Iterative Receiver For Coded Modulation Schemes

- This section investigates the performance of coded modulation schemes over frequency selective Rayleigh fading channels.

- The combined receiver doing joint equalization and decoding of coded signals which significantly cancels out the detrimental effect of intersymbol interference is presented.

- The proposed receiver is applied to trellis coded modulation and turbo trellis coded modulation schemes. The performance of the new receiver is compared with the conventional approach (i.e. separate equalizer and decoder) through simulation. It is shown that the new receiver considerably improves the error performance after only a small number of iterations.

Introduction

- The introduction of turbo codes (**Berrou** et al., 1993) has resulted in various applications in wireless communications, deep space communications, etc. With the remarkable performance of turbo codes, it is natural to combine turbo codes with multilevel modulation schemes in order to obtain large coding gains and high bandwidth efficiency over both AWGN and fading channels.

- Turbo trellis coded modulation (TTCM) proposed in (**Robertson** and **Woerz**, 1995) is the extension of the turbo codes where the component codes are replaced by Ungerboeck TCM codes in the recursive systematic forms to retain the advantages of both classical turbo codes and TCM codes.

- Turbo equalization introduced by authors in (**Douillard** et al., 1995) was the first effort to apply iterative principle to equalizing convolutionally coded signals transmitted over intersymbol interference channels.

- Recently, there have been some researches that were focusing on turbo equalization concept for convolutionally coded systems (**Bauch** et al., 1997), turbo coded systems (**Raphaeli** and **Zarai**, 1997), and serially concatenated systems (**Li** and **Mow**, 1999). In all of those, binary modulation was considered. The equalization and decoding blocks were basically based on soft-in soft-out (SISO) algorithms.

- In the work of **Glavieux** et al. (1997), multilevel modulation was considered but the equalizer used there was a kind of an adaptive one.

- It is well known that performance of the multilevel coded modulation schemes over frequency selective channels can be improved by employing SISO modules for both the equalizer and the decoder. In the conventional approach, the reliability information about the coded symbols from the equalizer is transferred to the decoder, but the equalizer does not utilize the corresponding soft values out of the decoder.

- In what follows, we use iterative decoding process to do the equalization and decoding of multilevel coded modulation signals over frequency selective Rayleigh fading channels.

- Since we are working at the symbol level, the decoder is modified to be able to output the soft information of both uncoded and coded symbols.

- In the multilevel coded modulation schemes, since the systematic bits are transmitted together with the parity bits in the same symbol, noise perturbs simultaneously both parity component and systematic component.

- The soft outputs of the decoder must contain soft information of the coded symbols to be used at the equalizer as the a priori information. The decoder is modified from the conventional SISO decoder. It is desirable to keep the new receiver similar to the conventional one as much as possible.

- In the decoder, a functional block is added to give out the coded symbols' extrinsic soft information. For the SISO blocks, the low-complexity, sub-optimum Max-Log-MAP algorithm is used.

- The remainder of this section is organized as follows. Section 2 describes the system model. The explicit expansion of the SISO-MAP algorithm for the equalizer and decoder is discussed in section 3.

2 System Model

Fig. 1 shows the block diagram of the communication system we are going to investigate.

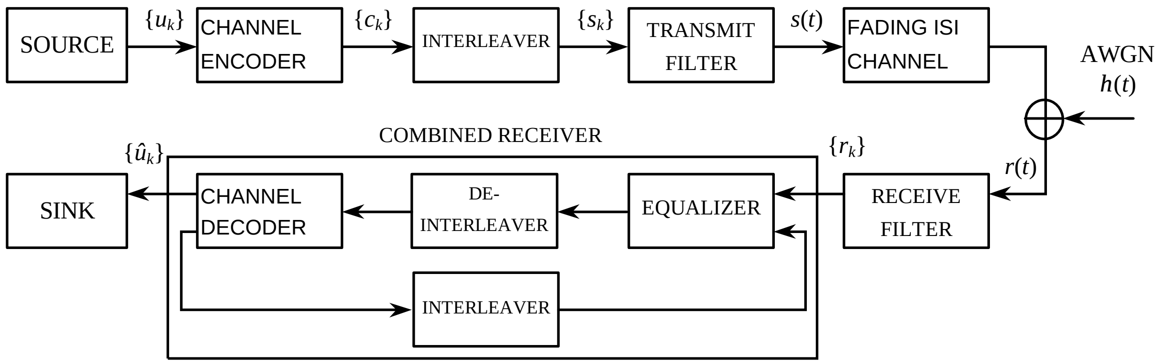

Fig. 1. System model for communication system with combined channel equalizer and channel decoder using iterative process

- The information sequence $\{u_k\}$, with u_k takes on values 1 or 0, which is supposed to be equally likely distributed, is coming our of the source. The information sequence is encoded by a coded modulation encoder, which can be TCM or TTCM.

- The output symbol sequence $\{c_k\}$, where c_k is taken from one of M possible values (M-ary modulation), is block interleaved to break up burst errors caused by the fading channel before being transmitted into the channel.

- We assume that the interleaving depth is infinite, so that the fading is uncorrelated. In reality, even though this assumption cannot be met, by proper use of interleaving depth greater than the maximum fade duration, the correlation is negligible.

- The transmit and receive filters are used for spectrum shaping. They are chosen to be Nyquist filters and are matched to each other. Let $p(t)$ be the impulse response of the filters, the transmitted signal can be expressed as

$$s(t) = \sum_k s_k p(t - kT) \tag{1}$$

- where T is the symbol interval, s_k is the interleaved version of the TCM coded signal constellation point c_k.

- The overall channel model consists of a transmit filter, the fading channel, and a receive filter. It can often be modeled as a finite impulse response (FIR) filter having (L+1) taps $g(k) = (g_0(k), g_1(k), \ldots, g_L(k))$ at symbol instant k (**Proakis**, 1995).

The channel output at time instant kT is corrupted by the AWGN h_k, and the received signal r_k is then given by

$$r_k = \sum_{i=0}^{L} s_{k-i} g_i(k) + h_k . \tag{2}$$

- The additive Gaussian noise, after being filtered by the receive filter, becomes correlated in general. We still assume that the correlation is negligible so that $\{h_k\}$ is the sequence of statistically independent and identically distributed (i.i.d.) complex Gaussian noise samples.

- The probability density function (pdf) of each sample is given by

$$r(h_k) = \frac{1}{2ps_N^2} \exp\left(-\frac{|h_k|^2}{2s_N^2} \right) \tag{3}$$

- where s_N^2 is the noise variance in each signal space coordinate and $|h_k|$ is the magnitude of the complex noise sample h_k.

3 Iterative Equalization and Decoding

- Since we are going to deal with trellis diagrams, the following definitions are used to describe the channel as well as the encoder trellis diagrams. These definitions mostly follow (**Forney**, 1973).

State: A state s_k at time k is defined as

$$s_k = (s_{k-L}, s_{k-L+1}, \ldots, s_{k-2}, s_{k-1}), \text{ where } s_i = 0 \text{ while } i < 0. \tag{4}$$

Transition: A transition x_k is defined as

$$x_k = (s_k, s_{k+1}) = (s_{k-L}, s_{k-L+1}, ..., s_{k-2}, s_{k-1}, s_k), \quad \text{where } s_i = 0 \text{ while } i < 0 \\ = (s_k, s_k). \tag{5}$$

Additive Branch Metric (ABM): An ABM is the *squared Euclidean distance* caused by the transition x_k

$$l_{k+1}(x_k) = l_{k+1}(s_k, s_{k+1}) = \left| r_k - \sum_{i=0}^{L} s_{k-i} g_i(k) \right|^2 . \tag{6}$$

Multiplicative Branch Metric (MBM): An MBM of branch x_k is defined as

$$m_{k+1}(x_k) = m_{k+1}(s_k, s_{k+1}) = p(s_{k+1}, r_k \mid s_k) \\ = p(r_k \mid s_k, s_{k+1}) P(s_{k+1} \mid s_k) \\ = r(l_{k+1}(x_k)) P(s_{k+1} \mid s_k) \tag{7}$$

where $r(l_{k+1}(x_k)) = p(r_k \mid s_k, s_{k+1})$.

SISO-MAP Module

- Consider a trellis code with *m*-tuple input and *n*-tuple output. Let a group of *m* information bits at time *k* be represented by a symbol u_k, and a group of *n* output bits be represented by a symbol c_k. We simply call u_k and c_k the *uncoded* and *coded* symbols respectively.

- If the code is systematic, c_k is composed of u_k and the parity bits. Associating with each transition x_k is the uncoded symbol u_k and the coded symbol c_k.

- Basically, an SISO module is a four port device with two inputs and two outputs. The inputs to the module are the (logarithm of) a priori probability sequences of the uncoded and coded symbols denoted by $Li(u_k)$ and $Li(c_k)$ respectively. The outputs of it are the (logarithm of) extrinsic probability sequences of the uncoded and coded symbols, which are correspondingly denoted by $Lo(u_k)$ and $Lo(c_k)$.

Equalizer SISO-MAP Module

- Clearly, we can treat the equivalent discrete-time channel as a rate one feed forward time variant convolutional encoder.

- The equalizer SISO module does the estimation of the transmitted channel symbols s_k basing on the complete received signal sequence $\{r_k;\ k=1, 2 .., K\}$. It calculates the following a posteriori probability (**Bahl** et al., 1974)

$$P\left(s_k \mid r_1^K\right) = H_1 \cdot \sum_{s_k, s_{k+1}: s_k} p\left(s_k, r_1^{k-1}\right) \cdot p\left(s_{k+1}, r_k \mid s_k\right) \cdot p\left(r_{k+1}^K \mid s_{k+1}\right) \quad (8)$$

- where the summation is taken over all pairs of states (s_k, s_{k+1}) so that the transitions between them contain the symbol s_k and H_1 is a normalization constant.

Let

$$a_k(s_k) = p\left(s_k, r_1^{k-1}\right);\ m_{k+1}(x_k) = p\left(s_{k+1}, r_k \mid s_k\right);\ b_k(s_k) = p\left(r_k^K \mid s_k\right).$$

We have the following relations:

- **Transition probabilities**

$$m_{k+1}(x_k) = m_{k+1}(s_k, s_{k+1}) = r(\, l_{k+1}(x_k))P(s_{k+1} \mid s_k)$$
$$= \begin{cases} r(\, l_{k+1}(x_k))P(s_k), & \text{while } x_k \text{ contains } s_k \\ 0, & \text{elsewhere.} \end{cases} \quad (9)$$

- **Forward recursion**

$$a_k(s_k) = \sum_{s_{k-1}} p\left(s_{k-1}, r_1^{k-2}\right) \cdot m_k(x_{k-1}) \qquad (10)$$
$$= \sum_{s_{k-1}} a_{k-1}(s_{k-1}) \cdot m_k(s_{k-1}, s_k).$$

- **Backward recursion**

$$b_k(s_k) = \sum_{s_{k+1}} p\left(r_{k+1}^K \mid s_{k+1}\right) \cdot p\left(s_{k+1}, r_k \mid s_k\right) \qquad (11)$$
$$= \sum_{s_{k+1}} b_{k+1}(s_{k+1}) \cdot m_{k+1}(s_k, s_{k+1}).$$

- The output of the equalizer SISO-MAP can be obtained from the MBM of each branch x_k and from the forward and backward recursions as

$$p\left(s_k \mid r_1^K\right) = H_1 \cdot \sum_{s_k, s_{k+1}: s_k} a_k(s_k) \cdot m_{k+1}(s_k, s_{k+1}) \cdot b_{k+1}(s_{k+1}). \qquad (12)$$

- If we assume that the trellis states stay at zero state at the beginning as well as at the end of the received signal sequence, the initializations of $a_k(s_k)$ and $b_k(s_k)$ are as follows:

$$a_1(0) = 1 \text{ and } a_1(s_k) = 0, \quad \forall s_k \neq 0. \qquad (13)$$

$$b_K(0) = 1 \text{ and } b_K(s_k) = 0, \quad \forall s_k \neq 0. \qquad (14)$$

- From Eqs. (9)-(12), we observe that the a priori probability $P(s_k)$ can be extracted from the summation in Eq. (12).

We define the new quantity

$$P_e\left(s_k \mid r_1^K\right) = H_2 \frac{P\left(s_k \mid r_1^K\right)}{P(s_k)} \qquad (15)$$

where H_2 is a normalization constant.

- It is clear that $P_e\left(s_k \mid r_1^K\right)$ does not depend on the a priori probability $P(s_k)$. In the nature of turbo decoding, $P_e\left(s_k \mid r_1^K\right)$ is called extrinsic probability.

- Now, we define the scaled transition probability $m_{k+1}^*(x_k)$ as

$$m_{k+1}^*(x_k) = \frac{m_{k+1}(x_k)}{P(s_k)} = \frac{r(\, l_{k+1}(x_k))P(s_{k+1} \mid s_k)}{P(s_k)}$$
$$= \begin{cases} r(\, l_{k+1}(x_k)), & \text{while } x_k \text{ contains } s_k \\ 0, & \text{elsewhere.} \end{cases} \qquad (16)$$

- The extrinsic probability $P_e\left(s_k \mid r_1^K\right)$ is calculated through the following equation

$$p_e\left(s_k \mid r_1^K\right) =$$
$$H_1 \cdot H_2 \cdot \sum_{s_k, s_{k+1}: s_k} a_k(s_k) \cdot m_{k+1}^*(s_k, s_{k+1}) \cdot b_{k+1}(s_{k+1}). \qquad (17)$$

- Because of the *monotonicity* of the logarithm function, we can simplify all the previous probability calculations by taking the natural logarithm of them.

- Define the following

$$Li\left(s_k\right) = L\mathbf{og}P\left(s_k\right). \qquad (18)$$

$$Lo(s_k) = LogP_e\left(s_k \mid r_1^K\right). \qquad (19)$$

$$\bar{a}_k(s_k) = Log\,a_k(s_k). \tag{20}$$

$$\bar{b}_k(s_k) = Log\,b_k(s_k). \tag{21}$$

$$\bar{m}_{k+1}(s_k, s_{k+1}) = Log\,m_{k+1}(s_k, s_{k+1}). \tag{22}$$

$$\bar{m}^*_{k+1}(s_k, s_{k+1}) = Log\,m^*_{k+1}(s_k, s_{k+1}). \tag{23}$$

$$L^c(s_k) = -\frac{l_{k+1}(x_k)}{2s_N^2} - Log\left(2ps_N^2\right) \quad \text{(channel information).} \tag{24}$$

- The relation between $\bar{m}^*_{k+1}(x_k)$ and the channel information $L^c(s_k)$ is as follows:

$$\bar{m}^*_{k+1}(x_k) = \begin{cases} L^c(s_k), & \text{while } x_k \text{ contains } s_k \\ -\infty, & \text{elsewhere.} \end{cases} \tag{25}$$

- When working at medium high SNR regions, we can use the approximation

$$Log(e^{d_1} + e^{d_2} + \cdots + e^{d_n}) \approx \max_{i \in \{1,\ldots,n\}} d_i \tag{26}$$

- to arrive to Max-Log-MAP algorithm (**Robertson** et al., 1995), (**Ha** and **Rajatheva,** 1999)

$$\bar{m}_{k+1}(s_k, s_{k+1}) = -\frac{l_{k+1}(x_k)}{2s_N^2} - Log\left(2ps_N^2\right) + Log P(s_{k+1} \mid s_k)$$
$$= \begin{cases} Li(s_k) + L^c(s_k), & \text{while } x_k \text{ contains } s_k \\ -\infty, & \text{elsewhere.} \end{cases} \tag{27}$$

$$\bar{a}_k(s_k) = \max_{s_{k-1}} \{\bar{a}_{k-1}(s_{k-1}) + \bar{m}_k(s_{k-1},\ s_k)\}. \qquad (28)$$

$$\bar{b}_k(s_k) = \max_{s_{k+1}} \{\bar{b}_{k+1}(s_{k+1}) + \bar{m}_{k+1}(s_k, s_{k+1})\}. \qquad (29)$$

$$Lo(s_k) = \max_{s_k, s_{k+1}: s_k} \left\{\bar{a}_k(s_k) + \bar{m}^*_{k+1}(s_k, s_{k+1}) + \bar{b}_{k+1}(s_{k+1})\right\} + h_1 \qquad (30)$$

where h_1 is a constant used to avoid numerical overflow.

- *We summarize the Max-Log-MAP SISO algorithm for the equalizer as follows:*

(1) Initialization:

$$\bar{a}_1(0) = 0 \text{ and } \bar{a}_1(s_k) = -\infty, \quad \forall s_k \neq 0. \qquad (31)$$

$$\bar{b}_K(0) = 0 \text{ and } \bar{b}_K(s_k) = -\infty, \quad \forall s_k \neq 0. \qquad (32)$$

(2) Forward recursion at each time k: Compute $\bar{a}_k(s_k)$ through Eq. (28).

(3) Backward recursion, when r_i^K has been completely received:
Compute $\bar{b}_k(s_k)$ through Eq. (29).

(4) Log extrinsic probability: Compute $Lo(s_k)$ through Eq. (30).

Decoder SISO-MAP Module

- The SISO-MAP module for the trellis decoder calculates the logarithm of extrinsic probabilities of the uncoded and coded symbols in the similar way as that of the equalizer, i.e., as Eq. (30).

$$Lo(u_k) = \max_{s_k, s_{k+1}: u_k} \left\{\bar{a}_k(s_k) + \bar{m}^{*,c}_{k+1}(s_k, s_{k+1}) + \bar{b}_{k+1}(s_{k+1})\right\} + h_2. \qquad (33)$$

$$Lo(c_k) = \max_{s_k, s_{k+1}: c_k} \left\{ \bar{a}_k(s_k) + \bar{m}^{*,u}_{k+1}(s_k, s_{k+1}) + \bar{b}_{k+1}(s_{k+1}) \right\} + h_3. \qquad (34)$$

- The main difference compared with the equalizer SISO-MAP is in the calculation of the new extrinsic probability for the coded symbols, and the transition probabilities due to the fact that, the decoder is not directly connected to the channel.

- The decoder SISO-MAP uses the available probabilistic information from the equalizer to calculate its transition probabilities as follows

$$\bar{m}_{k+1}(s_k, s_{k+1}) = \begin{cases} Li(c_k) + Li(u_k), & \text{while } x_k \text{ contains } u_k \text{ and } c_k \\ -\infty, & \text{elsewhere} \end{cases} \qquad (35)$$

\- where $Li(c_k)$ is set equal to the extrinsic value coming out of the equalizer, i.e. equal to $Lo(s_k)$ for $c_k \equiv s_k$.

- The scaled transition probabilities $\bar{m}^{*,c}_{k+1}(s_k, s_{k+1})$ and $\bar{m}^{*,u}_{k+1}(s_k, s_{k+1})$ in the summations from Eqs. (33) and (34) are calculated as

$$\bar{m}^{*,c}_{k+1}(x_k) = \begin{cases} Li(c_k), & \text{while } x_k \text{ contains } c_k \\ -\infty, & \text{elsewhere.} \end{cases} \qquad (36)$$

$$\bar{m}^{*,u}_{k+1}(x_k) = \begin{cases} Li(u_k), & \text{while } x_k \text{ contains } u_k \\ -\infty, & \text{elsewhere.} \end{cases} \qquad (37)$$

\- The forward and backward recursions are still calculated the same as Eqs. (28) and (29) respectively.

The Combined Receiver for TCM

- Figure 2 shows the combined receiver applied to TCM.

- For the TCM scheme, the equalizer observes the distorted version of transmitted signal all the time. The equalizer SISO module does not need to include the constant $-Log\left(2ps_{N}^{2}\right)$ in the calculation of transition probabilities as far as the maximization is concerned.

- At the first iteration, we set the a priori probability to the equalizer equal zero. After that, the a priori information is extracted from the TCM decoder SISO-MAP.

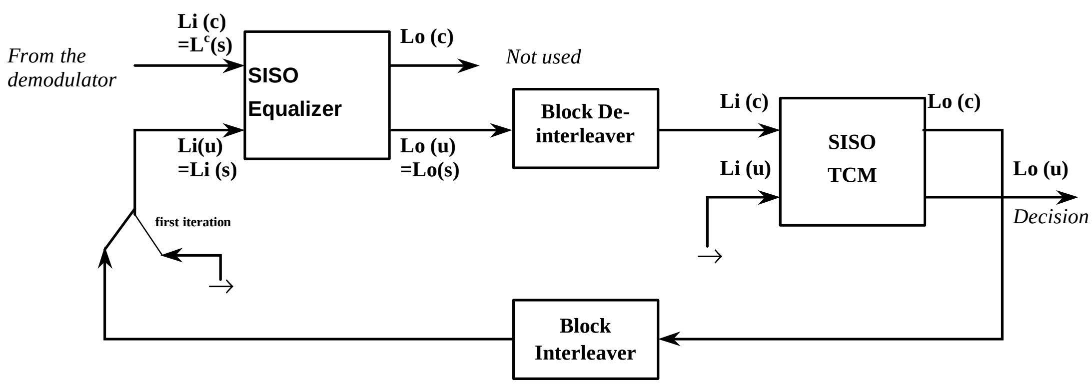

Fig. 2 Combined receiver for TCM

The Combined Receiver for TTCM

- Fig. 3 shows the combined receiver applied to TTCM. The equalizer alternatively observes the distorted version of the transmitted signals form the upper and lower component TCM encoders.

- We set the a priori probability to the equalizer SISO the same as TCM scheme. For the first decoding step of the upper decoder (half iteration), we set $Li(u) = -mLog2;$ where $m/(m+1)$ is the code rate of component TCM's.

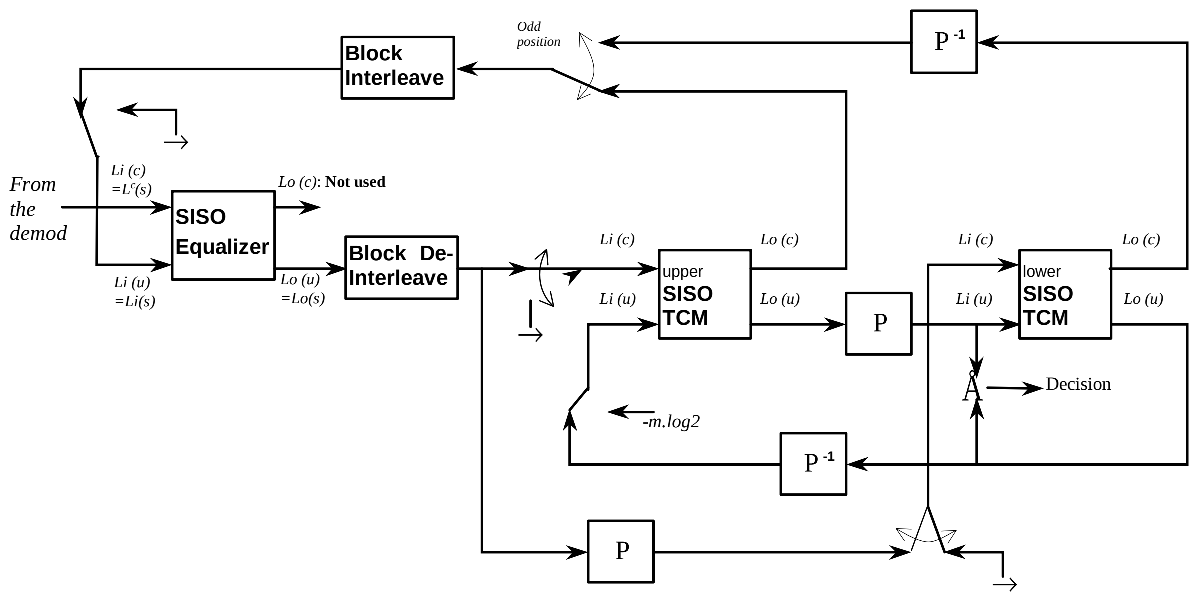

Figure 3 Combined receiver for TTCM

Examples and Results

- In the simulation, frequency selective Rayleigh fading channels with two equal paths are used.

- Different component codes using 4/8-PSK modulation schemes are investigated. The information sequence has the block lengths of 800 and 2000 bits. The block interleavers of sizes 20x20 and 25x40 are utilized to reduce the burst error effects of the fading.

- The channel state information is assumed perfectly known at the receiver. The selection of component TCM codes for the simulation is based on the optimum TCM's found so far for AWGN and flat fading channels. They are all in recursive systematic forms.

- Simulation was carried out with 3 and 6 iterations for TCM and TTCM schemes. Since our intention is not to design the optimum interleavers for the TTCM scheme, we set the interleaving index of the odd-even random interleavers to be changed after each block of input bits. This gives us the average simulation results over all possible interleavers.

- We first start with the QPSK scheme. Fig. 4 shows the performance of 4-state, QPSK-TCM with combined decoding. The fading is independent.

\- We should note that performance of the first iteration for the combined scheme is exactly the same as the case of the separate equalization and decoding receiver. We can see that at the second iteration, the error performance can be improved with about 1dB coding gain. The BER cannot be improved much after further iterations.

- Figures 5 and 6 show the performance of 8 state, 8PSK-TCM with block length 800 and 2000 respectively. The fading is independent. We observe that coding gains of 1.5-2 dB can be achieved after 2 iterations.

\- The coding gains are considerably improved to compare with the performance of the QPSK scheme. From this point, we can expect that our combined receiver can achieve more coding gains for coded modulation schemes using larger constellation sizes.

- Fig. 7 compares the performance 8 state, 8PSK-TCM with block lengths 800 and 2000 over the independent fading channel.

\- We observe that the performance of the scheme does not improve for the larger block length (2000). It can be explained as follows.

- Because the fading ISI channel is a kind of non-recursive convolutional encoder, the interleaver gain for the serially concatenated structure is not significant (**Benedetto** et al., 1998).

- Fig. 8 shows the performance of 16 state, 8PSK-TCM with block length 800.

- Fig. 9 shows the comparison between 8 state 16 state TCM's. In some SNR regions, the performance of the 16 state scheme is even worse than that of the 8 state scheme. This is because we do not use the optimum TCM encoders for the frequency selective channel. This justifies the need for code designs over this kind of channel.

- Fig. 10 shows the performance of 8 state, 8PSK-TTCM with block length 800. The receiver is the separate equalizer and decoder scheme.

- Fig. 11 illustrates the performance of 8 state, 8PSK-TTCM with block length 800. The receiver is the combined equalizer and decoder scheme. In both figures, the error performances are improved significantly after the third iteration.

- Fig. 12 shows that the proposed receiver outperforms the separate one for 8-state, 8PSK-TTCM, with block length 800. The coding gain of about 1.5 dB is seen. This is a good result because the BER against SNR curse goes down steeply within a small region of SNR.

- Similarly, Figures 13-16 show the performance of the proposed and separate receivers applied to TCM and TTCM over correlated Rayleigh fading with the maximum Doppler frequency of 105 Hz (mobile speed $v = 60\,\mathbf{km/h}$; carrier frequency $f_c = 1.9\,\mathbf{GHz}$).

- Performance degradations are about 7 dB for TCM scheme and about 9 dB for TTCM scheme to be compared with the independent fading case.

- We observe that the performance of the proposed receiver is quite modest for correlated channels, with coding gain of roughly about 0.8 dB at the third iteration to be compared with the separate one. Performance of the TTCM scheme does not improve much after 3 iterations. It is also seen that TTCM scheme with short interleaver size (=800 in our case) fails to outperform TCM when working over correlated fading channels.

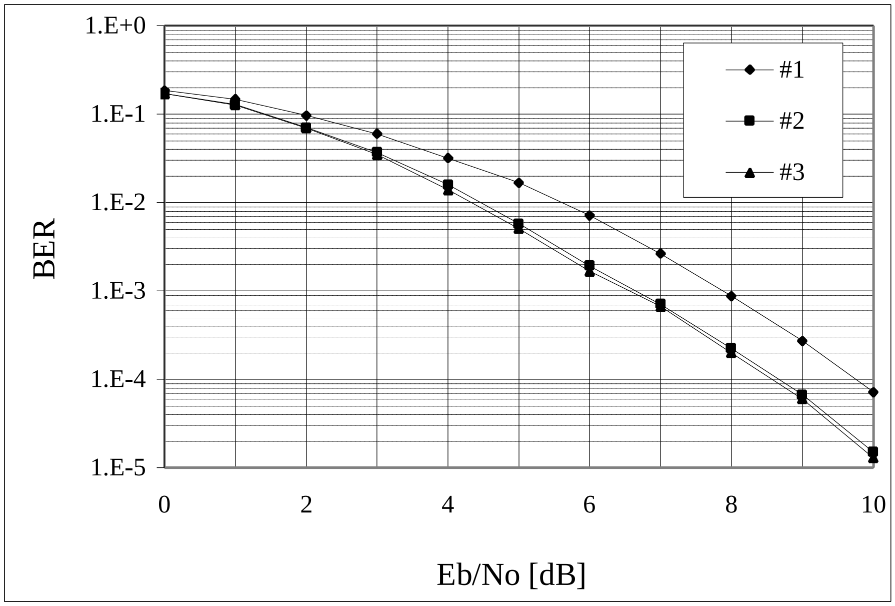

Fig. 4. Performance of 4-state, QPSK-TCM with h(0)=11, h(1)=02.
Block length=800. Independent fading.

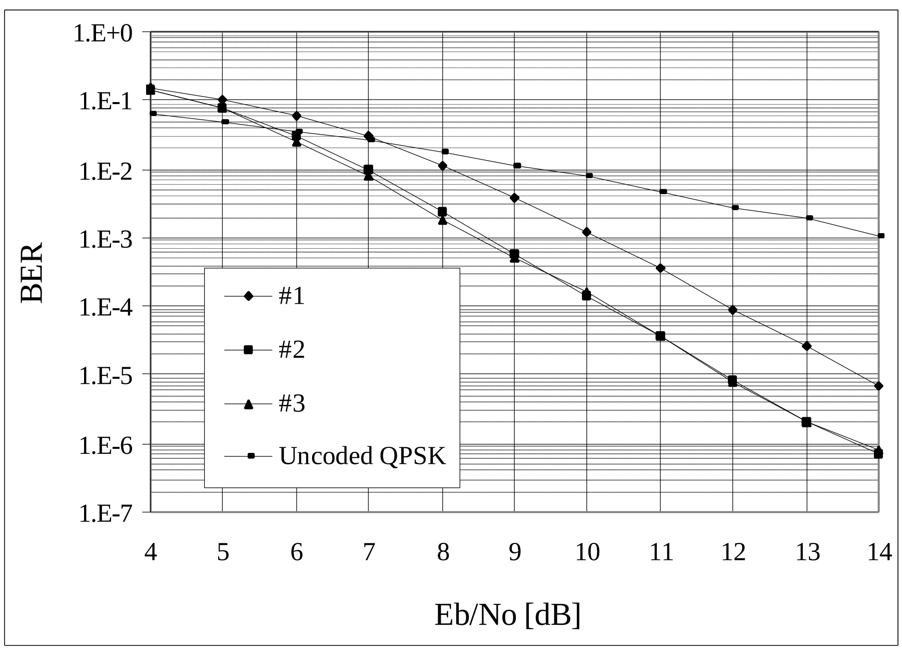

Fig. 5. Performance of 8-state, 8PSK-TCM with h(0)=11, h(1)=02, h(2)=04.
Block length=800. Independent fading.

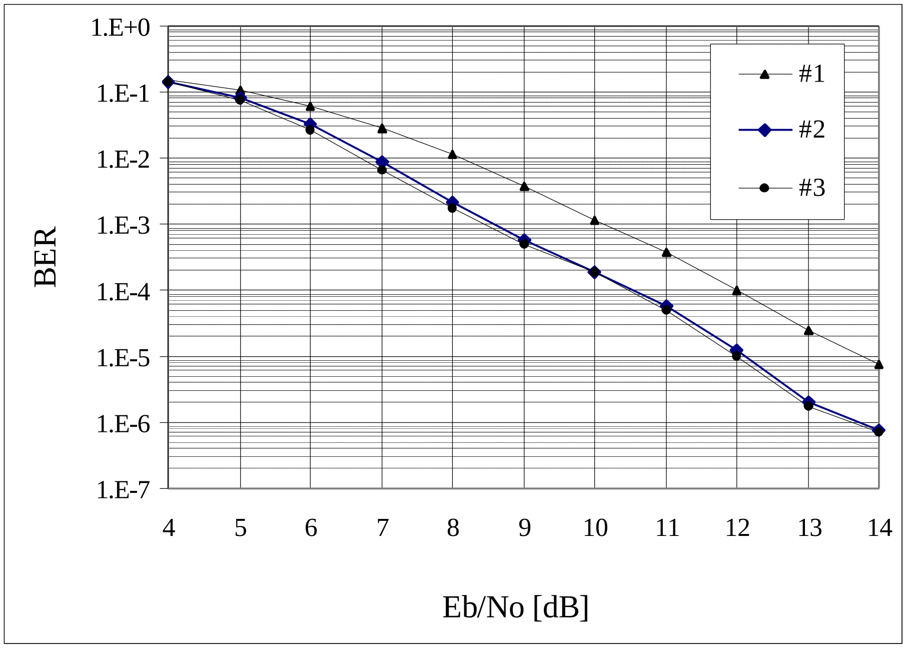

Fig. 6. Performance of 8-state, 8PSK-TCM with h(0)=11, h(1)=02, h(2)=04. Block length=2000. Independent fading.

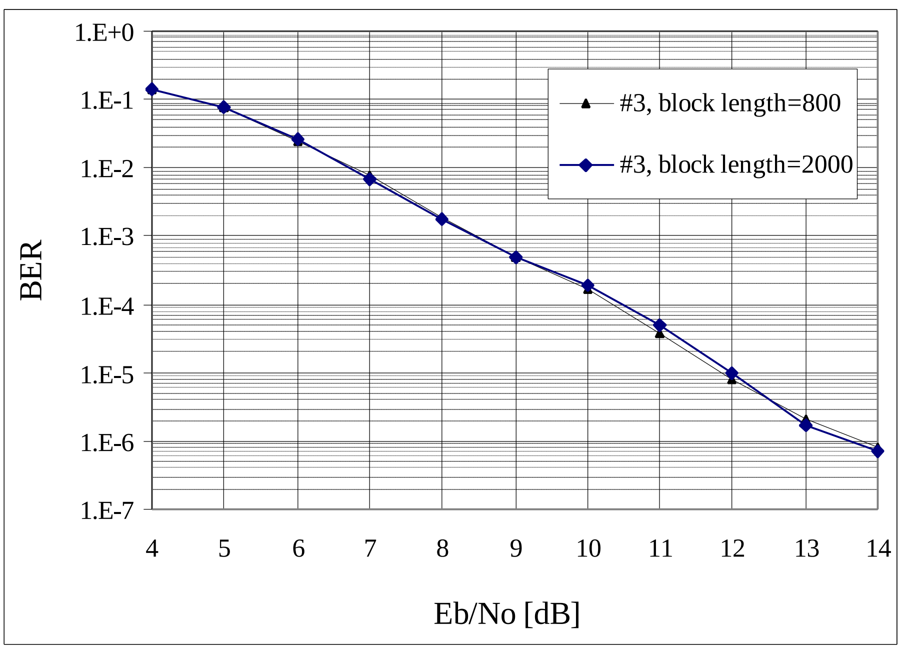

Fig. 7. Comparison of- 8 state, 8PSK-TCM with h(0)=11, h(1)=02, h(2)=04 having different block lengths. Independent fading.

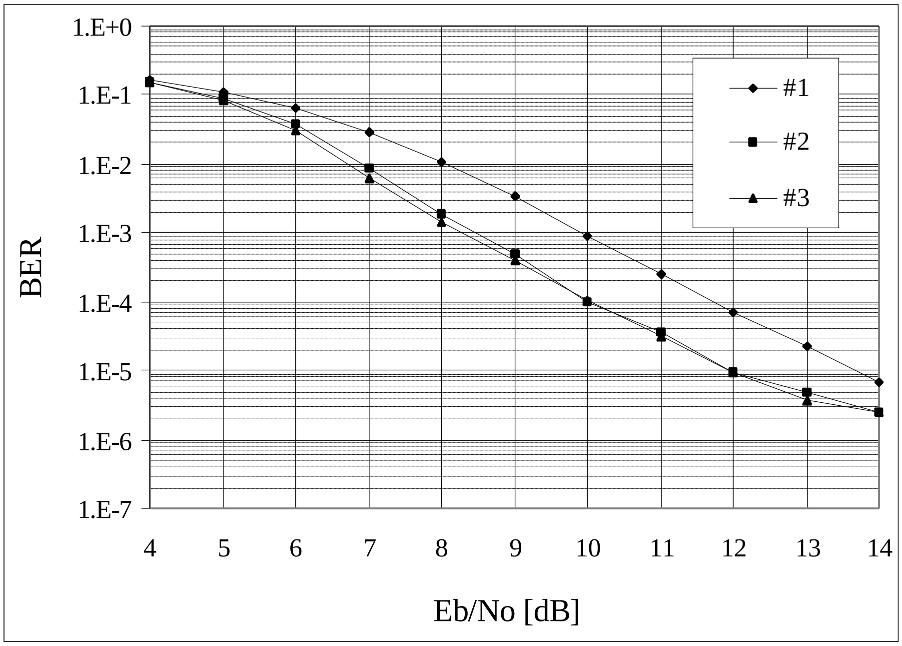

Fig. 8. Performance of 16-state, 8PSK-TCM with h(0)=21, h(1)=02, h(2)=04, h(3)=10. Block length=800. Independent fading.

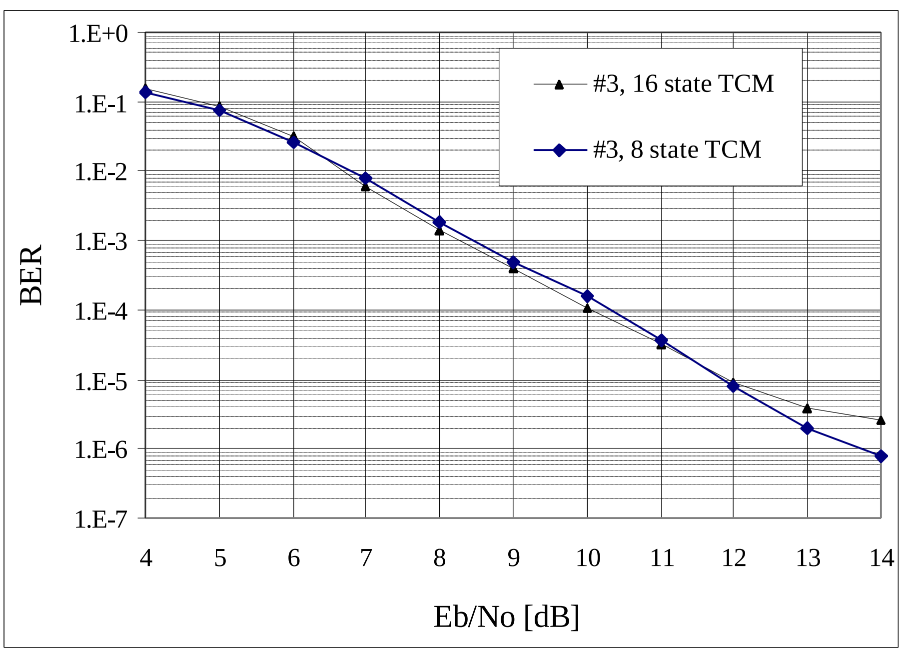

Fig. 9. Comparison of 8PSK-TCM, block length=800 with different states. Independent fading.

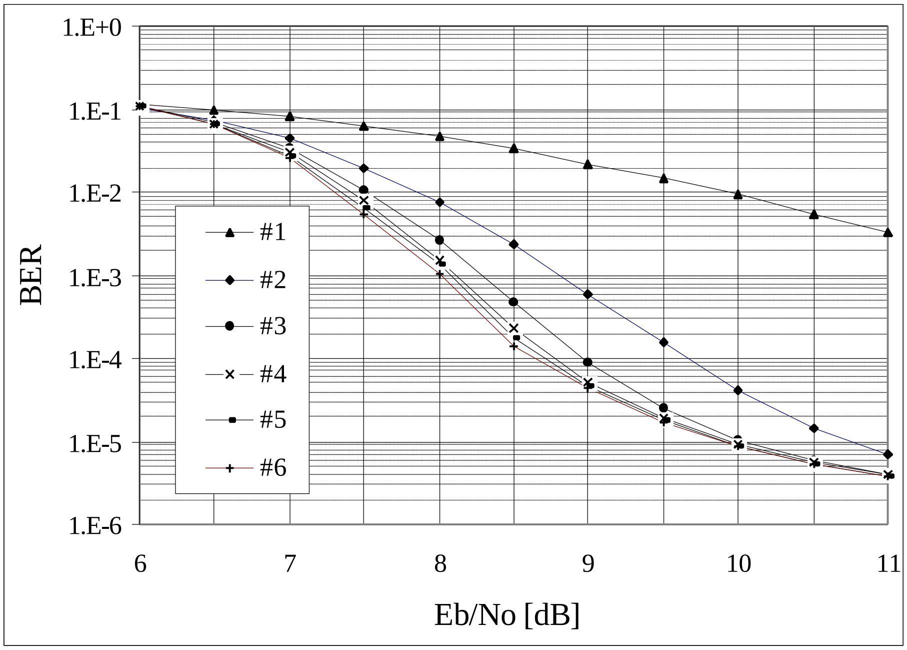

Fig. 10. Performance of separate receiver for TTCM with 8-state, 8PSK-TCM component codes having h(0)=11, h(1)=02, h(2)=04. Block length=800. Independent fading.

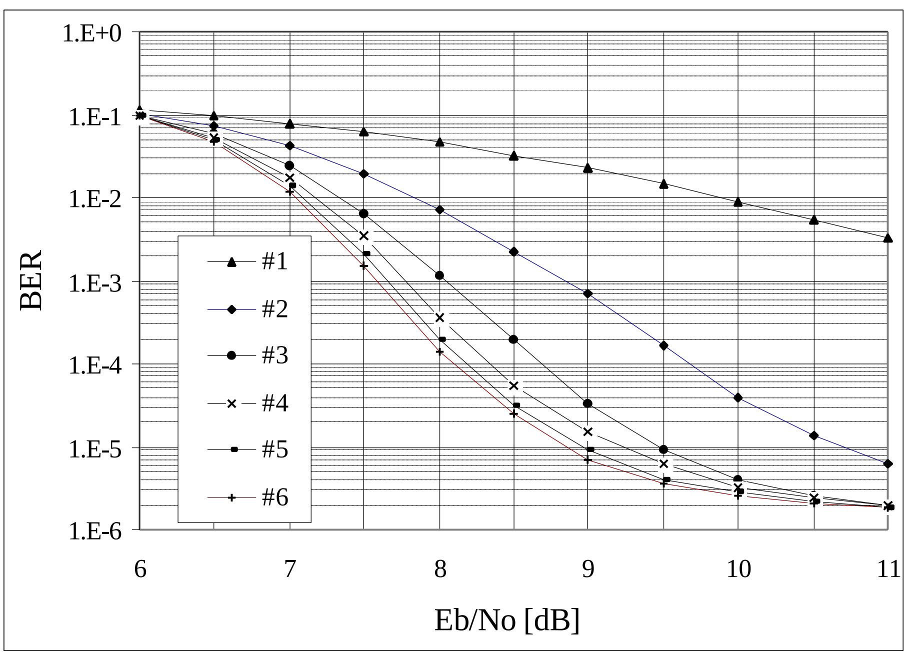

Fig. 11. Performance of combined receiver for TTCM with 8-state, 8PSK-TCM component codes having h(0)=11, h(1)=02, h(2)=04. Block length=800. Independent fading.

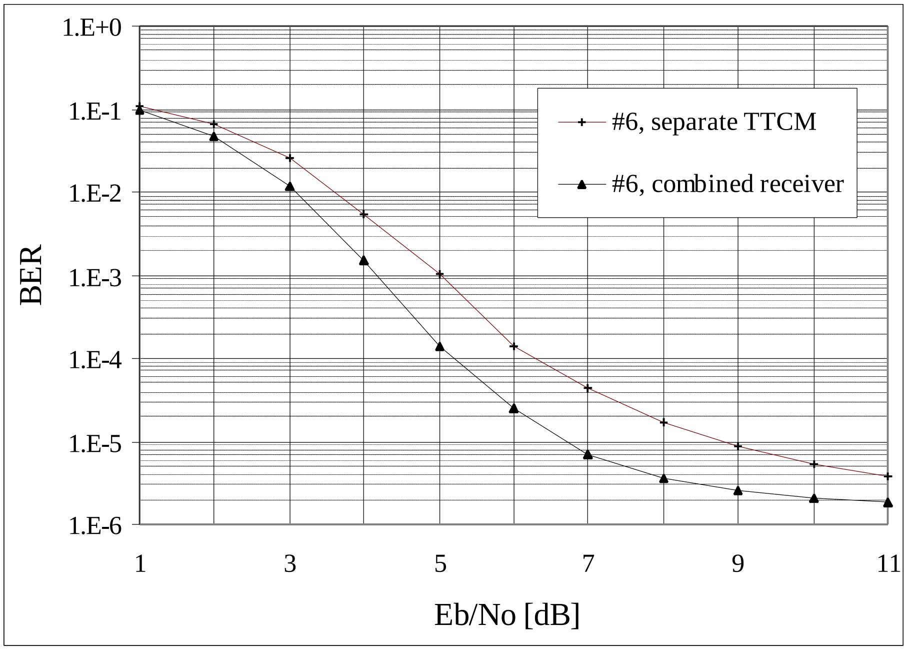

Fig. 12. Comparison between combined receiver and separate one for TTCM scheme having 8-state, 8PSK-TCM component codes, with h(0)=11, h(1)=02, h(2)=04. Block length=800. Independent fading.

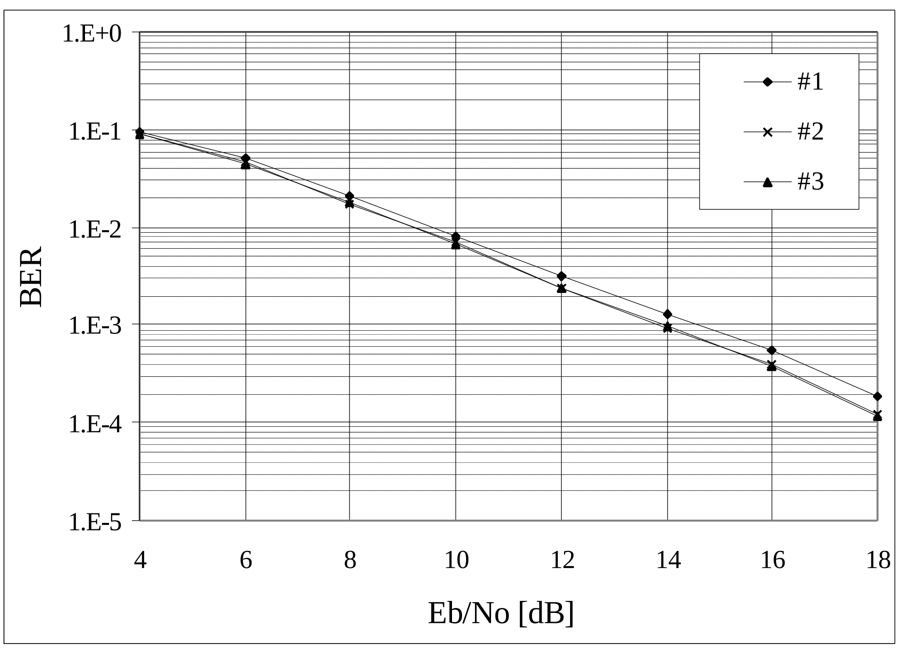

Fig. 13. Performance of 8-state, 8PSK-TCM with h(0)=11, h(1)=02, h(2)=04. Block length=800. Correlated fading with f_m=105 Hz.

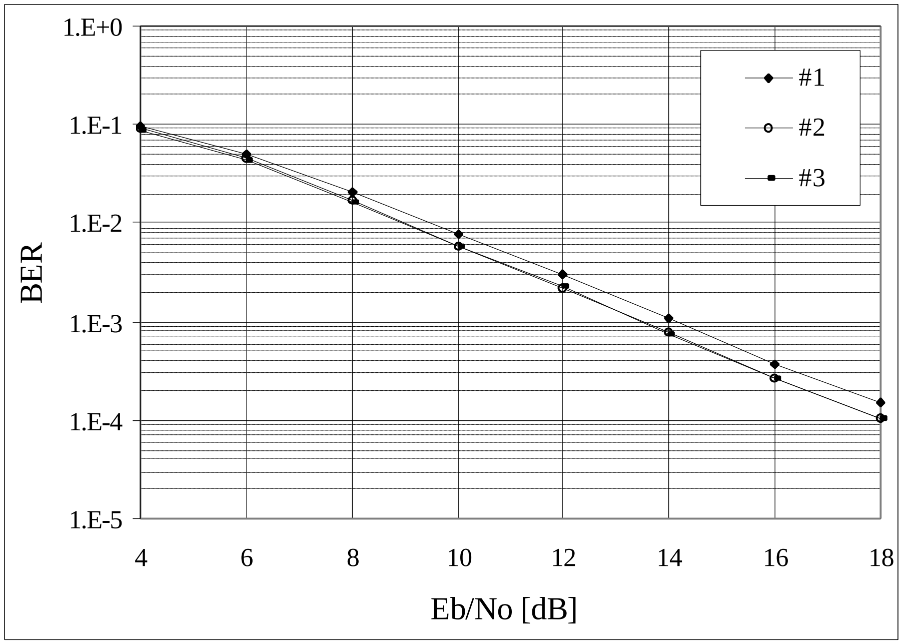

Fig. 14. Performance of 8-state, 8PSK-TCM with h(0)=11, h(1)=02, h(2)=04. Block length=2000. Correlated fading with f_m=105 Hz.

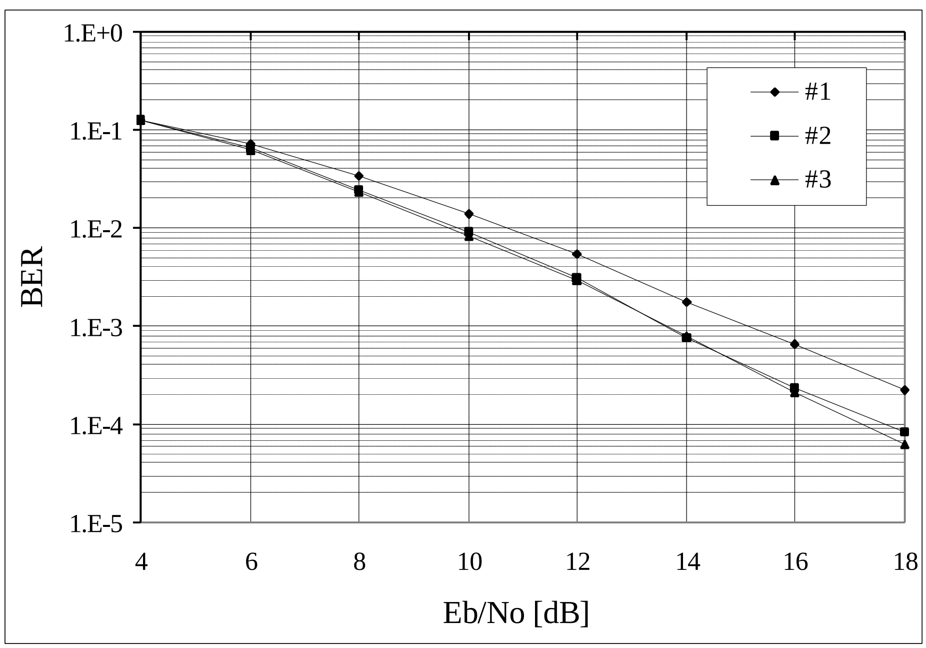

Fig. 15. Performance of separate receiver for TTCM with 8-state, 8PSK-TCM component codes having h(0)=11, h(1)=02, h(2)=04. Block length=800. Correlated fading with f_m=105 Hz.

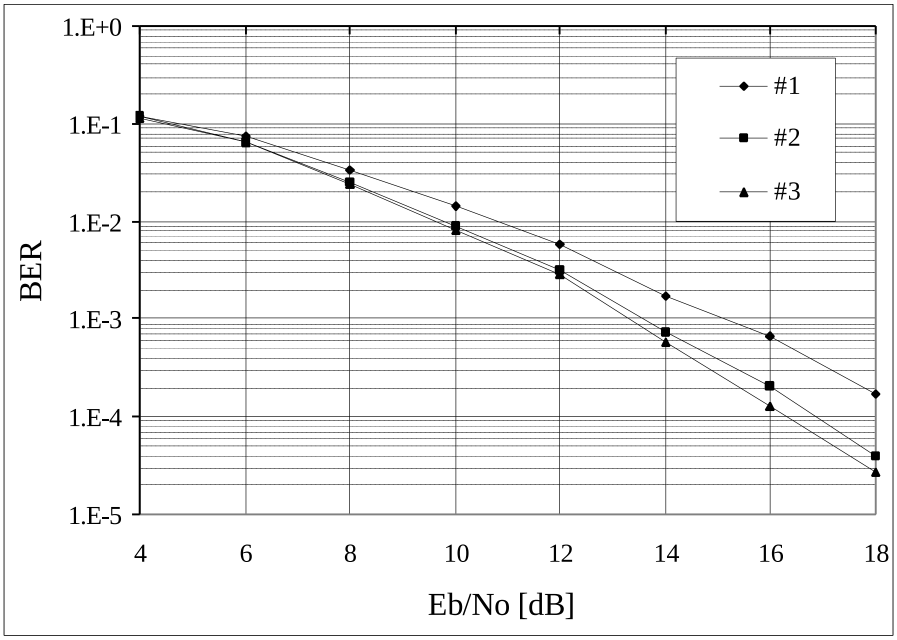

Fig. 16. Performance of combined receiver for TTCM with 8-state, 8PSK-TCM component codes having h(0)=11, h(1)=02, h(2)=04. Block length=800. Correlated fading with f_m=105 Hz.

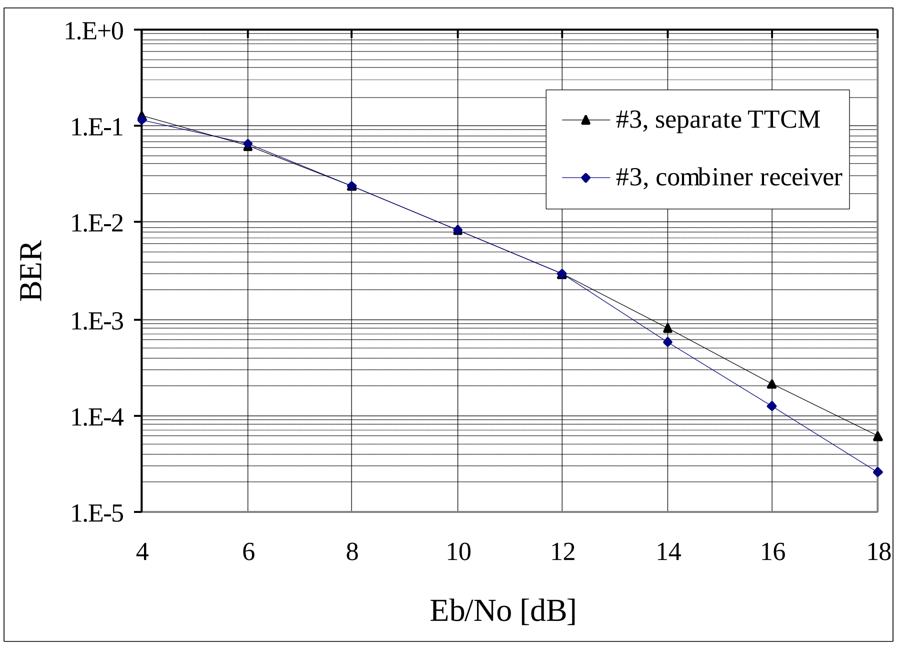

Fig. 17. Comparison between combined receiver and separate one for TTCM having 8-state, 8PSK-TCM component codes, with h(0)=11, h(1)=02, h(2)=04. Block length=800. Correlated fading with f_m=105 Hz.