

3. Turbo Equalization with *Pragmatic* Approach

- Figure 3.1 shows the overall transmission model for the system using combined turbo equalization and turbo decoding.

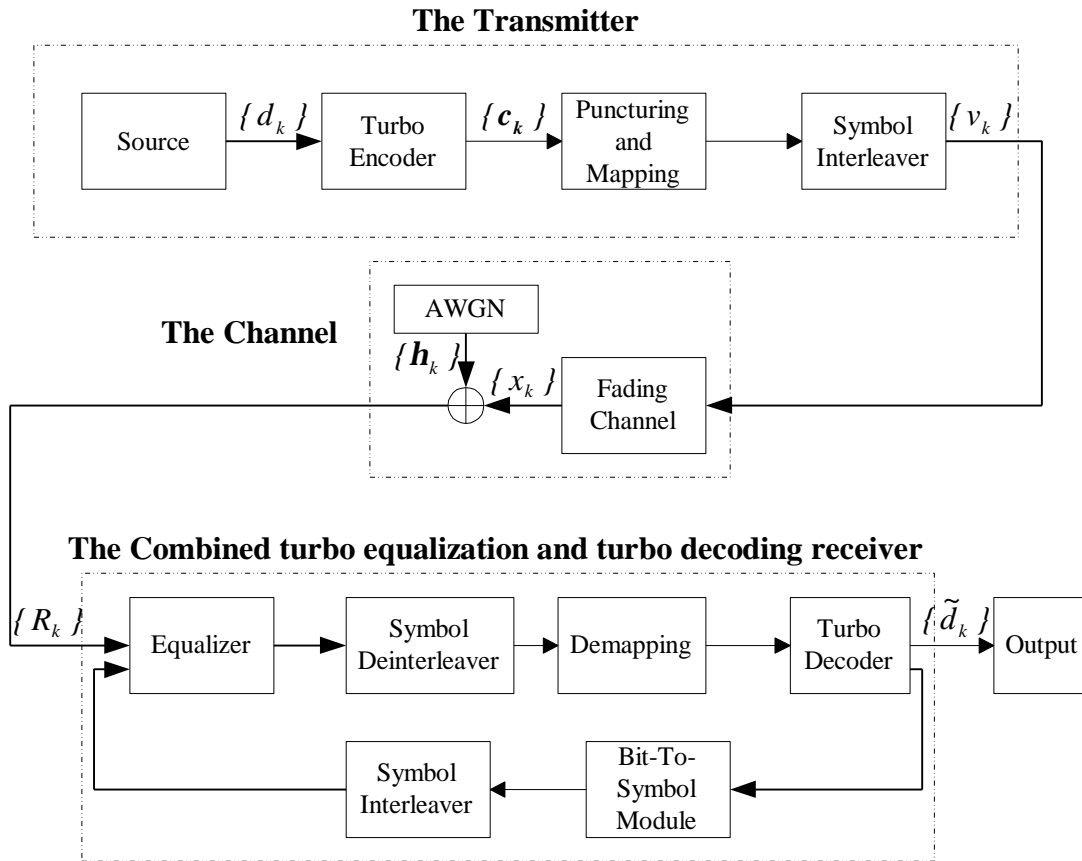


Figure 3.1 The Overall System Model with the Combined Turbo Equalization and Turbo Decoding Receiver

- An information bit sequence $\{d_k\}$ is encoded by turbo encoder. A set of the corresponding output bit sequence $\{c_k\}$, $c_k = (c_k^1, c_k^{21}, c_k^{22})$, will be punctured and mapped into high spectral efficiency modulation such as M-PSK or M-QAM using Gray mapping.
- Then, after block-mode interleaving, the symbol sequence $\{v_k\}$ will be transmitted through the fading channel before added by the additive white Gaussian noise sequence $\{h_k\}$.
- At the receiver, after receiving the whole sequence of the received symbol $\{R_k\}$, the combined turbo equalization and turbo decoding

receiver will estimate the information sequence $\{\tilde{d}_k\}$ at the output. Then, they will be compared with the information bit sequence $\{d_k\}$ from the source to evaluate the BER performance for every E_b/N_0 . The implementation for each module will be further clarified.

The Transmitter

- Figure 3.2 shows the transmitter model modified from (Goff et al., 1994). d_k is the binary data with the elements $\{0,1\}$, c_k^1 is the first coded bit, which is equal to the data bit d_k for systematic encoder. c_k^{2x} is the second coded bit from the encoder generator number x (RSC- x), $x=2$ for 2 encoder generators.
- The interleaver I_2 is inserted in order to reduce the effect of fading. The standard turbo encoder produces the coded bits c_k^1, c_k^{21} and c_k^{22} from the data d_k . They will be punctured and mapped into M-PSK or M-QAM ($M=2^m$) constellation using Gray mapping with rate $R=(m-\tilde{m})/m$ before transmitting through the ISI channel. The puncturing function is inserted in order to obtain a large code family, with various rates R .

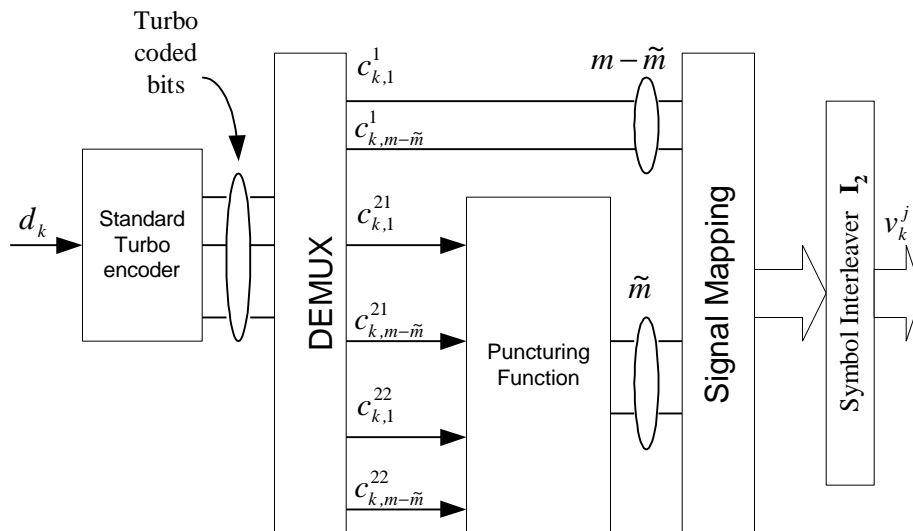


Figure 3.2 The Transmitter (Adapted from Goff et al., 1994)

The Proposed Combined Turbo Equalization and Turbo Decoding

- Figure 3.4 shows the proposed combined turbo equalization and turbo decoding receiver. In order to implement turbo equalization, both the equalizer and turbo decoder must be SISO.

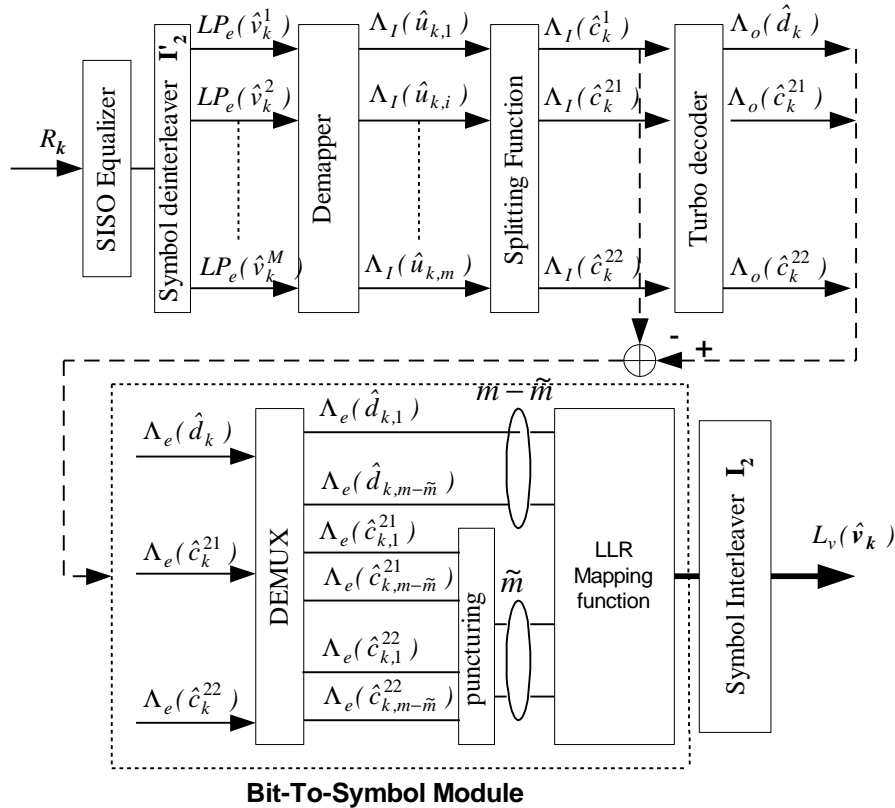


Figure 3.4 The Combined Turbo Equalization and Turbo Decoding Receiver

The SISO Equalizer

- For the SISO equalizer, in order to use the soft value extracted from the turbo decoder, which is in bit level in LLR format, the approach that transforms the information from bit level into symbol level proposed in (Franz and Bauch, 1999) is modified and implemented here.

- By defining the symbol corresponding to the $\log_2 M$ -tuple of zeros as a reference symbol, the LLR value of the symbol v_k^j is calculated as shown in (**Franz and Bauch, 1999**):

$$L_v(v_k^j) = \sum_{\substack{i=1 \\ v(i)=+1}}^{\log_2 M} L(c_{k,i}), \quad j=1\dots M \quad (1)$$

- where the LLR symbol value $L_v(v_k^j)$ is obtained by adding the LLR bit value $L(c_{k,i})$ for those bits $v(i)$ of the symbol v that are equal to +1.

Equivalent Channel Model

- This section shows how to apply the LLR value extracted from the Demapper to turbo decoder.
- The LLR $\Lambda_I(\hat{c}_k)$ extracted from the Demapper can be viewed mathematically as the LLR of an equivalent additive white noise channel having c_k as the input symbol taking values $\{0,1\}$ and y_k as the equivalent output symbol with the equivalent additive noise n_k .
- The equivalent channel can be represented by the following equation (**Raphaeli and Zarai, 1998**):

$$y_k = \mathbf{a} \cdot (2c_k - 1) + n_k \quad (2)$$

- where \mathbf{a} is the equivalent channel attenuation. From this equivalent channel model, the LLR can be calculated as:

$$\Lambda_I(\hat{c}_k) = \frac{2 \cdot \mathbf{a} \cdot y_k}{\mathbf{s}_q^2} \quad (3)$$

- where \mathbf{s}_q^2 is the variance of the equivalent additive white noise n_k .

- From this point of view, we can see that the LLR extracted from the Demapper can also be derived from the equivalent channel model.
- Thus, to follow the procedure mathematically, after getting the LLR value $\Lambda_I(\hat{c}_k)$, use it to estimate the variance of the equivalent noise source \mathbf{S}_q^2 as shown in (**Raphaeli and Zarai, 1998**).
- Then, translate the LLR value into the equivalent received symbols y_k using (3) and send them to turbo decoder implementing MAP or Log-MAP algorithm.
- As we can see, the MAP algorithm in turbo decoder will convert this y_k into soft value again which is equal to $\Lambda_I(\hat{c}_k)$ exactly. (The derivation of this assumption is shown in Appendix A.) In conclusion, we can apply the LLR value extracted from the Demapper to turbo decoder directly without any transformation.

Turbo Decoder

- As mentioned in the previous topic, the LLR value extracted from the Demapper (LAPP from the equalizer) can be directly used by turbo decoder. The Splitting Function will collect and split the the LLR values to their corresponding decoders (DEC1 or DEC2) as shown in Figure 3.5.
- These LLR values will be used for extracting the information for each data bit d_k by turbo decoder as follows:

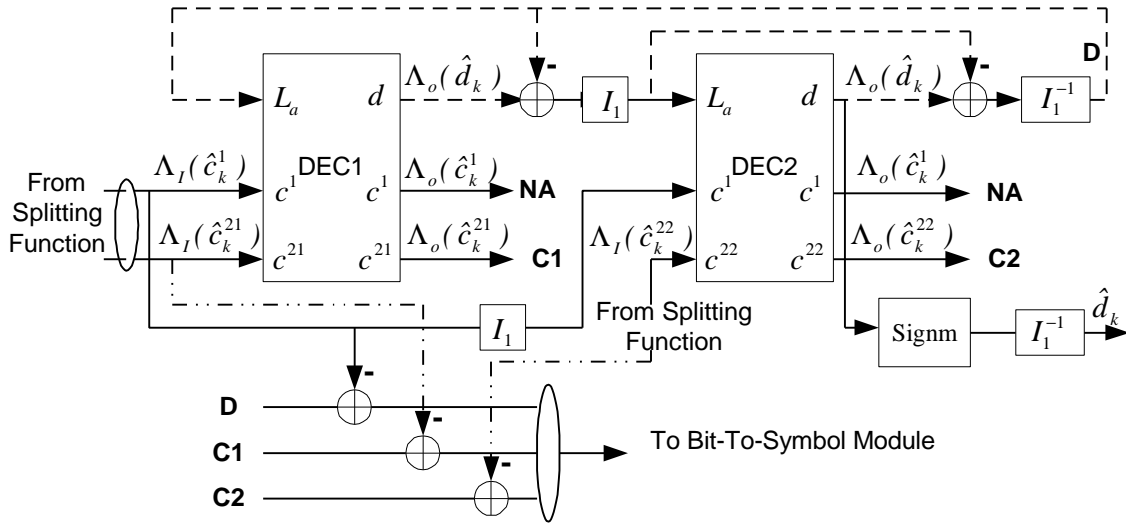


Figure 3.5 Turbo Decoder (Adapted from **Raphaeli and Zarai**, 1998)

- DEC1 and DEC2 are the SISO decoders which correspond to the first encoder (RSC1) and the second encoder (RSC2) respectively.
- They evaluate the APP-LLR of each uncoded d_k bit and its corresponding coded bits c_k^1, c_k^{2x} . I_1 and I_1^{-1} are the interleaver and the deinterleaver which correspond to that in turbo encoder.
- Because of the approximation in Max-log-MAP, this algorithm is suboptimal and yields an inferior soft-output than MAP algorithm. Log-MAP tries to avoid that approximation by adding some correction function (**Robertson et al.**, 1995) as shown:

$$\text{Log}(e^{c_1} + e^{c_2}) = \max(\mathbf{d}_1, \mathbf{d}_2) + \text{Log}(1 + \exp(-|\mathbf{d}_1 - \mathbf{d}_2|)) \quad (4)$$

- Due to the complexity advantage of Log-MAP, in turbo decoder, we will apply Log-MAP algorithm to both SISO decoders (DEC1 and DEC2) to estimate the soft value (APP-LLR) of each uncoded $\Lambda_o(\hat{d}_k)$ and coded $\Lambda_o(\hat{c}_k)$ bit.

- From the derivation of the Max-Log-MAP algorithm described earlier, the output of the SISO decoder number x , DEC- x ($x = 1, 2$) implementing Log-MAP algorithm are: (**Benedetto et al., 1996b**) :

$$\Lambda_o(\hat{d}_k) = \begin{array}{l} \text{Jac} \\ \mathbf{s}_k, \mathbf{s}_{k+1}; d_k = +1 \end{array} \left\{ \begin{array}{l} \bar{\mathbf{a}}_k(\mathbf{s}_k) + \bar{\mathbf{m}}_{k+1}^d(\mathbf{x}_k) \\ + \bar{\mathbf{b}}_{k+1}(\mathbf{s}_{k+1}) \end{array} \right\} - \begin{array}{l} \text{Jac} \\ \mathbf{s}_k, \mathbf{s}_{k+1}; d_k = 0 \end{array} \left\{ \begin{array}{l} \bar{\mathbf{a}}_k(\mathbf{s}_k) + \bar{\mathbf{m}}_{k+1}^d(\mathbf{x}_k) \\ + \bar{\mathbf{b}}_{k+1}(\mathbf{s}_{k+1}) \end{array} \right\} \quad (5)$$

$$\Lambda_o(\hat{c}_k^1) = \begin{array}{l} \text{Jac} \\ \mathbf{s}_k, \mathbf{s}_{k+1}; c_k^1 = +1 \end{array} \left\{ \begin{array}{l} \bar{\mathbf{a}}_k(\mathbf{s}_k) + \bar{\mathbf{m}}_{k+1}^1(\mathbf{x}_k) \\ + \bar{\mathbf{b}}_{k+1}(\mathbf{s}_{k+1}) \end{array} \right\} - \begin{array}{l} \text{Jac} \\ \mathbf{s}_k, \mathbf{s}_{k+1}; c_k^1 = 0 \end{array} \left\{ \begin{array}{l} \bar{\mathbf{a}}_k(\mathbf{s}_k) + \bar{\mathbf{m}}_{k+1}^1(\mathbf{x}_k) \\ + \bar{\mathbf{b}}_{k+1}(\mathbf{s}_{k+1}) \end{array} \right\} \quad (6)$$

$$\Lambda_o(\hat{c}_k^{2x}) = \begin{array}{l} \text{Jac} \\ \mathbf{s}_k, \mathbf{s}_{k+1}; c_k^{2x} = +1 \end{array} \left\{ \begin{array}{l} \bar{\mathbf{a}}_k(\mathbf{s}_k) + \bar{\mathbf{m}}_{k+1}^{2x}(\mathbf{x}_k) \\ + \bar{\mathbf{b}}_{k+1}(\mathbf{s}_{k+1}) \end{array} \right\} - \begin{array}{l} \text{Jac} \\ \mathbf{s}_k, \mathbf{s}_{k+1}; c_k^{2x} = 0 \end{array} \left\{ \begin{array}{l} \bar{\mathbf{a}}_k(\mathbf{s}_k) + \bar{\mathbf{m}}_{k+1}^{2x}(\mathbf{x}_k) \\ + \bar{\mathbf{b}}_{k+1}(\mathbf{s}_{k+1}) \end{array} \right\} \quad (7)$$

(the trellis diagram we consider here belongs to the RSC in turbo encoder)

- Jac denotes the Jacobian logarithm using (4), x denotes the number of the encoder generators, which is equal to 2 for standard turbo encoder. c_k^1 and c_k^{2x} are the coded bits corresponding to the data bit d_k .
- The quantities $\bar{\mathbf{m}}_{k+1}^d(\mathbf{x}_k)$, $\bar{\mathbf{m}}_{k+1}^1(\mathbf{x}_k)$ and $\bar{\mathbf{m}}_{k+1}^{2x}(\mathbf{x}_k)$ are the logarithm of the MBM for the uncoded bit d_k , coded bit c_k^1 and c_k^{2x} belonging to the transition \mathbf{x}_k respectively. $\Lambda_o(\hat{c}_k^{2x})$ is evaluated in the corresponding SISO decoder number x only, otherwise, it is equal to zero.

- The quantities $\bar{a}_k(\mathbf{s}_k)$ and $\bar{b}_k(\mathbf{s}_k)$ are obtained through the forward and backward recursion respectively.
- To compute the logarithm of the MBM $\bar{m}_{k+1}(\mathbf{x}_k)$, each SISO decoder uses the information of each transmitted bit extracted from the Demapper and the other SISO decoder to calculate the logarithm of the MBM as follows:

$$\bar{m}_{k+1}(\mathbf{x}_k) = [d_k(\mathbf{x}_k) \cdot \Lambda_I(\hat{d}_k)] + [c_k^1(\mathbf{x}_k) \cdot \Lambda_I(\hat{c}_k^1)] + [c_k^{2x}(\mathbf{x}_k) \cdot \Lambda_I(\hat{c}_k^{2x})] \quad (8)$$

- where d_k, c_k^1, c_k^{2x} are with the transition \mathbf{x}_k . $\Lambda_I(\hat{c}_k^1)$ and $\Lambda_I(\hat{c}_k^{2x})$ refer to the information of the coded bits c_k^1 and c_k^{2x} extracted from the Demapper respectively. $\Lambda_I(\hat{d}_k)$ refers to the *a priori* probability in LLR format of the data bit d_k , received from the other SISO decoder.

- Substituting (8) into (5), (6) and (7), they can be rewritten as:

$$\Lambda_o(\hat{d}_k) = \Lambda_I(\hat{c}_k^1) + \Lambda_I(\hat{d}_k) + \Lambda_e(\hat{d}_k) \quad (9)$$

$$\Lambda_o(\hat{c}_k^1) = \Lambda_I(\hat{c}_k^1) + \Lambda_e(\hat{c}_k^1) \quad (10)$$

$$\Lambda_o(\hat{c}_k^{2x}) = \Lambda_I(\hat{c}_k^{2x}) + \Lambda_e(\hat{c}_k^{2x}) \quad (11)$$

- The steps for calculating the APP-LLR of each uncoded and coded bit is similar to those explained in Max-Log-MAP equalizer.

- Since RSC1 and RSC2 are systematic, $d_k = c_k^1$ so that $\Lambda_o(\hat{d}_k) = \Lambda_o(\hat{c}_k^1)$. Thus, we will neglect (10) as denoted **NA** in Figure 3.5.
- After receiving the soft values from the Demapper, DEC1 will calculate $\Lambda_o(\hat{d}_k)$ and $\Lambda_o(\hat{c}_k^{21})$ using (9) and (11).
- Then, only the extrinsic information of $\Lambda_o(\hat{d}_k)$ (the information not received from the other decoder, DEC2) will be sent to the other SISO decoder (DEC2) where they will be used as the *a priori* probability.
- Next, after receiving the soft values of the information bit d_k , DEC2 will use them to evaluate $\Lambda_o(\hat{d}_k)$ and $\Lambda_o(\hat{c}_k^{22})$ where the former will be used for making a decision.
- Only the extrinsic term of $\Lambda_o(\hat{c}_k^{21})$, $\Lambda_o(\hat{c}_k^{22})$ and $\Lambda_o(\hat{d}_k)$ from DEC2 evaluated for each information bit d_k , denoted by $\Lambda_e(\hat{c}_k^{21})$, $\Lambda_e(\hat{c}_k^{22})$ and $\Lambda_e(\hat{d}_k)$ will be transmitted to the Bit-To-Symbol Module for transforming from bit level into symbol level.
- These terms will not include the information extracted from the equalizer or Demapper. Moreover, the extrinsic information of $\Lambda_o(\hat{d}_k)$ (the information not received from the other decoder, DEC 1) will be also fed back to DEC1 as the *a priori* probability for the next iteration.
- In conclusion, the operations carried out here to evaluate the soft value of each uncoded and coded bit are similar to those done in the Max-Log-MAP equalizer. The only difference lies in the computation of the logarithm of the MBM.

Bit-To-Symbol Module

- The LLR Mapping Function shown in Figure 3.4 transforms the LLR value of each uncoded and coded bit received from turbo decoder into their corresponding LLR symbol value, however it cannot be done directly.
- The number of LLR values received per one symbol interval is more than that required for mapping into one symbol. Thus, we need some modifications by including DEMUX and puncturing function in our Bit-To-Symbol Module and the implementation will be stated as follows:
- For each time instant, turbo decoder generates three LLR values that correspond to the information bit d_k : the LLR value of the information bit $\Lambda_e(\hat{d}_k)$, the LLR values of the second coded bit from the first and second decoder $\Lambda_e(\hat{c}_k^{21}), \Lambda_e(\hat{c}_k^{22})$.
- For the spectral efficiency $m - \tilde{m}$, DEMUX and puncturing function must operate in the same manner as in the encoder part to get $m - \tilde{m}$ LLR values of the information bits and \tilde{m} LLR values of the second coded bits entering the LLR Mapping function.
- For example, consider 2 bit/s/Hz spectral efficiency, DEMUX has to transmit two LLR values of the information bits and four LLR values of the second coded bit. Then, if we use 8-PSK modulation scheme, 3 out of 4 values of the second coded bits will be punctured, or if we use 16-QAM modulation scheme, only 2 out of 4 will be punctured.
- Thus, we have two LLR values of the information bits and one of the second coded bit for 8-PSK modulation, compared to 16-QAM modulation, where we have two of the second coded bits. Now, the total number of the LLR values is equal to those entering the Mapping function at the encoder, three for 8-PSK ($m = 3$) and four for 16-QAM ($m = 4$).
- Next, they will be mapped into the LLR Mapping Function in order to find their corresponding M LLR symbol values $L_v(\hat{v}_k)$. In order to

implement it, the mapping algorithm for each modulation must be available at this LLR Mapping function.

- Therefore, this imposes the receiver to have the same configuration as in the encoder part. Finally, these M LLR symbol values will be interleaved and fed back to the equalizer to continue the iterative process. Using this method $\Lambda_I(\hat{c}_k^1)$, $\Lambda_I(\hat{c}_k^{2x})$ and $\Lambda_I(\hat{d}_k)$ fed to the SISO module in the turbo decoder will be updated for every iteration.

4. Simulation Parameters

- The simulation model is the same as what is shown in Figure 3.1. The following table shows the overall parameters used for computer simulation.

• **Table 4.1** Simulation Configurations

Encoder generator	(37,21) and (7,5)
Information blocksize	640 bits and 1920 bits
Interleaver I_1	Random Interleaver 640 and 1920
Symbol Interleaver I_2	Block Interleaver 20×16 and 60×16
Mapping function	Gray mapping
Modulation scheme	8-PSK and 16-QAM
Code rate	2/3 for 8-PSK and 1/2 for 16-QAM
Spectral efficiency	2 bit/s/Hz
Channel model	A tapped delay line consisting of 2 independent equal strength taps with perfect channel estimation, tap delay equals to symbol duration T
Doppler shift	Not considered
Receiver	Combined turbo equalization and turbo decoding and non-turbo equalization receiver
Equalizer	Max-Log-MAP equalizer and SSE
Decoder	Iterative decoding using Log-MAP
No. of iteration	5
BER simulation	Monte Carlo simulation method

Standard Turbo Encoder

- In the transmitter part shown in Figure 3.2, encoder generators (7,5) and (37,21), the best rate $\frac{1}{2}$ CC for rate $\frac{1}{3}$ PCCC for constraint length 3 and 5 respectively, are selected according to its maximum effective free distance for constructing standard turbo encoder (**Benedetto and Montorsi, 1996b**). (The effective free distance plays a similar role as the minimum free distance in the convolutional codes.)
- Figure 4.1 shows the standard turbo encoder using the mentioned encoder generators in recursive systematic form connected through the fully interleaved random interleaver I_1 .

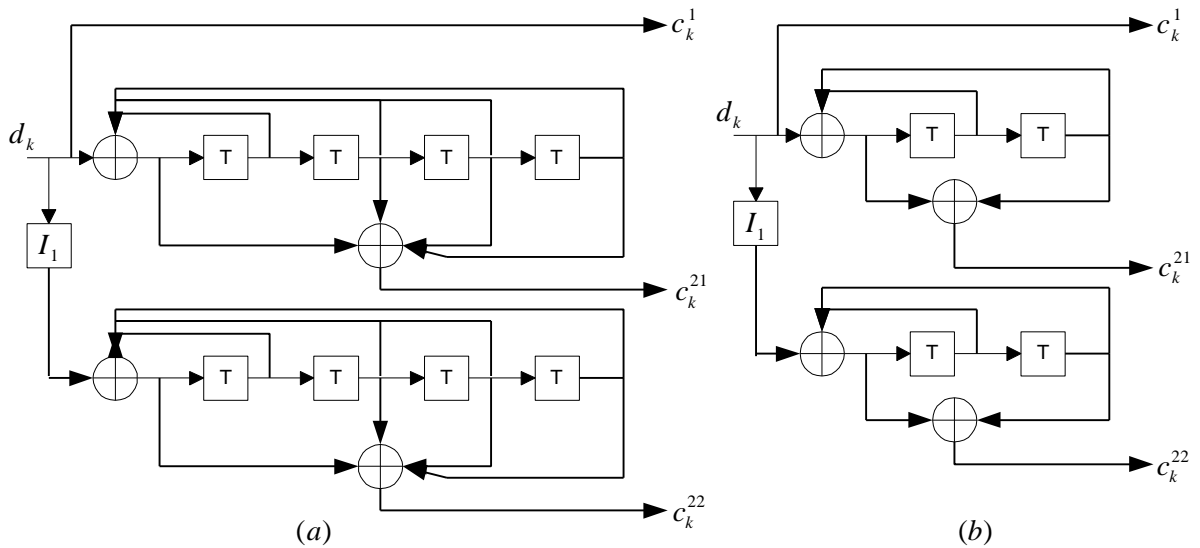


Figure 4.1 (a) Standard Turbo Codes Structure with Encoder Generator (37,21)
 (b) Standard Turbo Codes Structure with Encoder Generator (7,5)

- The information bit d_k taking on values 0 or 1; supposed to be equally likely distributed, is sent to the standard turbo encoder.
- Two types of information block sizes i.e. 640 and 1920 (three times the former) will be taken into account. After encoding for each information bit d_k , the coded bits c_k^1, c_k^{21} and c_k^{22} are obtained.

- They will be demultiplexed and punctured by DEMUX and Puncturing Function, respectively in order to achieve the required spectral efficiency.
- In this simulation, 2 bit/s/Hz is taken into account ($m - \tilde{m} = 2$) so that, for 8-PSK modulation ($m = \log_2 8 = 3$), 3 out of 4 LLR values of the second coded bits are punctured out ($\tilde{m} = 1$). In case of 16-QAM modulation, ($m = \log_2 16 = 4$), 2 from 4 values are punctured out ($\tilde{m} = 2$).
- Here, we consider only one puncturing pattern for each modulation scheme. For 2 bit/s/Hz spectral efficiency, there are 2 information bits for each transmitted symbol, and each information bit has two parity bits. For 8-PSK modulation, the second coded bit from the first encoder (c_k^{21}) of the first information bit is selected for each odd transmitted symbol and the second coded bit from the second encoder (c_k^{22}) of the other information bit is selected for each even transmitted symbol. For 16-QAM, we consider rate $\frac{1}{2}$ turbo codes so that two information bits with their corresponding parity bits are combined in one transmitted symbol.
- At the Bit-To-Symbol Module in the combined turbo equalization and turbo decoding receiver, this process must be carried out exactly in the same manner as in the encoder part for each puncturing pattern. After receiving the three LLR values from turbo decoder, they must be demultiplexed and puncturing like what has been explained in the previous paragraph. The only difference is that now the implementation is taken on the LLR values instead of bits.

Simulation Results

- The performance of turbo codes with multilevel modulation with combined turbo equalization and turbo decoding will be investigated in term of bit error rate (BER) as a function of E_b/N_0 over different types of information block size, encoder generator, modulation scheme and equalizer. For bit error rate simulation, Monte Carlo method is used. To ensure the reliability of the results, the number of data bits generated is in the order of $100/BER$.
- Most of the simulations carried out use the Max-Log-MAP equalizer. Performance of this system using the SSE is investigated in Figure 4.9 only.
- Figures 4.3 to 4.6 present the performance of turbo codes with 8-PSK modulation scheme over the frequency selective Rayleigh fading channel using combined turbo equalization and turbo decoding receiver with different types of encoder generators and information block sizes i.e. 640 and 1920 (three times the previous one). Max-Log-MAP equalizer is considered. Also shown is the performance of the uncoded-QPSK modulation over the same channel environment.

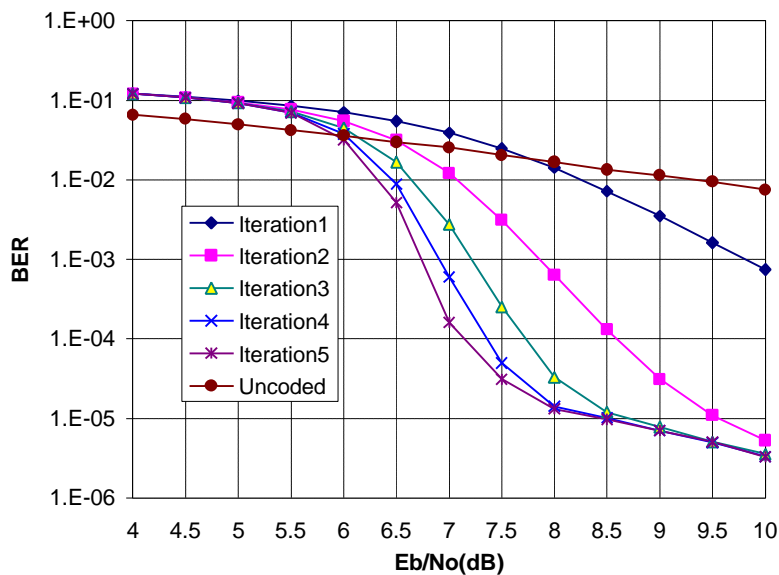


Figure 4.3 Performance of turbo codes with 8-PSK modulation, encoder generator (37,21), 1920-information block size.

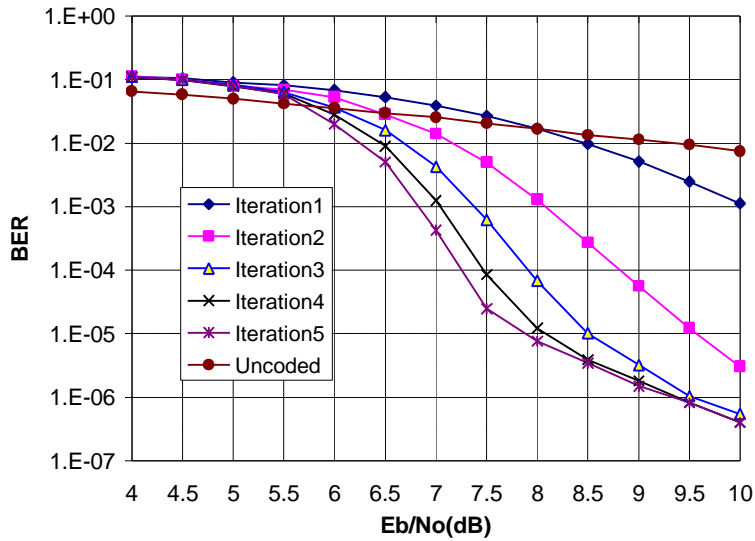


Figure 4.4 Performance of turbo codes with 8-PSK modulation, encoder generator (7,5), 1920-information block size.

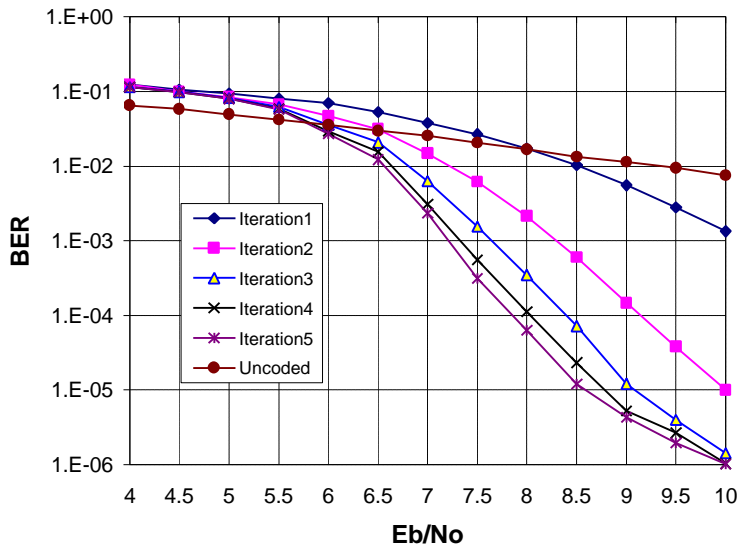


Figure 4.5 Performance of turbo codes with 8-PSK modulation, encoder generator (7,5), 640-information block size.

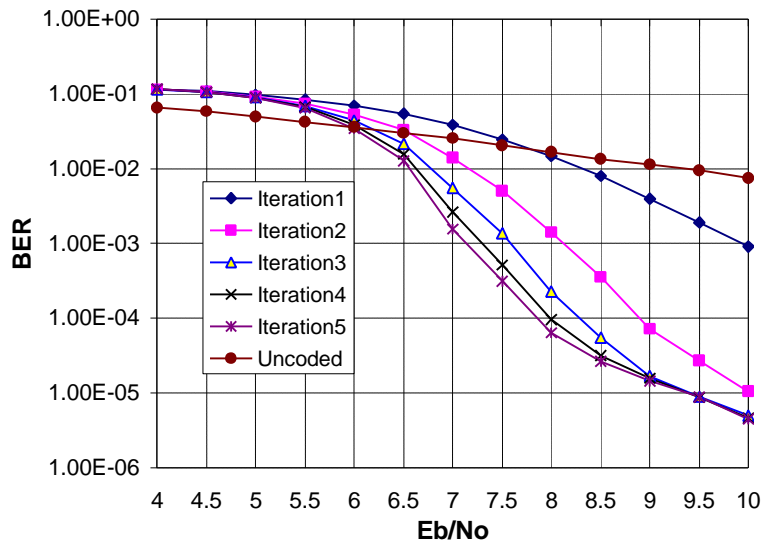


Figure 4.6 Performance of turbo codes with 8-PSK modulation, encoder generator (37,21), 640-information block size.

- From the simulation results, it is seen that we can achieve significant improvement after 2 iterations and 3 iterations are sufficient to obtain a very good BER performance.
- However, after 3 iterations, not much improvement is seen. Moreover, in the system with 8-PSK modulation scheme, a BER floor can be noticed when E_b/N_0 is greater than 8.0 dB.
- Figure 4.7 compares the performance of the system with 8-PSK modulation scheme at the 5th iteration with different types of encoders and information block sizes using the Max-Log-MAP equalizer.

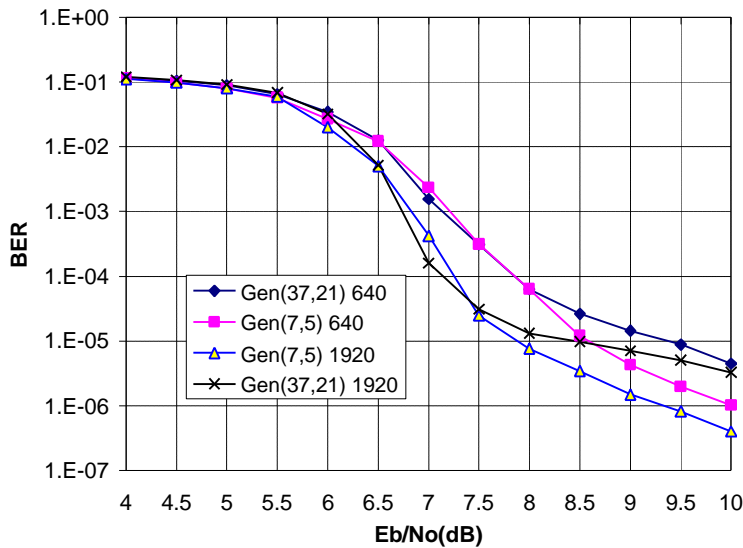


Figure 4.7 Comparison of the performance of turbo codes with 8-PSK modulation between two different types of encoder generator and information block size at the 5th iteration.

- Figure 4.7 shows that, when comparing between the systems with different information block sizes which correspond to different interleaver sizes in both types of the encoders, interleaving gain can be achieved when increasing the information block size. It is about 0.5-1.0 dB and tends to decrease at high E_b/N_0 with generator (37,21) but consistent about 0.5 dB with generator (7,5).
- Moreover, when comparing between the system with different types of the encoder generators in both types of information block sizes, an irregular result is noticed.
- The performance of turbo codes with encoder generator (37,21) is worse than that with encoder generator (7,5) at high E_b/N_0 in both types of the information block sizes.
- The performance with encoder generator (37,21) looks slightly better than that with encoder generator (7,5) when E_b/N_0 is greater than 6.5 dB however, at high E_b/N_0 the latter clearly outperforms the former. This difference occurs at lower E_b/N_0 for 1920 information block size. *Thus, we can conclude that increasing the constraint length of the generator of turbo encoder in this system does not necessarily improve the BER performance.*

- Figure 4.8 shows the performance of turbo codes using non-turbo equalization receiver over the frequency selective Rayleigh fading channel with 8-PSK modulation, encoder generator (7,5) and 1920 information block size. Max-Log-MAP equalizer is considered. Non-turbo equalization means that there is no feed back to the equalizer from the turbo decoder.

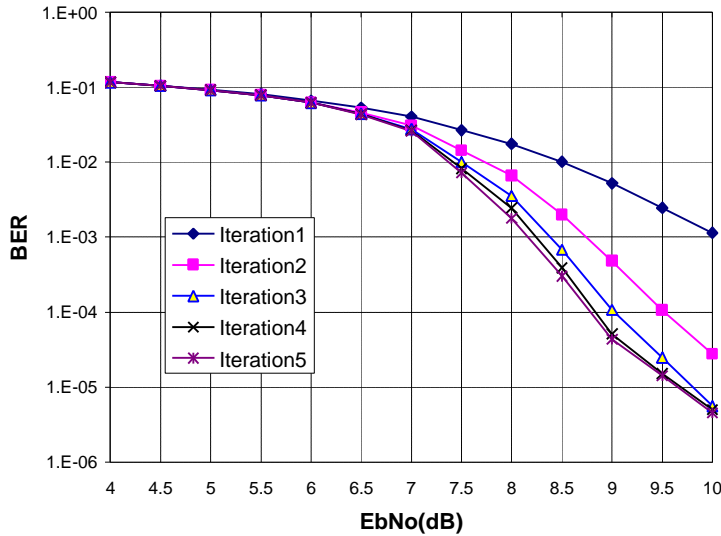


Figure 4.8 Performance of turbo codes with 8-PSK modulation using non-turbo equalization, encoder generator (7,5), 1920-information block size.

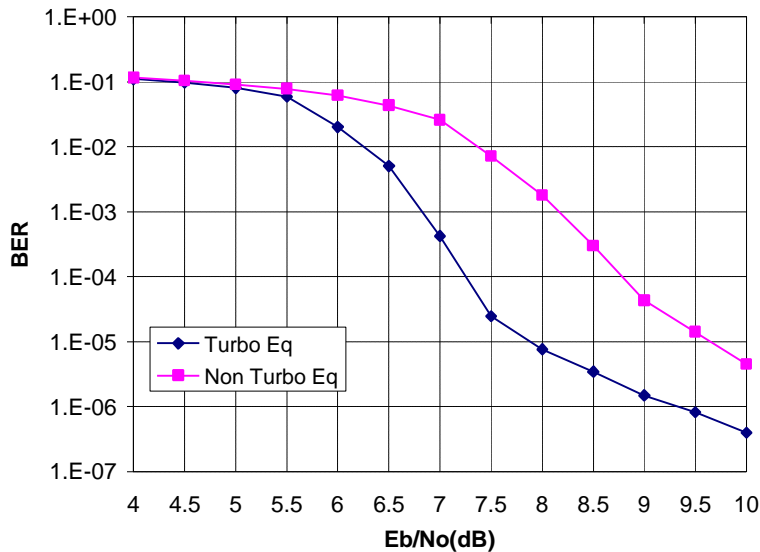


Figure 4.9 Comparison of the performance of turbo codes with 8-PSK modulation between the receiver implementing turbo equalization and non-turbo equalization at the 5th iteration.

- Figure 4.9 compares the performance of turbo codes between the system implementing turbo equalization in Figure 4.4 and non-turbo equalization in Figure 4.8 at the 5th iteration.
- From the simulation result, a significant improvement about 1.5-2.0 dB can be seen with turbo equalization.
- Figure 4.10 illustrates the performance of the system implementing turbo equalization using SSE for 8-PSK modulation scheme and 1920 information block size. For the SSE, the decision delay is equal to 5 since according to (Li et al., 1995) a delay of five times channel memory is usually sufficient.

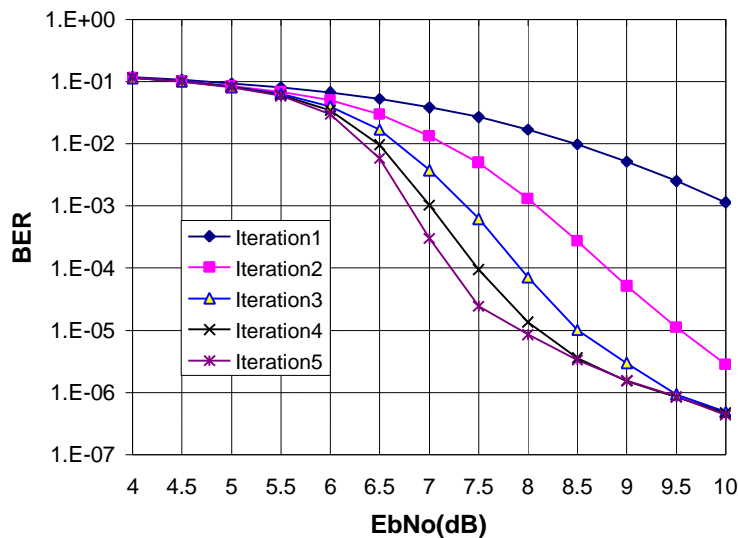


Figure 4.10 Performance of turbo codes with 8-PSK modulation, using SSE, encoder generator (7,5), 1920-information block.

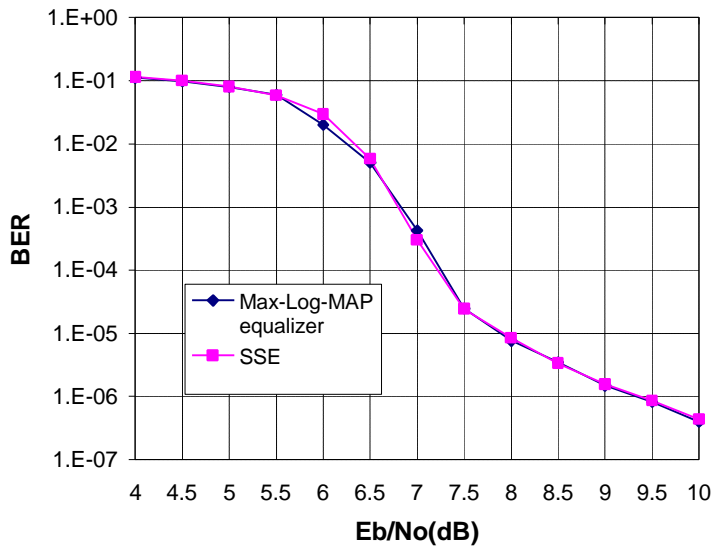


Figure 4.11 Comparison of the performance of turbo codes between Max-Log-MAP equalizer and SSE with 8-PSK modulation, encoder generator (7,5) and 1920 information block size at the 5th iteration.

- Figure 4.11 compares the performance of the system between the Max-log-MAP equalizer from Figure 4.4 and the SSE from Figure 4.10 at the 5th iteration.
- From Figure 4.11, the Max-Log-MAP equalizer and the SSE give virtually the same performance with turbo equalization. However, the computational complexity for each time instant of the SSE is higher than that of the Max-Log-MAP algorithm as shown in Table 4.2. In contrast, the memory requirement for the Max-Log-MAP equalizer is much greater than that for the SSE since it depends on the sequence length. In this table, we follow the simulation parameters used in Figure 4.11.
- **Table 4.2 Comparison of the Computational Complexity in One Interval for 1920 Information Block Size**

Computational Complexity	Max-Log-MAP	SSE
◆ No. of additions	256	2112
◆ No. of comparisons	168	1848
◆ No. of memory	76800	256

- Figures 4.12-4.15 show the performance of turbo codes with 16-QAM modulation scheme over the frequency selective Rayleigh fading channel using combined turbo equalization and turbo decoding receiver. Its performance is investigated over different types of information block size and encoder generator. Max-Log-MAP equalizer is considered. Also shown is the performance of the uncoded 4-QAM under the same channel environment.

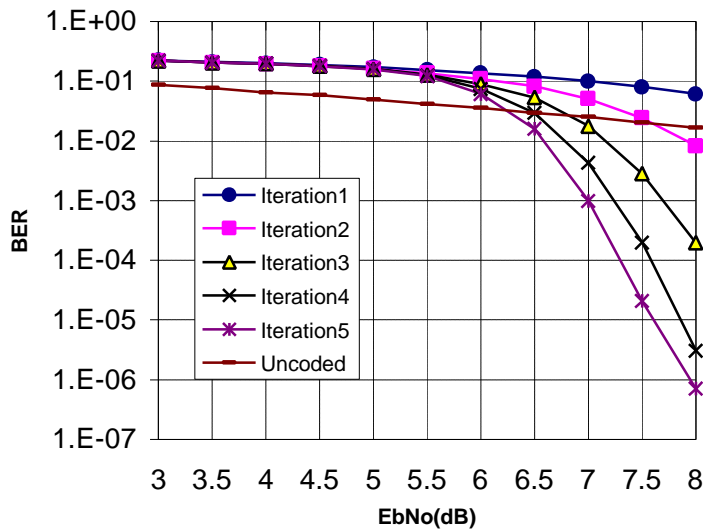


Figure 4.12 Performance of turbo codes with 16-QAM modulation, encoder generator (7,5), 1920-information block size.

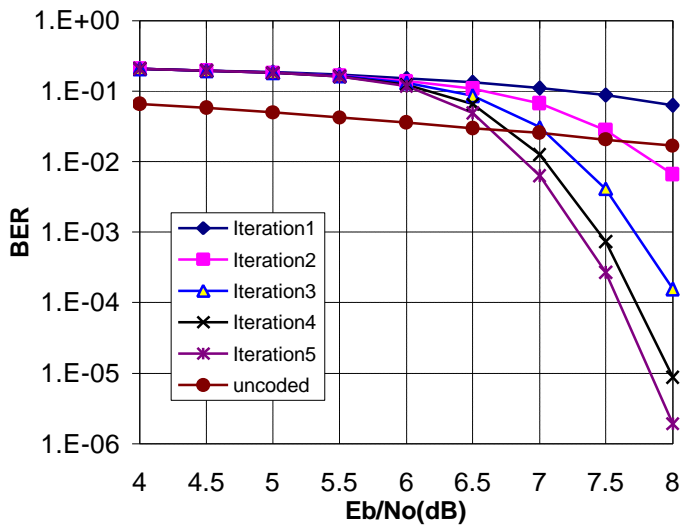


Figure 4.13 Performance of turbo codes with 16-QAM modulation, encoder generator (37,21), 1920-information block size.

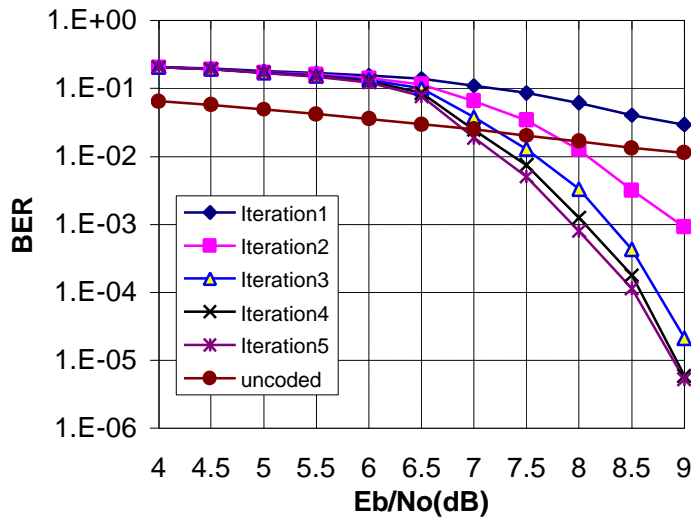


Figure 4.14 Performance of turbo codes with 16-QAM modulation, encoder generator (37,21), 640-information block size.

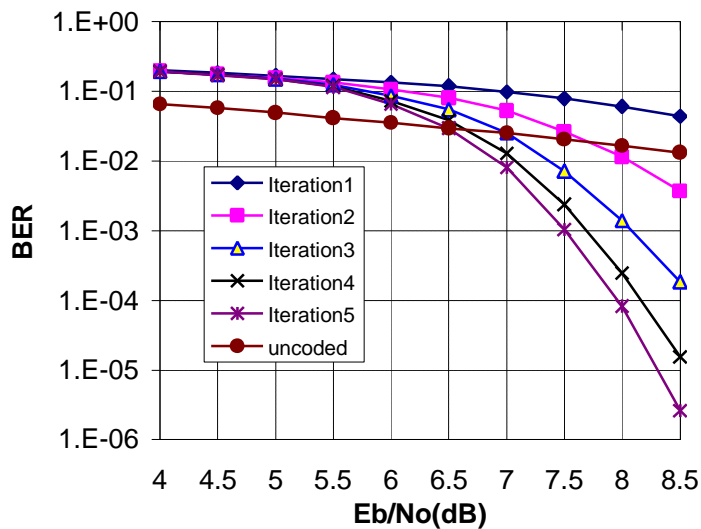


Figure 4.15 Performance of turbo codes with 16-QAM modulation, encoder generator (7,5), 640-information block size.

- From the simulation results, unlike 8-PSK modulation scheme, after 3 iterations there is still some improvement when increasing the number of iterations and an excellent performance is obtained at the 5th iteration.

- Figure 4.16 compares the performance of turbo codes with 16-QAM modulation between different types of encoder generators and information block sizes at the 5th iteration.

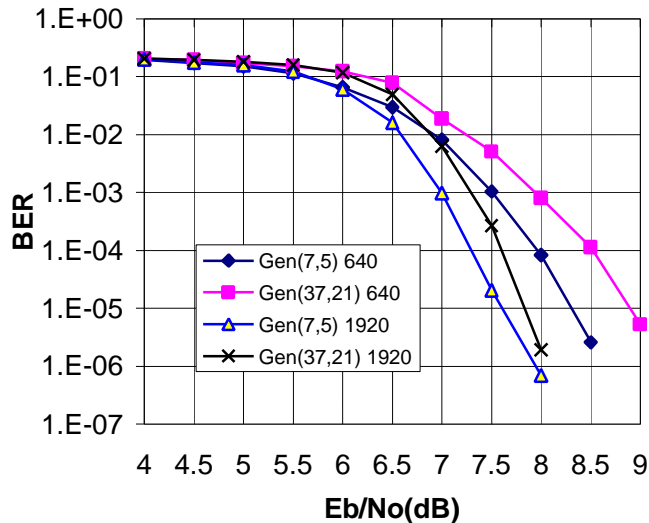


Figure 4.16 Comparison of the performance of turbo codes between two different types of encoder generator and information block size at the 5th iteration using 16-QAM modulation scheme.

- From the results, when comparing between the system with different information block sizes with both types of encoder generators, the results correspond to those obtained when comparing in 8-PSK modulation scheme in Figure 4.7.
- The interleaver gain can also be achieved in this system. This gain tends to be higher than that obtained in 8-PSK modulation scheme in both types of the encoder generator at high E_b/N_0 values. About 0.8 dB interleaver gain is obtained for encoder generator (7,5) and about 1.0 dB or higher for encoder generator (37,21).
- Moreover, when comparing between different encoder generators, encoder generator (7,5) still outperforms encoder generator (37,21). However, in case of 1920 information block size, the performance of the generator (37,21) tends to match that of (7,5).

- Figure 4.17 compares the performance between different types of modulation schemes for each type of encoder generator and information block sizes for the same spectral efficiency.

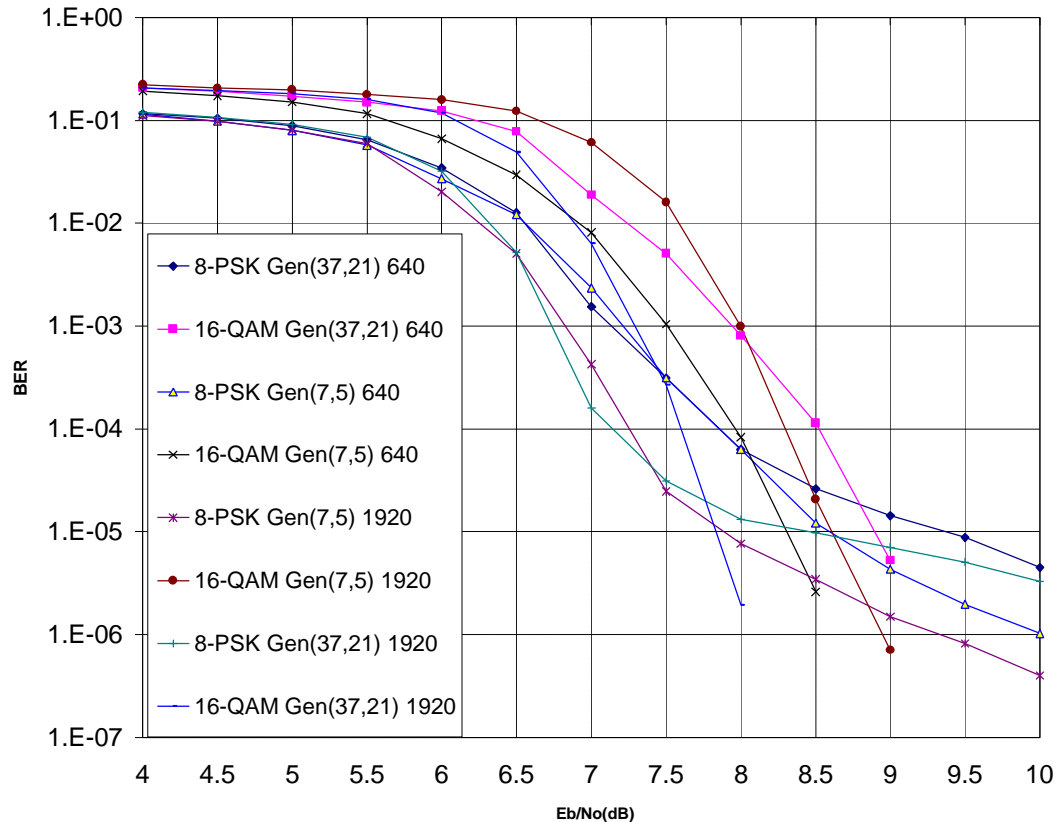


Figure 4.17 Comparison of the performance of turbo codes between two different types of modulation scheme at the 5th iteration.

- The performance of turbo codes with 8-PSK modulation is better than that with 16-QAM in the low E_b/N_0 range i.e. about 4.0-7.5 dB. After that while 8-PSK modulation tends to reach their BER floor at E_b/N_0 higher than 8.0 dB, 16-QAM does not show that effect in this E_b/N_0 interval.
- Hence, it can be concluded that at low E_b/N_0 range the performance of turbo codes with 8-PSK modulation is superior to that with 16-QAM modulation. However, 16-QAM outperforms 8-PSK at high E_b/N_0 range i.e. higher than 8.0 dB.

- The degradation for 8-PSK modulation scheme at high E_b/N_0 range is mainly due to less punctured parity bits since 3 out of 4 parity bits are punctured in order to achieve 2 bit/s/Hz spectral efficiency compared to 16-QAM where only half of them are punctured.
- This comparison is taken over the fixed puncturing patterns for each modulation scheme. For 8-PSK modulation even after changing the puncturing pattern, the ratio of the data bit to the parity bit in one transmitted symbol is still lower than that for 16-QAM. Hence, we can conclude that in this system, 16-QAM outperforms 8-PSK at high E_b/N_0 range in any puncturing scheme for 2 bit/s/Hz spectral efficiency.