<u>Space Time Equalization-*space time codes*</u>

System Model for STCM

- The system under consideration consists of ST encoder, fading channel model with AWGN, *two transmit antennas, one receive antenna*, Viterbi equalizer with ideal CSI, deinterleaver, and decoder.

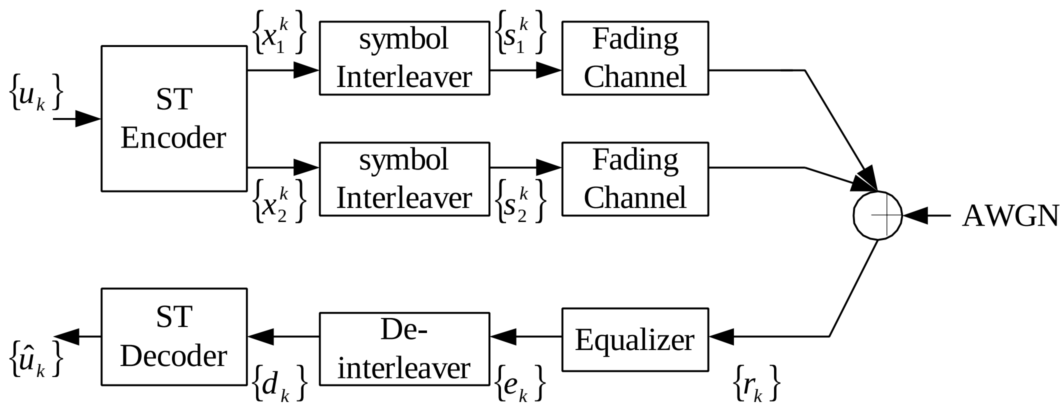


Figure 1 The system model for 2 space-time coded modulation scheme

- The information bits $\{u_k\}$ is passed to the ST encoder, which outputs the symbol sequences $\{x_1^k\}$ and $\{x_2^k\}$. The symbol sequence $\{x_1^k\}$ is transmitted over the first transmit antenna and the symbol sequence $\{x_2^k\}$ is transmitted over second transmit antenna.

- Both of symbol sequences are interleaved individually before sending through each channel. Interleaving is the typical way to mitigate the effect of burst errors in a fading channel.

- There are two fading channels due to two transmit antennas.

- The fading channel is modeled as a discrete-time channel as will be described later. Two symbol sequences are combined with AWGN before entering receive antenna.

- Then, the received symbol sequences $\{r_k\}$ enter the Viterbi equalizer that provides hard-output $\{e_k\}$ or enter to Suboptimum Soft-Output equalizer that provides soft-output for decoding process. The hard-output or soft-output information is deinterleaved to keep the original order of the sequence $\{d_k\}$. The ST decoder, maximum likelihood decoder, combines and employs received hard-output or soft-output information for decoding to the received information bits sequence $\{\hat{u}_k\}$.

ST Encoder

- The ST encoder employs delay scheme. The signal constellation is 4-PSK, where the signal points are labeled by 0,1,2 and 3. This encoder has a very simple description in term of sequence (b_k, a_k) of binary inputs at time *k*. From Appendix A, a diversity gain $2 \times 1 = 2$ places an upper bound on the transmission rate of R_b = 2 bit/s/Hz. The ST encoder and trellis diagram are shown in Figure 2 and Figure 3, respectively.

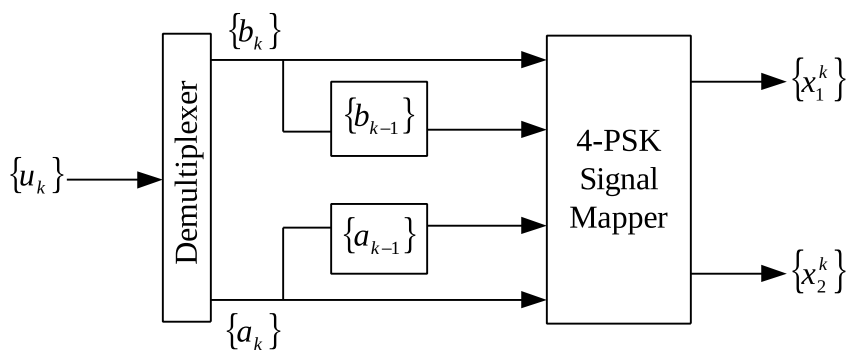


Figure 2 ST encoder system for 4-PSK

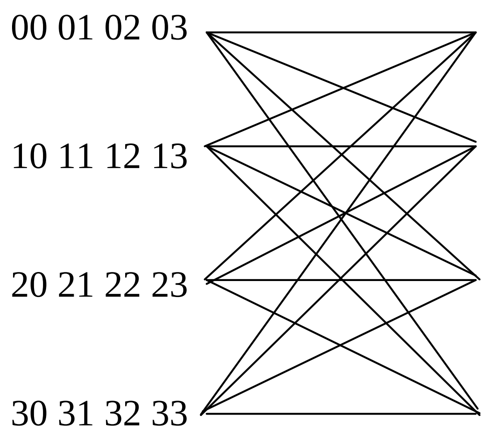


Figure 3 Four-state trellis diagram for STCM [**Tarokh et al.**, 1997]

- The information bits are first separated into two bit streams. In this simulation, total 400 information bits input to demuliplexer, each bit streams is 200 information bits. At time instant *k*, two bits $\{b_{k-1}\}$, $\{a_{k-1}\}$ from the memory and another two bits $\{b_k\}$, $\{a_k\}$ from demuliplexer enter the QPSK signal mapper froming two transmitted symbols $\{x_1^k\}$ and $\{x_2^k\}$. Then each 200 transmitted symbols entered the following symbol interleaver, it means that total transmitted bits are 800 bits, so the code rate is $1/2$ (see Example 1).

<u>Example 1</u>

- The information bits are 110110001011. After passing through demultiplexer, the information bits are divided into two information sequences, $\{b_k\} = \{101011\}$ and $\{a_k\} = \{110001\}$. Assuming that initial values of $\{b_{k-1}\}$ and $\{a_{k-1}\}$ are {0} and {0}, so at time *k*, 4 bits enter mapper at the same time, $\{x_1^k\}$ and $\{x_2^k\}$ are shown below,

$(x_1^k, x_2^k) = b_{k-1}(2,0) + a_{k-1}(1,0) + b_k(0,2) + a_k(0,1)$ [**Tarokh et al.**, 1997]

$\{b_{k-1}\}$	$\{a_{k-1}\}$	$\{b_k\}$	$\{a_k\}$	$\{x_1^k\}$	$\{x_2^k\}$
0	0	1	1	0	3
1	1	0	1	3	1
0	1	1	0	1	2
1	0	0	0	2	0
0	0	1	0	0	2
1	0	1	1	2	3
1	1			3	

- From this result, this is a delay scheme, it means that $\{x_1^k\}$ delays $\{x_2^k\}$ one symbol.

Symbol Interleaver and Deinterleaver

- The symbol interleaver is assumed to be a block interleaver with 20 rows and 10 columns. The symbols from the encoder are written in by row and read out by column. The proposed interleaver is shown in Figure 4. So, the interleaving delay per one block is 200-symbol duration. The deinterleaver which reverses the function of the interleaver are located before the decoder. However, in this research the interleaver is 20×10 block symbol interleaver, but the deinterleaver is 20×10 block bi-symbol deinterleaver.

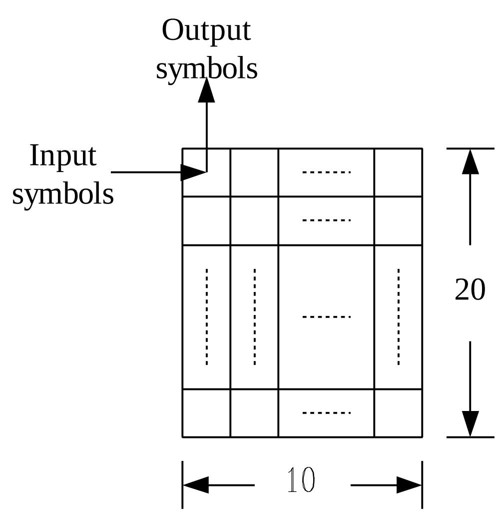


Figure 4 Interleaver of STCM system

Equalizer

- After two symbol sequences pass through frequency-selective Rayleigh fading channel individually, both of symbol sequences are combined together with AWGN. So, there are 16 possible bi-symbols in the signal constellation, where the signal points are labeled by 0,1,2,3,4,5,6,7,8,9,10,11,12,13,14 and 15 (see Table 1).

- Then, the bi-symbol sequences enter one equalizer due to one received antenna. The equalizer needs 16-state trellis diagram to get hard-output information for VE and get soft-output information for SSE, it is shown in Figure 4.5. The transition branches are labeled by A, B, C, D, E, F, G, H, I, J, K, L, M, N, O and P, there are 16 branches in each state, and 16 states have same labeled branches. We used the decision delay $d = 5$ for SSE.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
00	01	02	03	10	11	12	13	20	21	22	23	30	31	32	33

Table 1 The label of signal constellation

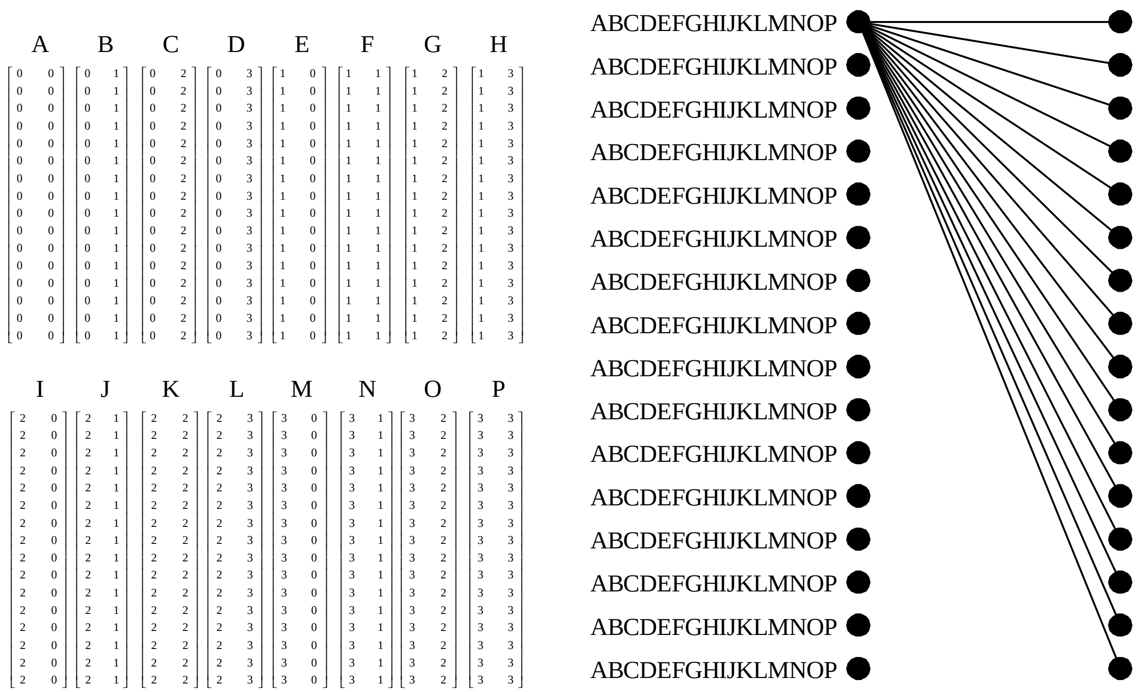


Figure 5 16-state trellis diagram for STCM equalizer

The branch metric for Viterbi equalizer (VE)

- Assuming that r_k^j is the received signal at the received antenna *j* at the time *k*, and the signals from the transmitted antenna *i* transmit through *p*-memory channel. The branch metric for a transition labeled $q_k^1 q_k^2 q_k^3 \cdots q_k^n q_{k-1}^1 \cdots q_{k-L}^1 \cdots q_{k-L}^n$ is given by

$$\sum_{j=1}^{m}\left| r_k^j - \sum_{p=0}^{L}\sum_{i=1}^{n} g_{p,k}^{i,j} q_{k-p}^i \right|^2$$

- The Viterbi algorithm is the used to compute the path with the lowest accumulated metric in the Viterbi equalizer. Note that q_{k-p}^i are labeled by M-PSK.

<u>The branch metric for Suboptimum Soft-output equalizer (SSE)</u>

Let D is the decision delay, and two transmitted antennas, one received antenna. The branch metric of SSE is same as the one of VE. The Suboptimum Soft-output algorithm (SSA) will select the path with the minimum APM. Therefore, in this research the soft-output corresponds to two symbols, one from each antenna. The soft-output from SSE is also called information packet:

$$\left(\min_{p_k \in Q_k(D,0,0)} (\Gamma_{k+1}(p_k)), \min_{p_k \in Q_k(D,0,1)} (\Gamma_{k+1}(p_k)), \ldots, \min_{p_k \in Q_k(D,M-1,M-1)} (\Gamma_{k+1}(p_k)) \right)$$

Decoder

Maximum likelihood decoder is employed with the received soft-output information from the equalizer. To use the Viterbi algorithm (VA), trellis diagram of the coding scheme must be known.

A path through the trellis from the initial node to the final node corresponds to a codeword. Maximum likelihood deciding chooses the path with the minimum path metric to be the decoded codeword.

The decoder uses VA for the cases of STCM-VE and STCM-SSE. But, the branch metric for both of them is different. For VE, at time k, the branch metric is given by

$$\sum_{i=1}^{n}\left| x_k^i - q_k^i \right|^2$$

where x_k^i is hard-output that corresponds to two symbols in this research, one from each antenna *i=2*. And, the branch metric of Viterbi decoder for SSE is soft-output from the equalizer corresponding to the symbols on a branch in the decoder trellis, that is shown in Eq. 4-2. Note that q_k^i are labeled by *M*-PSK.

The trellis complexity of the space-time code is at least $2^{b(r-1)} = 2^{2\times(2-1)} = 4$, so in this simulation, the trellis complexity of the space–time code is 4 states.

<u>Example 2</u>

- In non-error situation, hard-output bi-symbol codeword from equalizer is {804020}, and this codeword enters decoder. Then, output symbol sequences of decoder are {033112200113}. As ST encoder is a delay scheme, the original information bits can be obtained by QPSK signal constellation and deleting one of the repetition symbol. It is shown below,

0 3 3 1 1 2 2 0 0 1 1 3

↑ ↑ ↑ ↑ ↑ ↑

11 01 10 00 10 11

Simulation Model for RS-STCM

- The system components consists of RS-outer code encoder, bit interleaver, ST-inner code encoder, symbol interleaver, 2 channel model with AWGN due to 2 transmit antennas, Viterbi equalizer, symbol deinterleaver, ST-inner code decoder, deinterleaver and RS-inner code decoder. Note that demodulatior is omitted here because the lowpass equivalent discrete-time channel model will be used. Figure 6 shows the simulation system.

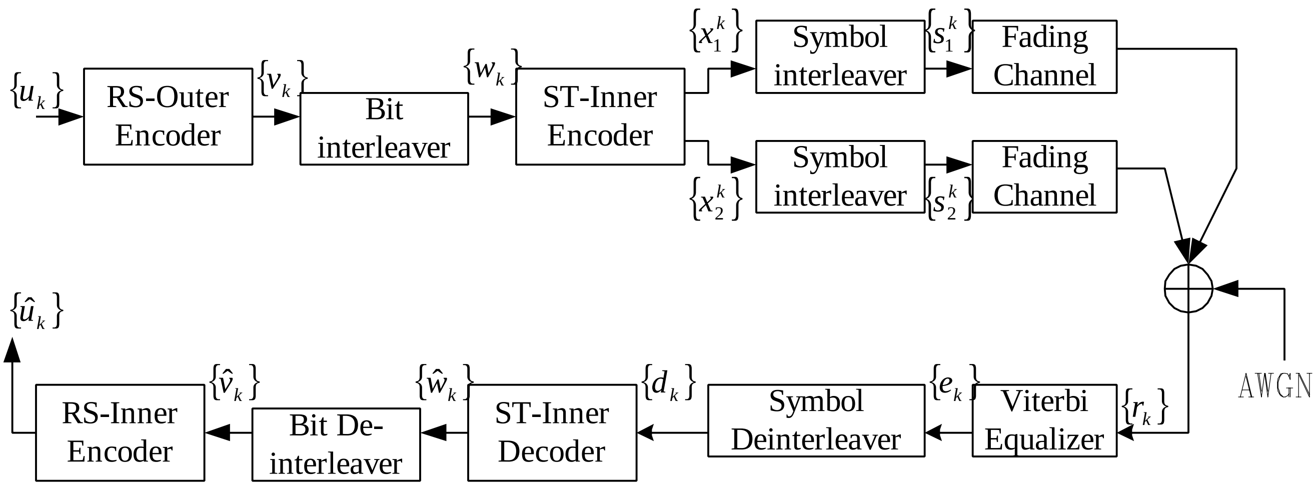


Figure 6 The simulation system for RS-ST concatenated coding scheme

- The hard-output information is deinterleaved first and then used in the ST-inner decoder that is Viterbi decoder. It outputs hard-decision bits sequence $\{\hat{w}_k\}$. To keep the original order of the sequence, $\{\hat{w}_k\}$ is deinterleaved to $\{\hat{v}_k\}$. The RS-outer code decoder then decodes the coming bit stream to the corresponding information bits $\{\hat{u}_k\}$.

Reed-Solomon (RS) Outer Code Encoder and Decoder

The RS scheme is decided to be RS(63,57). First, the information sequence is divided into $57 \times 3 = 171$symbols of 6-bit each. These 171 symbols are applied to the input of RS encoder that will generate 189 output symbols. The output symbols sequence then enter the following bit-interleaver.

For the selected RS scheme, the minimum distance will be n-k+1=63-57+1=7. Hence, it can correct up to (n-k)/2=(63-57)/2=3 6-bit symbols error.

The RS outer decoder employs an algebraic method, Berlekamp-Massey algorithm, which gets the hard-decision output bytes from the STCM inner decoder after bit deinterleaver.

Bit Interleaver and Deinterleaver

To improve the error correcting performance of RS, the bit-interleaver is introduced. It is an array of 3 rows by 63 columns. Each array component contains one 6-bit byte. The 63 bytes of RS codeword enter the interleaver by columns and read out by rows. It shows in Figure 7. The deinterleaver is applied before the outer decoder. The number of bits required to fill the interleaver is $63 \times 6 \times 3 = 1134$ bits.

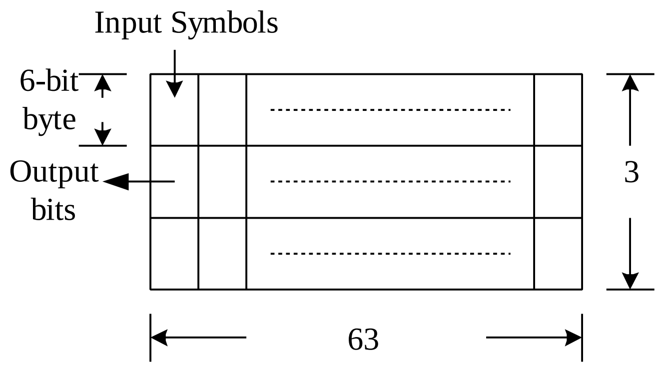
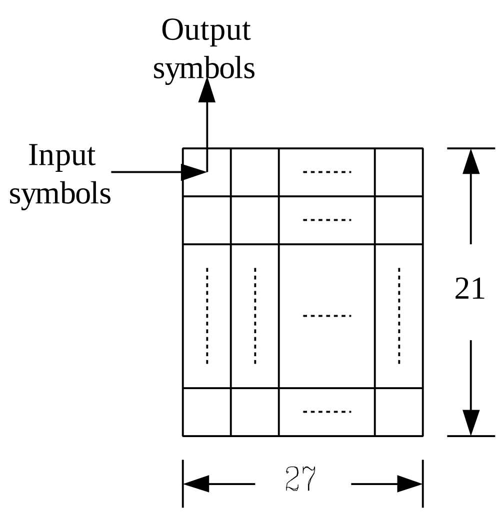


Figure 7 Bit-interleaver of RS-ST scheme

Symbol Interleaver and Deinterleaver

The symbol interleaver is assumed to be a block interleaver with 21 rows and 27 columns. The symbols from the encoder are written in by row and read out by column. The proposed interleaver is shown in Figure 8. The interleaving delay per one block is 567-symbol duration. The deinterleaver which reverses



the function of the interleaver are located before the decoder.

Figure 8 Symbol interleaver of RS-ST scheme

Channel Model

- The frequency-selective Rayleigh fading channel can be interpreted as N-path flat fading channel with ISI. So, it is modeled by tapped delay line with complex coefficients. The coefficients are complex Gaussian random variables. The amplitude of the coefficient is Rayleigh distributed and the phase is uniform distributed. The received signal r_k for the kth symbol can be shown as

$$r_k = \sum_{j=1}^{2} \sum_{i=0}^{L} s_{k-i}^{j} g_i^j(k) + n_k$$

Where the tap coefficients $g_i^j(k)$ are uncorrelated zero-mean complex Gaussian random processes and $\left|g_i^j(k)\right|^2$ are Rayleigh distributed at any time instant k. The tap coefficients of the overall time-varying channel are assumed to be perfectly estimated by receiver and the phase of received signal is fully compensated. In Eq., n_k is the complex zero-mean Additive White Guassian noise with identical variance $2 s_N^2$. The variance of the simulated Gaussian source for a specified E_b/N_0 (in dB) is

$$s_N^2 = \frac{1}{4} \times 10^{-\frac{1}{10}\left(\frac{E_b}{N_0}\right)}$$

Note that, with QPSK, $E_s = 2E_b$.

In this research, two-tap delay line model is used and shown in Figure 9. The delay is equal to one symbol duration. Two signal paths are assumed to have equal average power. Doppler effect will not be considered. Each channel is normalized to unit variance.

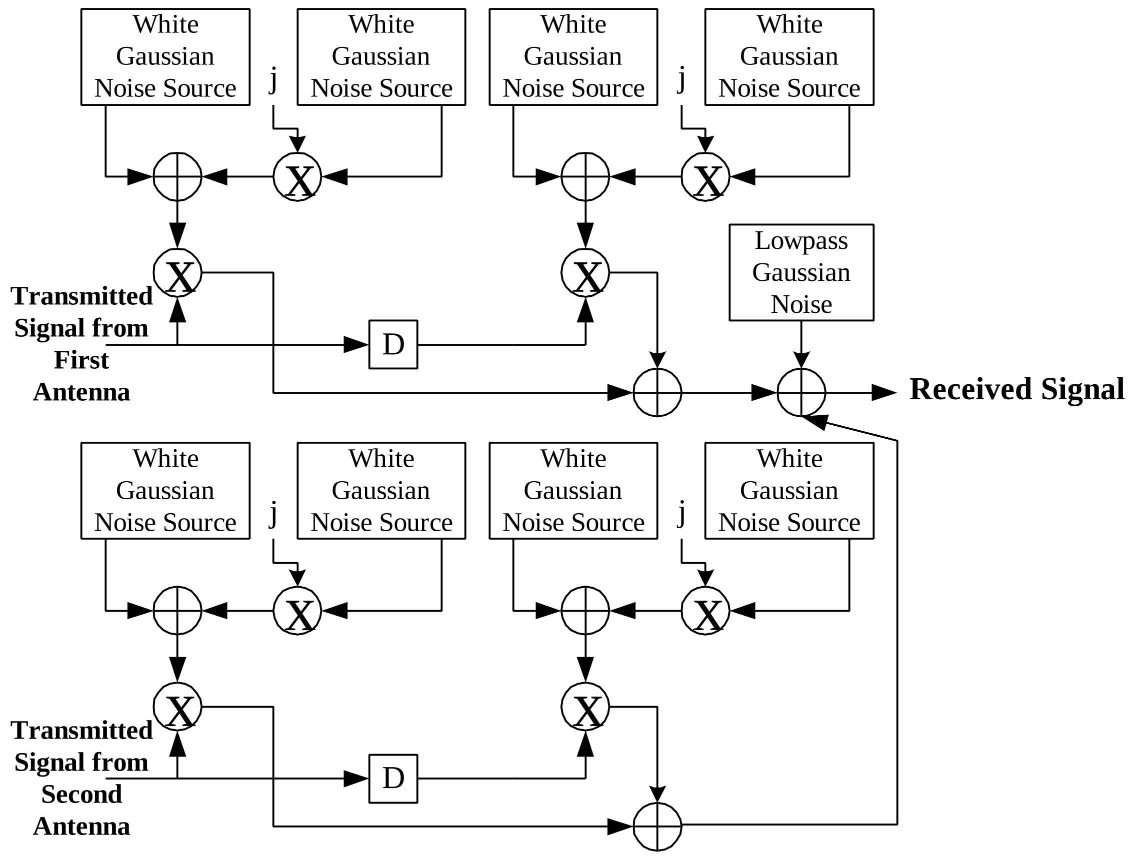
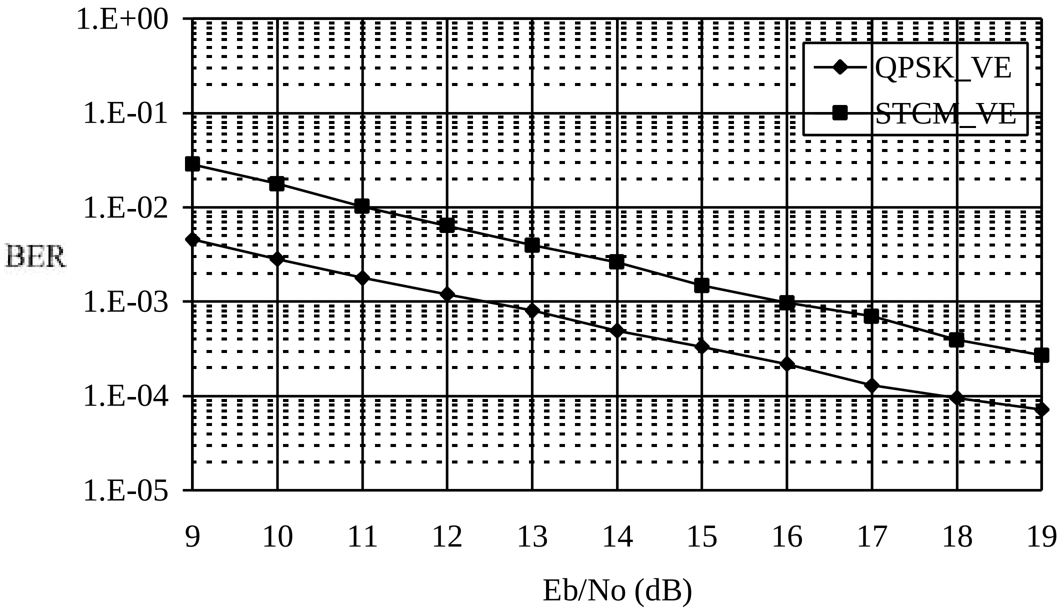


Figure 9 The frequency-selective Rayleigh fading channel model with 2-tap delay and without Doppler effect

From Eq. it is reasonable to assume that variance of the tap coefficients $g_i^j(k)$ is 0.5 for STCM, RS-STCM schemes and uncoded-QPSK scheme.

Simulation Results and Discussion

Three STCM schemes mentioned in earlier sections will be studied in a frequency-selective Rayleigh fading channel. Doppler effect is not included in this section. Monte-Carlo simulation will be carried out until one hundred or more error bits are found. Figures 10 to 12 present the simulation results for the STCM schemes with two types of equalizer and with RS outer code. The



uncoded QPSK are plotted as the reference system. The bit error rate (BER) is plotted versus signal-to-noise ratio (E_b/N_0) in decibel.

Figure 10 BER performance of STCM codes with VE over frequency-selectiveRayleigh fading channel, which compared with uncoded-QPSK

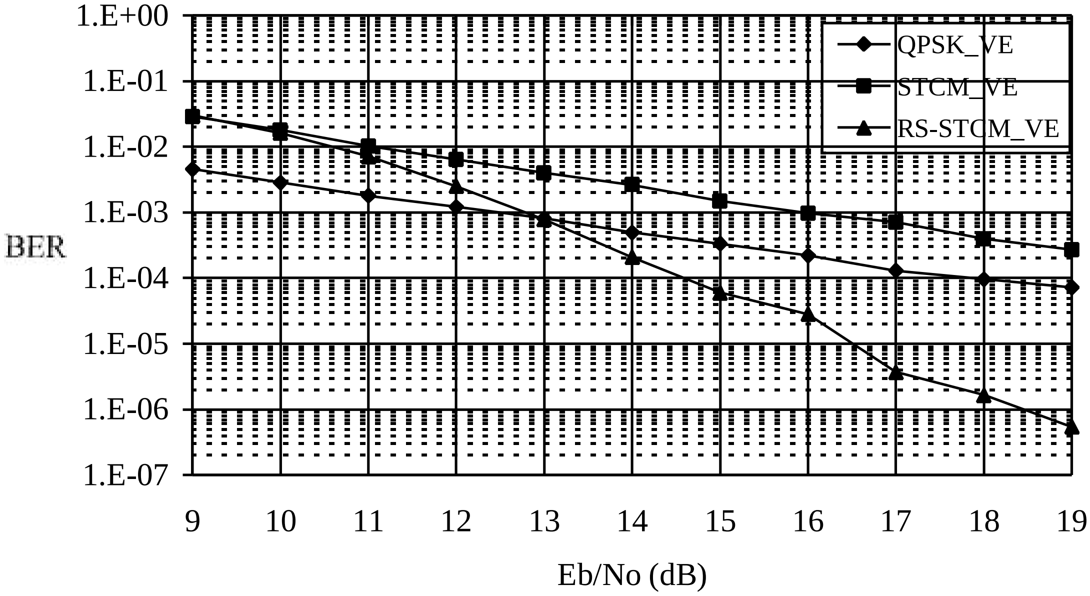


Figure 11 BER performance of STCM codes over frequency-selective Rayleighfading channel, which compared with RS-STCM

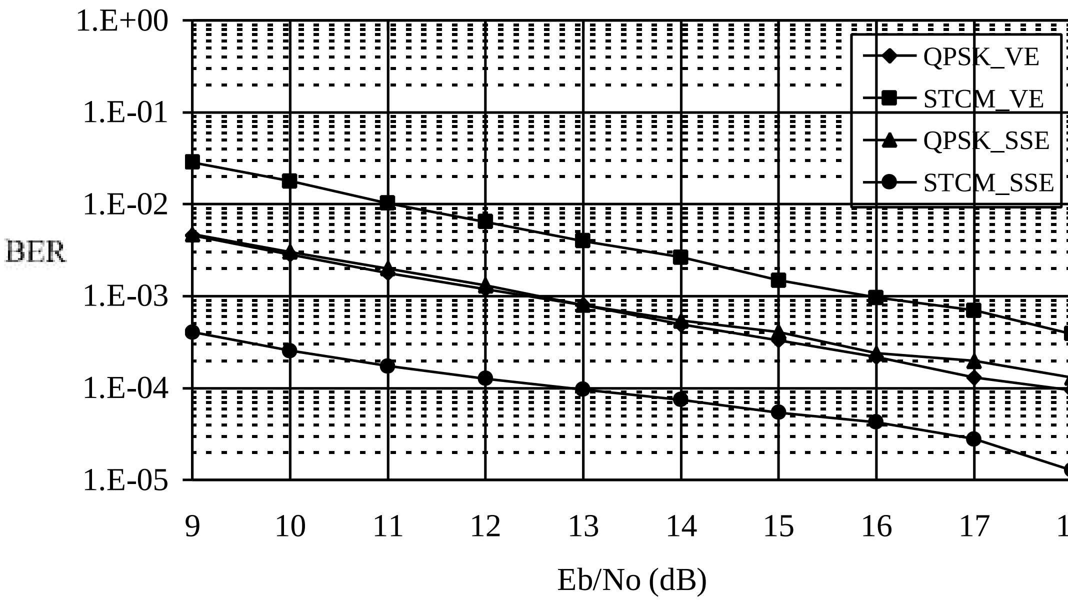


Figure 12 BER performance of STCM codes with SSE over frequency-selective Rayleigh fading channel, which compared with uncoded-QPSK

From the simulation results illustrated in Figure 10-12 we observe the following.

- The performance of STCM with Viterbi equalizer is worse than the performance of uncoded-QPSK. In Figure 10 the equalizer for QPSK system is VE.
- As expected, the performance of RS-STCM is better than the performance of uncoded-QPSK. At BER level 10^{-3} it gives 3dB gain over STCM. Note that in Figure 4.11 the equalizer for RS-STCM system is VE.
- From Figure 10 and Figure 11, at the low SNR the uncoded system outperforms RS-STCM system. At high SNR (greater than 13dB) the error performance is reversed.
- The system performance enhancement due to using a better and soft-output equalizer can be observed from Figure 12. When SSE is employed, the performance of STCM gives about 5-6dB coding gain over QPSK. And, more than 9dB improvement over STCM-VE at any BER level.

To summarize, the performance of STCM over frequency selective Rayleigh fading channel has been presented. Our simulation results show that uncoded QPSK performance in the proposed system outperforms STCM scheme with VE. The system model that combines interleaver, hard-decision STCM Viterbi decoder and hard-output equalizer is not suitable for STCM system. It is seen that soft-decision STCM Viterbi decoder and soft-output equalizer (SSE) show a 5-6dB coding gain in SNR. In the paper [**Bauch and Naguib**, 1999], it overcomes the problem of equalization space-time codes with transmit diversity to employ MAP equalization. The performance of SEE is close to that of MAP [**Tarasak** and **Rajatheva** 2000]. It can be explained that why we can employ SSE for STCM scheme. As expected, at high SNR the performance of RS-STCM outperforms uncoded QPSK.

References:

1. **Bauch G. and Naguib A. F.,** (1999). "MAP Equalization of Space-Time Coded Signals over Frequency Selective Channels", *IEEE Wireless Communications and Networking Conference,* Vol. 1, pp. 261-265.

2. **Tarasak P. and Rajatheva R.M.A.P.,** "Comparison of TCM and BCM schemes on Frequency-Selective Rayleigh Fading Channels with Soft-output Algorithms", ICC 2000, New Orleans, June.

3. **Tarokh, V., Seshadri, N., and Calderbank, A. R.,** (1997a). "Space-Time Codes for Wireless Communication: Code Construction", *IEEE VTC*.

4. **Tarokh, V., Seshadri, N., and Calderbank, A. R.,** (1997b). "Space-Time Codes for High Data Rate Wireless Communication: Performance Criterion and Code Construction", *IEEE Transaction on Information Theory*, Vol. 44, No. 2, pp. 744-765, March.

5. **Jung C. C.**, "Performance Of Space-Time Coded Modulation Over Frequency-Selective Rayleigh Fading Channels", *Research Report*, *Asian Institute of Technology*, *1999*.