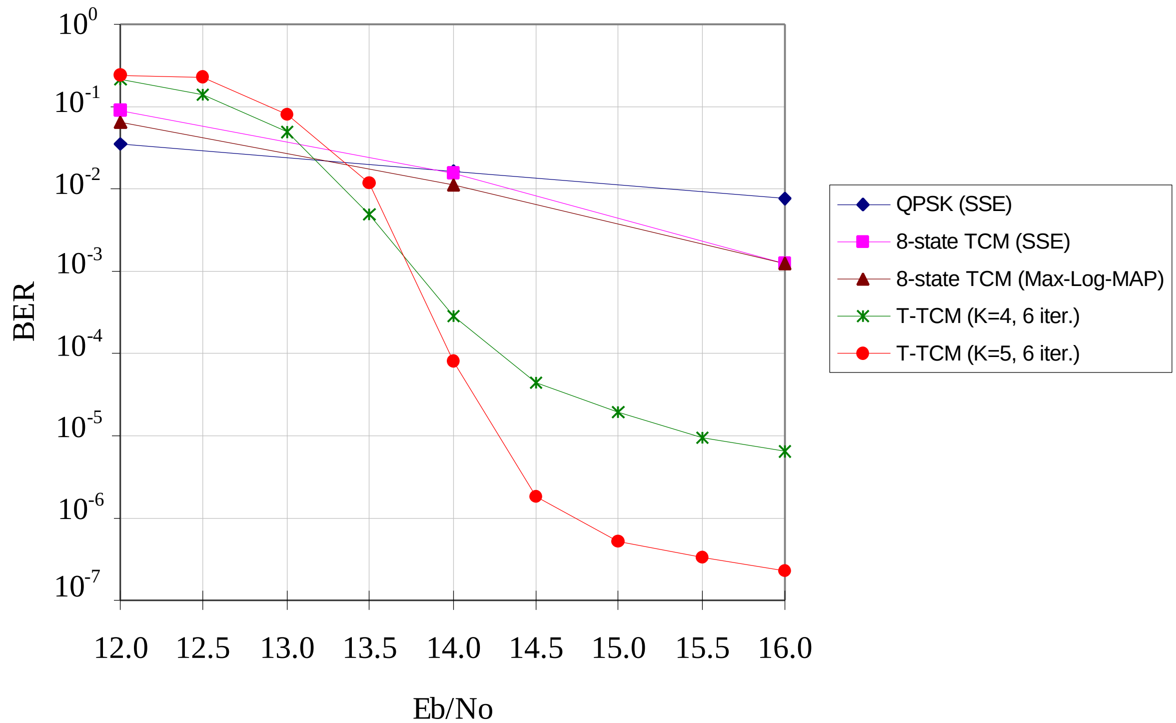


**Fig. 16.** Comparison between various spectral efficiency modulation schemes over frequency-selective Rayleigh fading channel.

- The frequency-selective fading channel including transmit and receive filters in the simulation consists of two equal strength Rayleigh fading paths and the delay between paths equals to symbol duration *T*.

- The tap coefficients of the overall time-varying channel are assumed to be perfectly estimated by the receiver. Equalizer employs Max-Log-MAP algorithm to deliver the logarithm of probability of each symbol to the T-TCM decoder. The block length of infomation sequence is 2400 bits (hence 1200 symbols). Iterative decoding is simulated with 8 iteration and the BER is calculated after every iteration.

- The improvement in performance of T-TCM over frequecy-selective fading channel due to iterative process can be seen from Fig. 14 and 15. Similar to the results of T-TCM over AWGN channel, a large coding gain is achieved after the second iteration and very little gain can be seen after the forth iteration.

- The superiority of T-TCM over conventional TCM schemes having the same spectral efficiency is shown in Fig. 16. In that figure the performances of two T-TCM schemes after 6 iterations and of 8-state 8-PSK Ungerboeck TCM scheme over frequency-selective fading channel are compared. Also plotted in Fig. 16 is the performance of uncoded QPSK scheme.

- There are two performance curves of 8-state 8-PSK scheme in Fig. 16. One corresponds to the case when SSE and Viterbi decoder for TCM are used in the receiver, the other one corresponds to the case when Max-Log-MAP algorithm is used in both equalizer and TCM decoder.

- At BER level of $10^{-3}$ a coding gain of 2.5dB can be easily achieved by T-TCM schemes over TCM scheme. More coding gain is possible at lower level of BER. However, the price paid for this performance improvement is the increase in complexity.

- In conclusion, a system model was successfully developed to employ turbo coding technique for frequency-selective Rayleigh fading channel. T-TCM schemes benefit from the advantages of both iterative decoding technique and coded-modulation technique. Our simulation results show that T-TCM schemes give a very good performance compared to other efficient bandwidth coded modulation schemes, i.e. TCM. The system model is applicable to any other T-TCM scheme.