Fading Models

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Physical Layer Methods in Wireless Communication Systems

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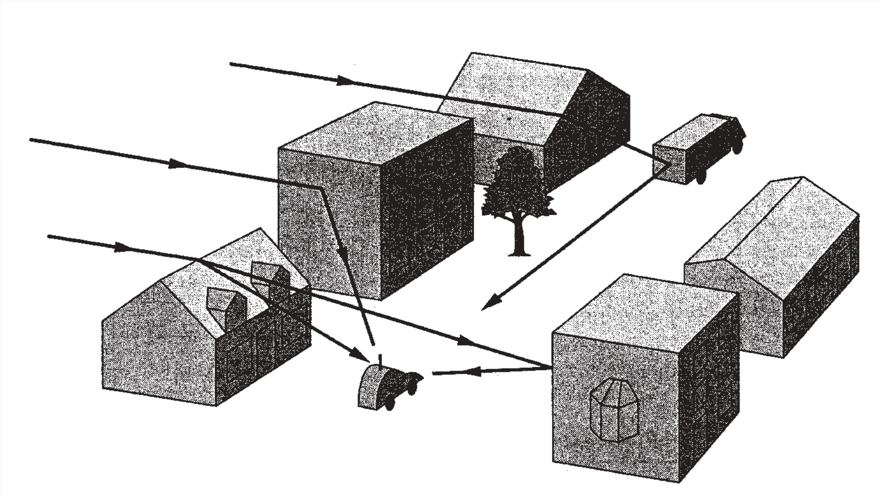
Motivation

- Most mobile communication systems are used in and around center of population.
- The transmitting antenna or Base Station (BS) are located on top of a tall building or tower and they radiate at the maximum allowed power.
- The mobile antenna or Mobile Station (MS) is well below the surrounding buildings.
- Wireless communication phenomena are mainly due to scattering of electromagnetic waves from surfaces or diffraction over and around buildings.
- The design goal is to make the received power adequate to overcome background noise over each link, while minimizing interference to other more distant links operating at the same frequency.

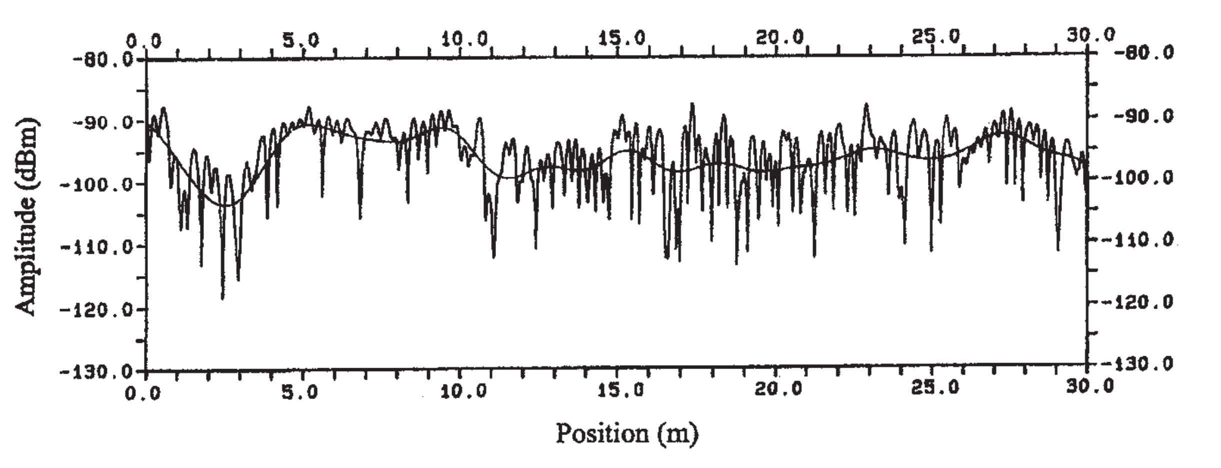
Fading Types

- Multiple propagation paths or *multipaths* have both slow and fast aspects.
- The received signal for narrowband excitation is found to exhibit three scales of spatial variation
  - Fast Fading
  - Slow Fading
  - Range Dependence

  as well as temporal variation and polarization mixing.



Fading Types



- As the MS moves along the street, rapid variations of the signal are found to occur over distances of about one-half the wavelength $\lambda = \frac{c}{f}$. Here c is the speed of light and $f = 910$ MHz.
- Over a distance of few meter the signal can vary by 30-dB.
- Over distances as small as $\frac{\lambda}{2}$, the signal is seen to vary by 20-dB.
- Small scale variation results from the arrival of the signal at the MS along multiple ray paths due to reflection, diffraction and scattering.

Fading Types

- The phenomenon of *fast fading* is represented by the rapid fluctuations of the signal over small areas.
- The multiple ray set up an interference pattern in space through which the MS moves.
- When the signals arrive from all directions in the plane, fast fading will be observed for all directions of motion.
- In response to the variation in the nearby buildings, there will be a change in the average about which the rapid fluctuations take place.
- This middle scale over which the signal varies, which is on the order of the buildings dimensions is known as *shadow fading*, *slow fading* or *log-normal fading*.

Flat Fading

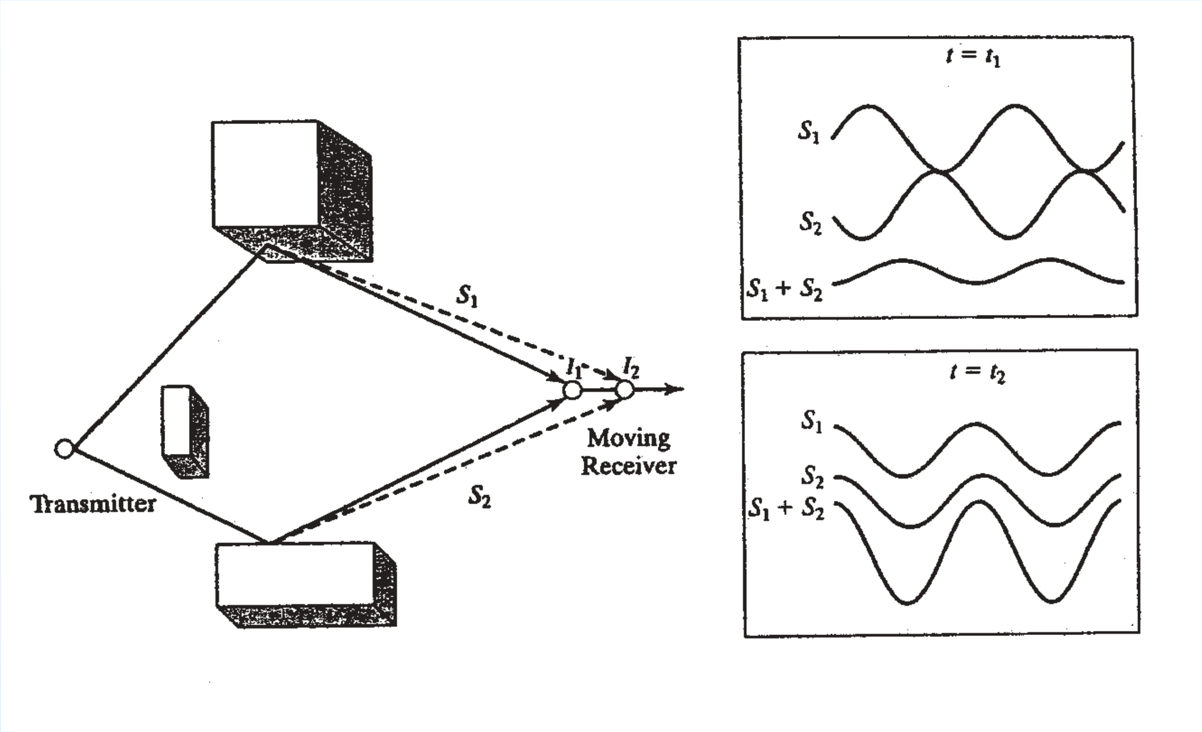
- The wireless channel is said to be flat fading if it has constant gain and linear phase response over a bandwidth which is greater than the bandwidth of the transmitted signal.
- In other words, flat fading occurs when the bandwidth of the transmitted signal B is smaller than the coherence bandwidth of the channel $B_m \Rightarrow B \ll B_m$.
- The effect of flat fading channel can be seen as a decrease of the Signal-to-Noise ratio.
- Since the signal is narrow with respect to the channel bandwidth, the Flat fading channels are also known as amplitude varying channels or narrowband channels.

Fast Fading

- Fast fading occurs if the channel impulse response changes rapidly within the symbol duration.
- In other works, fast fading occurs when the coherence time of the channel T_D is smaller than the symbol period of the the transmitted signal $T \Rightarrow T_D \ll T$.
- This causes frequency dispersion or time selective fading due to Doppler spreading.
- Fast Fading is due to reflections of local objects and the motion of the objects relative to those objects.

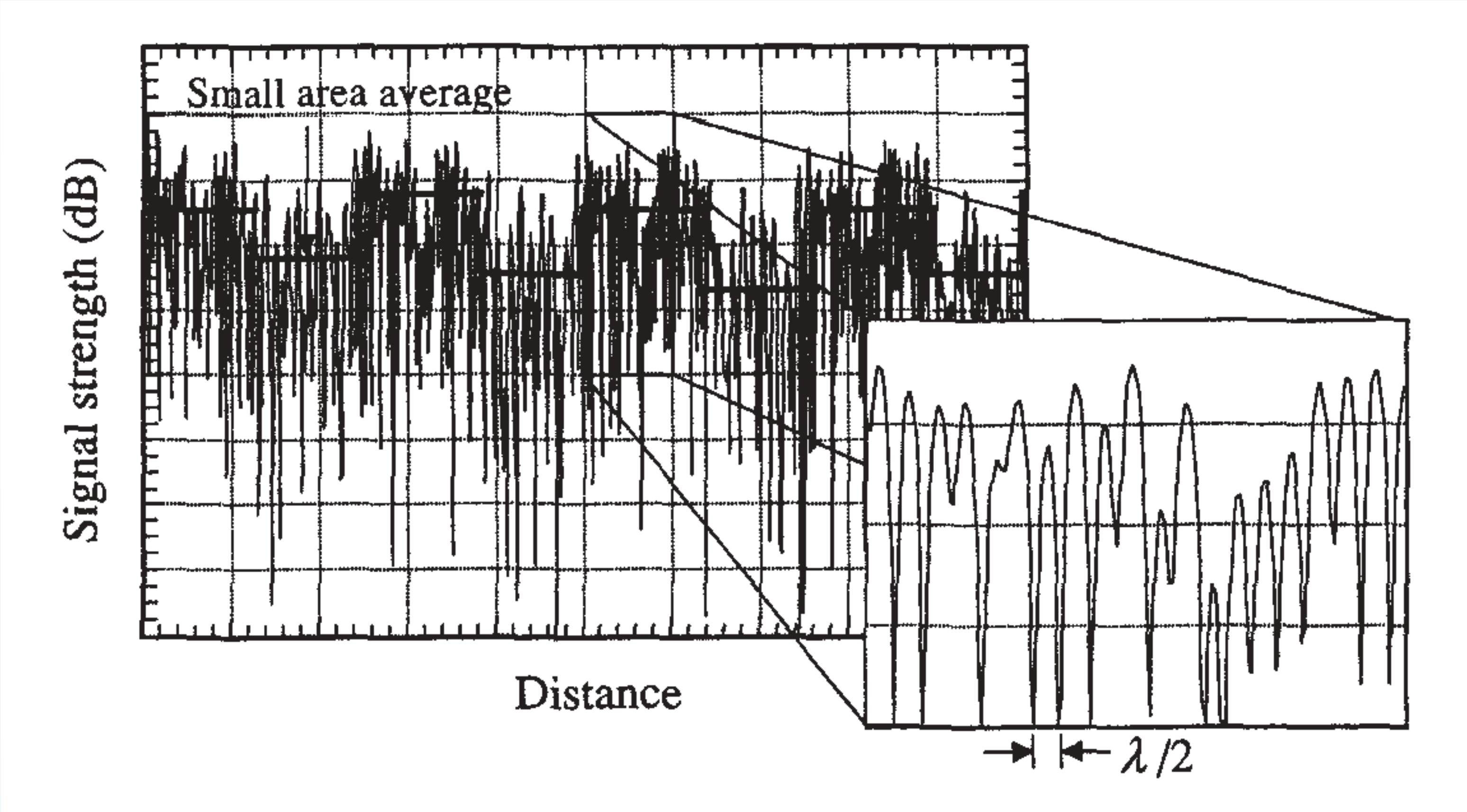
Fast Fading

- The receive signal is the sum of a number of signals reflected from local surfaces, and these signals sum in a constructive or destructive manner $\Rightarrow$ relative phase shift.
- Phase relationships depend on the speed of motion, frequency of transmission and relative path lengths.



Fast Fading

- To separate out fast fading from slow fading ⇒ the envelope or magnitude of the RX signal is averaged over a distance (e.g. 10-m).
- Alternatively, a sliding window can be used.



Small Scale Fading

- Received pass-band signal without noise after transmitted unmodulated carrier signal $\cos(2\pi f_c t)$

$$x(t) = \sum_i \alpha_i(t)\cos(2\pi f_c(t-\tau_i(t))) = \Re\{\sum_i \alpha_i(t)e^{-j2\pi f_c \tau_i(t)}e^{j2\pi f_c t}\}$$

where $\Re$ represent the real part, $\alpha_i(t)$ is the time-varying attenuation factor of the i^{th} propagation delay, $\tau_i(t)$ is the time-varying delay and f_c is the carrier frequency. Assume that $\tau_i(t) \ll T =$ symbol time.

- Equivalent baseband signal: $h(t) = \sum_i \alpha_i(t)e^{-j2\pi f_c \tau_i(t)}$.
- If the delays $\tau_i(t)$ change in a random manner and when the number of propagation path is large, central limit theorem applies and $h(t)$ can be modelled as a complex Gaussian process.

Rayleigh distribution

- When the components of $h(t)$ are independent the probability density function of the amplitude $r = |h| = \alpha$ assumes Rayleigh pdf:

$$f(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \qquad \text{where} \qquad E\{r^2\} = 2\sigma^2 \qquad \text{and} \qquad r \geq 0$$

- This represents the worst fading case because we do not consider to have Line-of-Sight (LOS).
- The power is exponentially distributed.
- The phase is uniformly distributed and independent from the amplitude.
- This is the most used signal model in wireless communications.

Ricean distribution

- In case the channel is complex Gaussian with non-zero mean (there is LOS), the envelope $r = |h|$ is Ricean distributed.
- Here we denote $h = \alpha e^{j\phi} + \nu e^{j\theta}$ where α follows the Rayleigh distribution and $\nu > 0$ is a constant such that ν^2 is the power of the LOS signal component. The angles ϕ and θ are assumed to be mutually independent and uniformly distributed on $[-\pi, \pi)$.
- Ricean pdf:

$$f(r) = \frac{r}{\sigma^2} e^{-\frac{r^2+\nu^2}{2\sigma^2}} I_0\left(\frac{r\nu}{\sigma^2}\right) \qquad \text{and} \qquad r \geq 0$$

  where I_0 is the modified Bessel function of order zero and $2\sigma^2 = E\{\alpha^2\}$.
- The Rice factor $K = \frac{\nu^2}{2\sigma^2}$ is the relation between the power of the LOS component and the power of the Rayleigh component.
- When $K \to \infty$, no LOS component and Rayleigh=Ricean.

Nakagami distribution

- In this case we denote $h = re^{j\phi}$ where the angle ϕ is uniformly distributed on $[-\pi, \pi)$.
- The variable r and ϕ are assumed to be mutually independent.
- The Nakagami pdf is:

$$f(r) = \frac{2}{\Gamma(k)} \left(\frac{k}{2\sigma^2}\right)^k r^{2k-1} e^{-\frac{kr^2}{2\sigma^2}} \qquad \text{and} \qquad r \geq 0$$

  where $2\sigma^2 = E\{r^2\}$, $\Gamma(\cdot)$ is the Gamma function and $k \geq \frac{1}{2}$ is the fading figure (degrees of freedom related to the number of added Gaussian random variables).
- It was originally developed empirically based on measurements.
- Instantaneous receive power is Gamma distributed.
- With $k = 1$ Rayleigh=Nakagami.

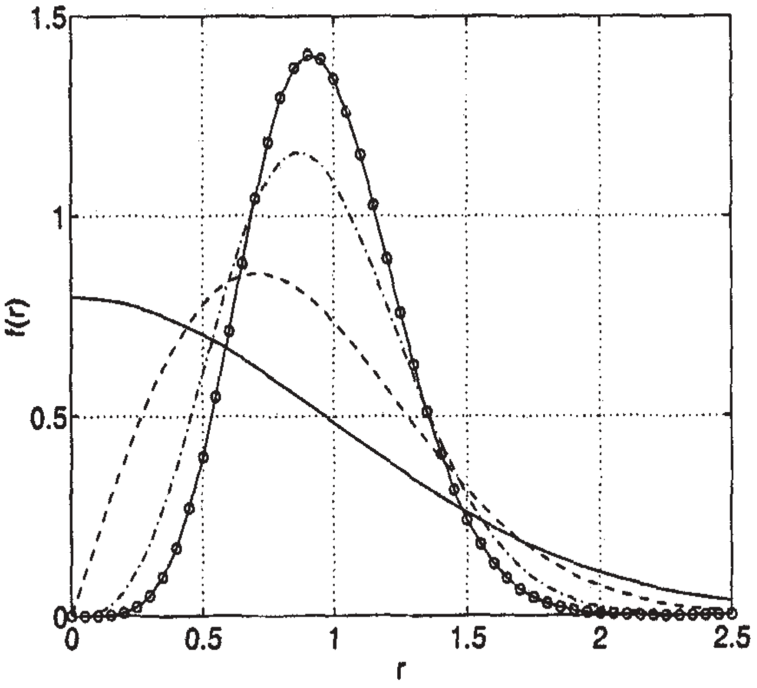
Weibull distribution

- Weibull distribution represents another generalization of the Rayleigh distribution.
- When X and Y are i.i.d. zero-mean Gaussian variables, the envelope of $R = (X^2 + Y^2)^{\frac{1}{2}}$ is Rayleigh distributed.
- However, is the envelope is defined as $R = (X^2 + Y^2)^{\frac{1}{k}}$, the corresponding pdf id Weibull distributed:

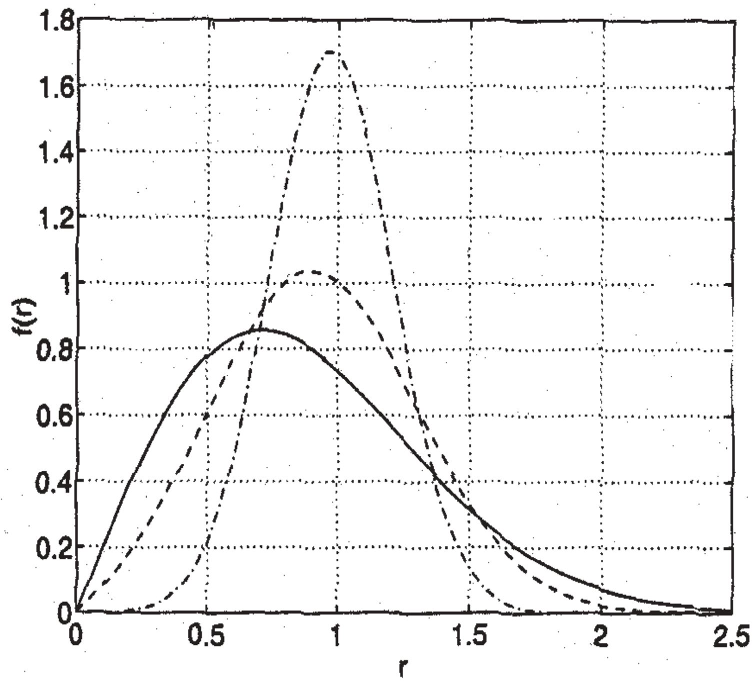
$$f(r) = \frac{kr^{k-1}}{2\sigma^2} e^{-\frac{r^k}{2\sigma^2}}$$

where $2\sigma^2 = E\{r^2\}$.

Distributions



Nakagami probability density function, (—) $\kappa = \frac{1}{2}$, (- -) $\kappa = 1$ (Rayleigh), (- · -) $\kappa = 2$, (∘) $\kappa = 3$



Ricean probability density function, (—) $K = -\infty$, (- -) $K = 3$ dB, (- · -) $K = 9$ dB.

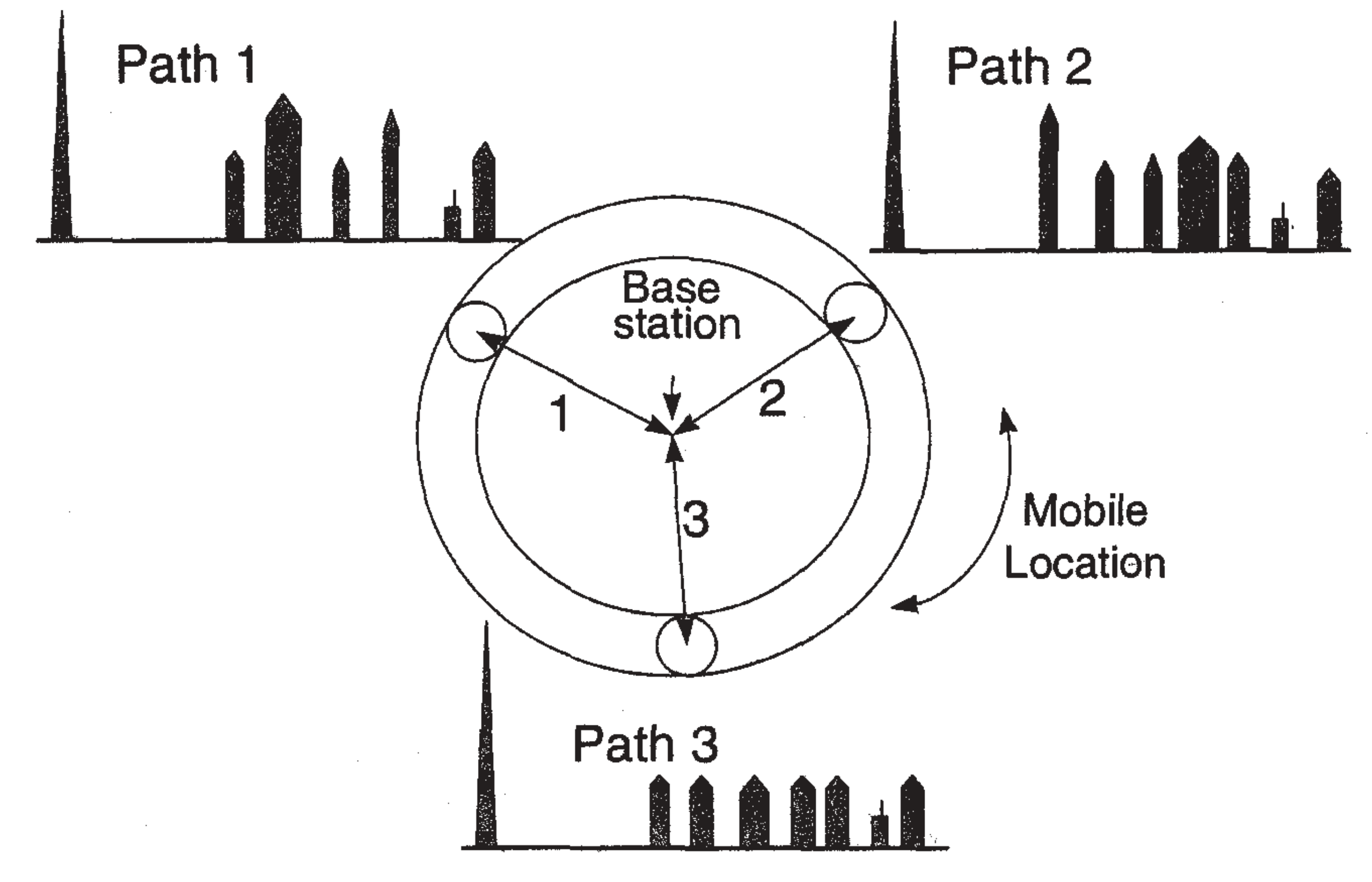
Slow Fading

- Slow fading is the result of shadowing by buildings, mountains, hills, and other objects.
- The average within individual small areas also varies from one small area to the next in an apparently random manner.
- The variation of the average if frequently described in terms of average power in decibel (dB): $U_i = 10\log(V^2(x_i))$ where V is the voltage amplitude and the subscript i denotes different small areas.
- For small areas at approximately the same distance from the Base Station (BS), the distribution observed for U_i about its mean value $E\{U\}$ is found to be close to the Gaussian distribution

$$p(U_i - E\{U\}) = \frac{1}{\sigma_{SF}\sqrt{2\pi}} e^{-\frac{(U_i - E\{U\})^2}{2\sigma_{SF}^2}}$$

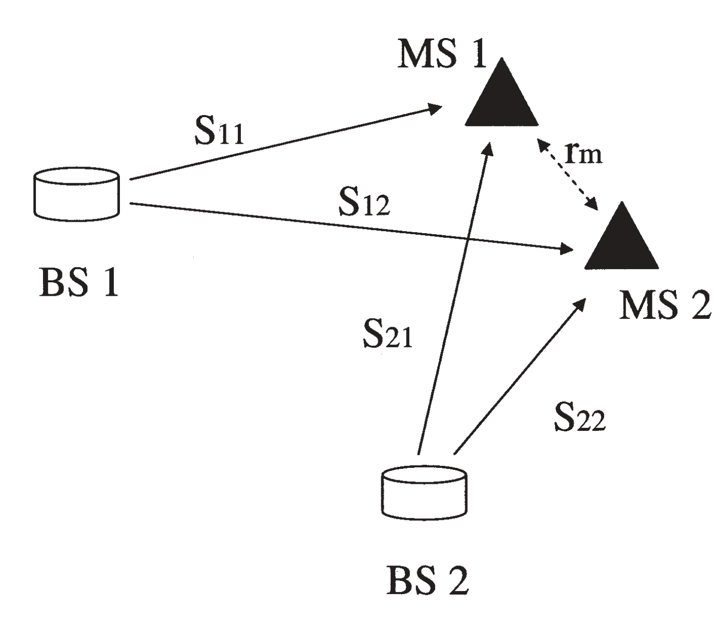
where σ_{SF} is the standard deviation or local variability of the shadow fading.

Slow Fading



Correlated Shadowing

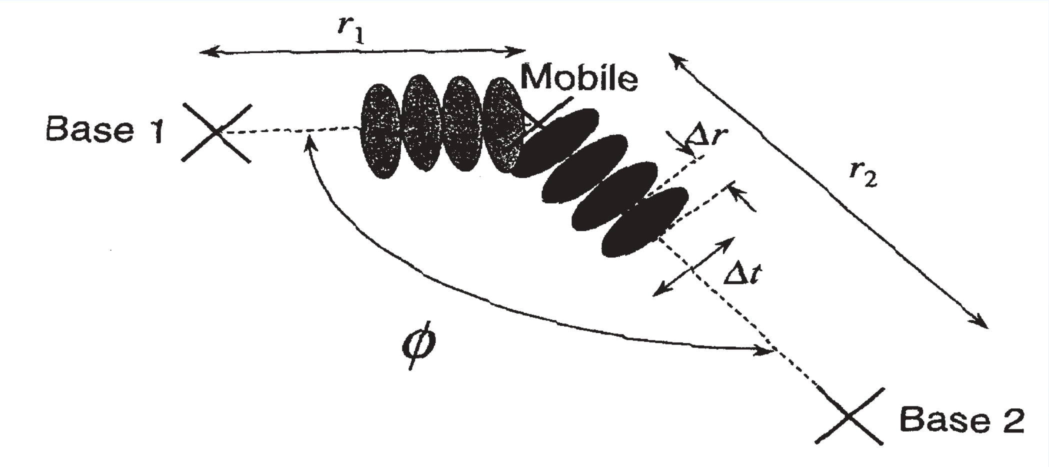
- *Serial Correlation*: auto-correlations between two mobile locations, receiving signals from a single Base Station, such as between S_{11} and S_{12} or between S_{21} and S_{22}: $\rho(r_m) = \frac{E\{S_{11}S_{12}\}}{\sigma_1\sigma_2}$.
- *Site-to-Site correlation*: cross-correlations between two base station locations as received at a single mobile location, such as between S_{11} and S_{21} or between S_{12} and S_{22}: $\rho(r_m) = \frac{E\{S_{11}S_{21}\}}{\sigma_1\sigma_2}$.



Serial & Site-to-Site Correlation

- There is currently no well-agreed model for the correlations.
- A model should includes two key variables:

1) the angle ϕ between the two paths from the BS's and MS. The correlation should decrease with increasing angle-of-arrival difference.
2) the relative values of the two path lengths. If $\phi = 0$, the correlation is expected to be one when the path length are the same. If one of the path length increases, then the correlation should decrease.



Serial & Site-to-Site Correlation

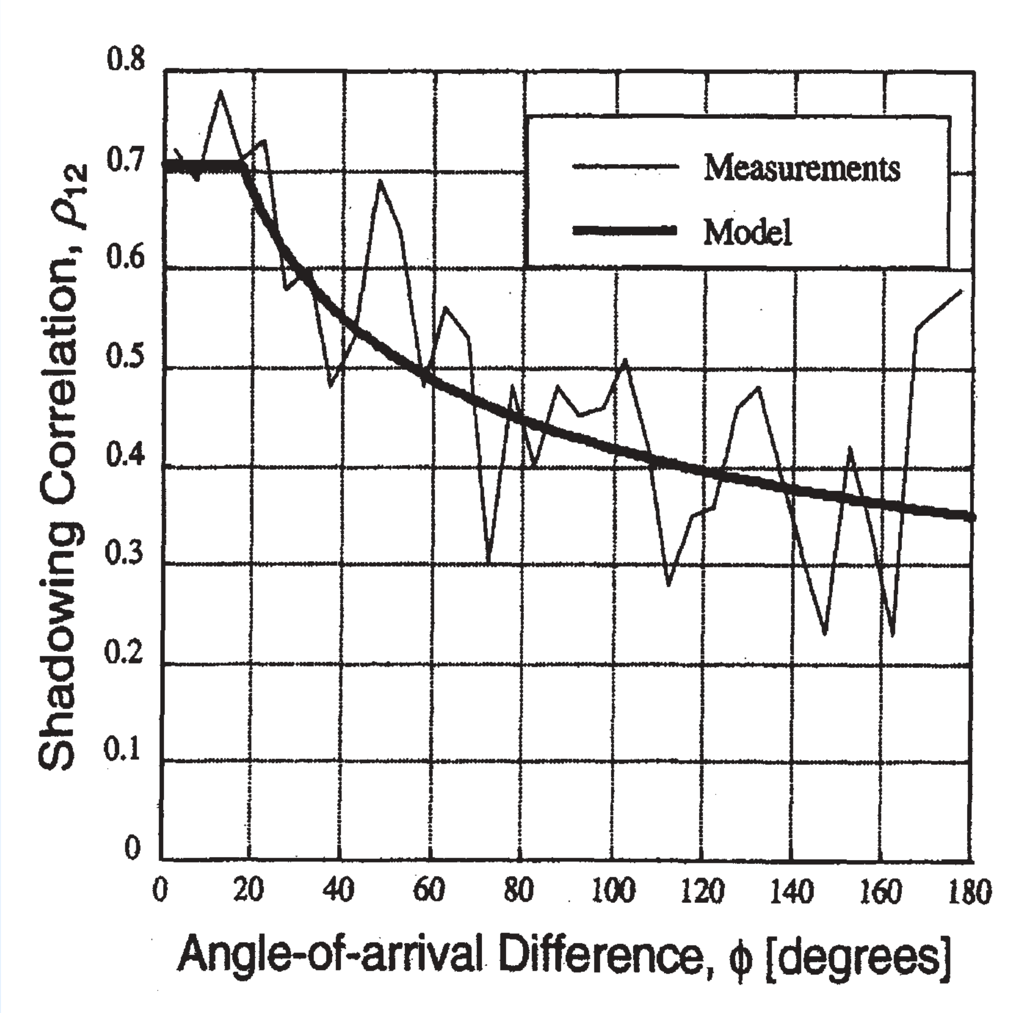
- The model can be expressed as

$$\rho = \begin{cases} \sqrt{\frac{r_1}{r_2}} & \text{for} \quad 0 \leq \phi \leq \phi_T \\ \left(\frac{\phi_T}{\phi}\right)^{\gamma} \sqrt{\frac{r_1}{r_2}} & \text{for} \quad \phi_T \leq \phi \leq \pi \end{cases}$$

  here $\phi_T = 2 \sin^{-1} \left(\frac{r_c}{2r_1}\right)$ where r_c is the serial correlation distance and γ is smooth correlation function.

- The serial correlation distance or shadowing correlation distance is the distance taken for the normalized autocorrelation to fall to 0.37 (1/e).
- The smooth function takes into account the size and heights of the terrain and clutter, and according to the heights of the Base Station antennas relative to them.

Serial & Site-to-Site Correlation



Conclusions

- In this presentation we have introduced the fading models.
- In particular flat, fast and slow fading have been considered.
- The main distribution for small scale fading have be presented.
- Finally, the correlations problem with related to shadowing have been shown.

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Homeworks

- Please, explain what is a Fast and Slow Fading process.
- Explain briefly the main differences between Serial and Site-to-Site correlation for shadowing.