Fading Models

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Abstract— **In this paper fading models are considered. In particular we divide the models into three classes by separating the received signal in three scale of spatial variation such as fast fading, slow fading (shadowing) and path loss. Moreover, several models for small scale fading are considered such as Rayleigh, Ricean, Nakagami and Weibull distributions. Slow fading is also investigated as well as serial and site-to-site correlations are compared.**

Index Terms— **Fast and slow fading, flat fading, correlated shadowing, small scale fading distributions.**

I. Introduction

Most mobile communication systems are used in and around center of population. The transmitting antenna or Base Station (BS) are located on top of a tall building or tower and they radiate at the maximum allowed power. In the other hand, the mobile antenna or Mobile Station (MS) is well below the surrounding buildings. Consequently, the radio channel is influenced by the surrounding structures such as cars, buildings,...

The wireless channel can be described as a function of time and space and the received signal is the combination of many replicas of the original signal impinging at receiver (RX) from many different paths [1],[2],[5],[6]. The signal on these different paths can constructively or destructively interfere with each other. This is referred as multipath. If either the transmitter or the receiver is moving, then this propagation phenomena will be time varying, and fading occurs. In addition to propagation impairments, the other phenomena that limit wireless communications are noise and interference. It is interesting to notice that wireless communication phenomena are mainly due to scattering of electromagnetic waves from surfaces or diffraction, e.g. over and around buildings.

The design goal is to make the received power adequate to overcome background noise over each link, while minimizing interference to other more distant links operating at the same frequency.

In this paper we describe the fading models by splitting the received signal in three scale of spatial variation such as fast fading, slow fading (shadowing) and path loss. Here we consider several models for small scale fading known as Rayleigh, Ricean, Nakagami and Weibull distributions. Slow fading is also investigated and the problem of serial and site-to-site correlations is analyzed.

This paper is organized as follows. First we introduce the fading types by giving a general description. In Section III we define the flat fading concept. In Section IV fast fading is introduced. In Section V we investigate the small scale fading effect by considering several kind of distribution functions. In Section VI the slow fading concept is described. In Section VII we talk about correlated shadowing. In particular we compare serial and site-to-site correlation. Finally, in Section VIII, we give our conclusions.

II. Fading types

Researches have shown that multiple propagation paths or multipaths have both slow and fast aspects [1],[2]. The received signal for narrowband excitation is found to exhibit three scales of spatial variation such as Fast Fading, Slow Fading and Range Dependence. Moreover temporal variation and polarization mixing can be present.

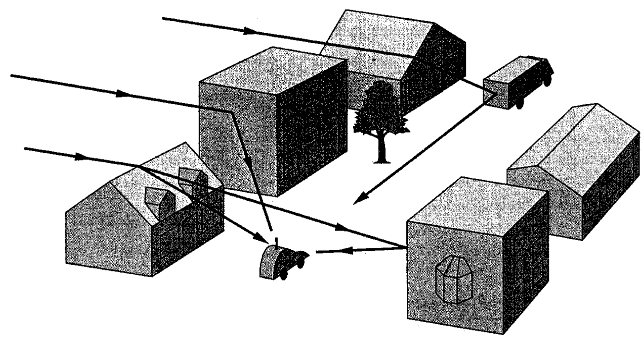

Fig. 1. Example of multipath due to diffraction effects in a urban environment.

In Fig. 1 we depict an example of multipath environment where, since the receiver (RX) is surrounded by objects, replicas of the original signal reach the RX from different paths with different delays. As the Mobile Station (MS) moves along the street, rapid variations of the signal are found to occur over distances of about one-half the wavelength.

The measured signal in Fig. 2 contains both fast and slow components [2]. For this particular example the frequency used was $f = 910$-MHz, the wavelength $\lambda = \frac{c}{f}$ where c is the speed of light. By analyzing the picture we can see that over a distance of few meter the signal can vary by 30-dB. Over distances as small as $\frac{\lambda}{2}$, the signal is seen to vary by 20-dB. Small scale variation results from the arrival of the signal at the MS along multiple ray paths due to reflection, diffraction and scattering [4]-[6].

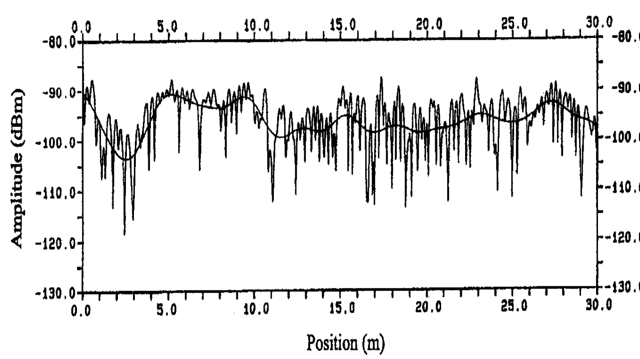

Fig. 2. Amplitude of the received signal as a function of the range dependence. Here the slow and fast fading are represented.

The phenomenon of fast fading is represented in Fig. 2 as rapid fluctuations of the signal over small areas. The multiple ray set up an interference pattern in space through which the MS moves. When the signals arrive from all directions in the plane, fast fading will be observed for all directions of motion. In response to the variation in the nearby buildings, there will be a change in the average about which the rapid fluctuations take place. This middle scale over which the signal varies, which is on the order of the buildings dimensions is known as shadow fading, slow fading or log-normal fading [1],[2],[7],[8].

III. Flat fading

The wireless channel is said to be flat fading if it has constant gain and linear phase response over a bandwidth which is greater than the bandwidth of the transmitted signal [1]-[4]. In other words, flat fading occurs when the bandwidth of the transmitted signal B is smaller than the coherence bandwidth of the channel B_m, i.e. $B \ll B_m$.

The effect of flat fading channel can be seen as a decrease of the Signal-to-Noise ratio (SNR). Since the signal is narrow with respect to the channel bandwidth, the flat fading channels are also known as amplitude varying channels or narrowband channels.

IV. Fast fading

Fast fading occurs if the channel impulse response changes rapidly within the symbol duration [1]-[4]. In other works, fast fading occurs when the coherence time of the channel T_D is smaller than the symbol period of the the transmitted signal T such as $T_D \ll T$. This causes frequency dispersion or time selective fading due to Doppler spreading [3],[4]. Fast Fading is due to reflections of local objects and the motion of the objects relative to those objects [2].

The receive signal is the sum of a number of signals reflected from local surfaces, and these signals sum in a constructive or destructive manner depending on the relative phase shift, see Fig.3. Phase relationships depend on the speed of motion, frequency of transmission and relative path lengths.

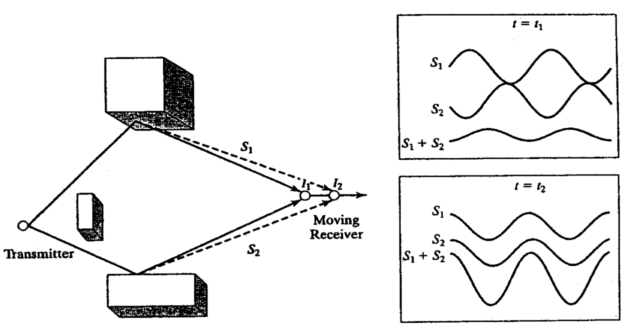

Fig. 3. Representation of constructive and destructive interference between two signals. The key role is played by the phase difference between S_1 and S_2.

In Fig. 4 we give an illustrative example of how to separate out fast fading from slow fading. It can be done by averaging the envelope or magnitude of the RX signal over a distance (e.g. 10-m). Alternatively, a sliding window can be used [2].

V. Small scale fading

The received pass-band signal without noise after transmitted unmodulated carrier signal $\cos(2\pi f_c t)$ can be written as [3]

$$\begin{aligned} x(t) &= \sum_i \alpha_i(t)\cos(2\pi f_c(t-\tau_i(t))) \\ &= \Re\{\sum_i \alpha_i(t) e^{-j2\pi f_c \tau_i(t)} e^{j2\pi f_c t}\}\ . \end{aligned} \quad (1)$$

Here $\Re$ represent the real part of the quantity within its brackets, $\alpha_i(t)$ is the time-varying attenuation factor of the i^{th} propagation delay, $\tau_i(t)$ is the time-varying delay and f_c is the carrier frequency. Assume that $\tau_i(t) \ll T$ where T is the symbol time.

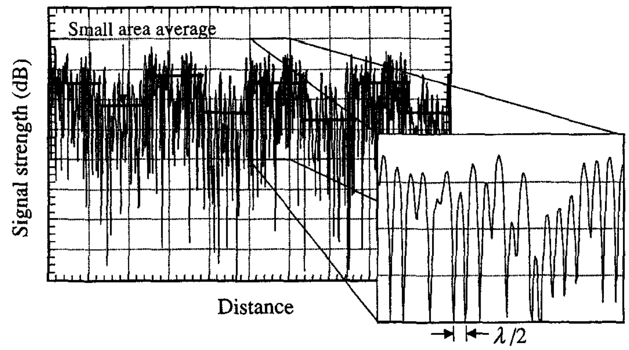

Fig. 4. Small-area obtained by averaging over discrete intervals. How the fast fading can be separate out from the slow fading.

The equivalent baseband signal can then be expressed as

$$h(t) = \sum_i \alpha_i(t) e^{-j2\pi f_c \tau_i(t)}. \quad (2)$$

If the delays $\tau_i(t)$ change in a random manner, and when the number of propagation path is large, central limit theorem applies and $h(t)$ can be modelled as a complex Gaussian process.

In the next paragraphs we introduce some distribution functions which can be used for modelling and designing wireless communication systems. Overall, the Rayleigh and the Ricean distribution are the most common used. Notice that in this paper we do not give the derivations of the distribution functions, however, they can be found in several course books, e.g. [1],[3],[4].

A. Rayleigh distribution

When the components of $h(t)$ are independent the probability density function of the amplitude $r = |h| = \alpha$ has Rayleigh pdf

$$f(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \quad (3)$$

where $E\{r^2\} = 2\sigma^2$ and $r \geq 0$. In Fig.5 the Rayleigh pdf is depicted.

This represents the worst fading case because we do not consider to have Line-of-Sight (LOS). The power is exponentially distributed. The phase is uniformly distributed and independent from the amplitude. This is the most used signal model in wireless communications.

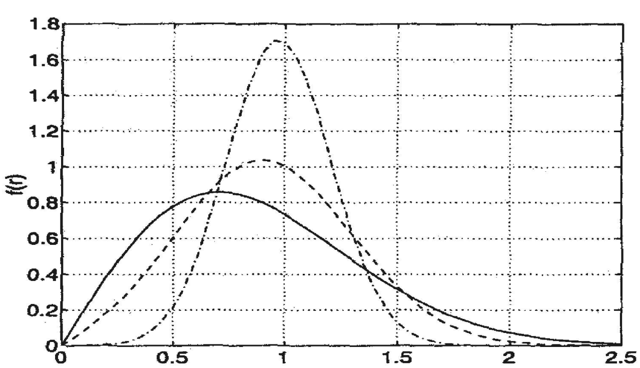

Fig. 5. Ricean probability density function, (-) $K = -\infty$, (- -) $K = 3$-dB, (-.-) $K = 9$-dB.

B. Ricean distribution

In case the channel is complex Gaussian with non-zero mean (there is LoS), the envelope $r = |h|$ is Ricean distributed. Here we denote $h = \alpha e^{j\phi} + \nu e^{j\theta}$ where α follows the Rayleigh distribution and $\nu > 0$ is a constant such that ν^2 is the power of the LOS signal component. The angles ϕ and θ are assumed to be mutually independent and uniformly distributed on $[-\pi, \pi)$. Ricean pdf can be written as

$$f(r) = \frac{r}{\sigma^2} e^{-\frac{r^2+\nu^2}{2\sigma^2}} I_0\Big(\frac{r\nu}{\sigma^2}\Big) \qquad \text{and} \qquad r \geq 0 \tag{4}$$

where I_0 is the modified Bessel function of order zero and $2\sigma^2 = E\{\alpha^2\}$. The Rice factor $K = \frac{\nu^2}{2\sigma^2}$ is the relation between the power of the LoS component and the power of the Rayleigh component. When $K \to \infty$, no LoS component and Rayleigh is equal to the Ricean pdf, see Fig.5.

C. Nakagami distribution

In this case we denote $h = re^{j\phi}$ where the angle ϕ is uniformly distributed on $[-\pi, \pi)$. The variable r and ϕ are assumed to be mutually independent. The Nakagami pdf can be expressed as

$$f(r) = \frac{2}{\Gamma(k)} \Big(\frac{k}{2\sigma^2}\Big)^k r^{2k-1} e^{-\frac{kr^2}{2\sigma^2}} \qquad \text{and} \qquad r \geq 0 \tag{5}$$

where $2\sigma^2 = E\{r^2\}$, $\Gamma(\cdot)$ is the Gamma function and $k \geq \frac{1}{2}$ is the fading figure (degrees of freedom related to the number of added Gaussian random variables). It was originally developed empirically based on measurements. Instantaneous receive power is Gamma distributed. With $k = 1$ the Rayleigh is equal to the Nakagami pdf, see Fig. 6.

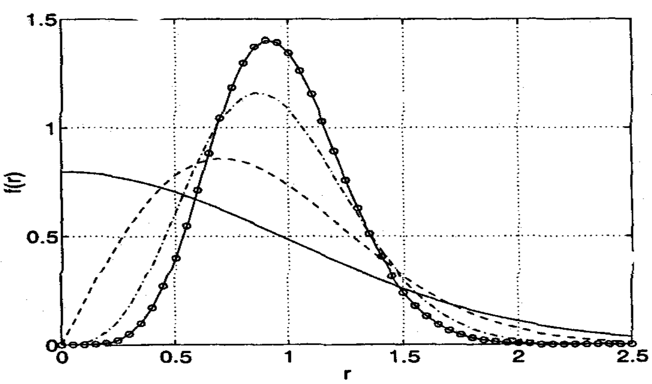

Fig. 6. Nakagami probability density function, (-) $K = -\frac{1}{2}$, (- -) $K = 1$ (Rayleigh), (-.-) $K = 2$ and (o) $K = 3$.

D. Weibull distribution

Weibull distribution represents another generalization of the Rayleigh distribution. When X and Y are i.i.d. zero-mean Gaussian variables, the envelope of $R = (X^2 + Y^2)^{\frac{1}{2}}$ is Rayleigh distributed. However, is the envelope is defined as $R = (X^2 + Y^2)^{\frac{1}{k}}$, the corresponding pdf is Weibull distributed:

$$f(r) = \frac{kr^{k-1}}{2\sigma^2} e^{-\frac{r^k}{2\sigma^2}} \tag{6}$$

where $2\sigma^2 = E\{r^2\}$.

VI. Slow fading

Slow fading is the result of shadowing by buildings, mountains, hills, and other objects [7],[8]. In Fig.7 we show the impact of shadowing on an object that is moving around a BS at a constant range. Some paths suffer increased loss, while others will be less obstructed and have an increased signal strength [1].

The average within individual small areas also varies from one small area to the next in an apparently random manner. The variation of the average if frequently described in terms of average power in decibel (dB)

$$U_i = 10 \log(V^2(x_i)) \tag{7}$$

where V is the voltage amplitude and the subscript i denotes different small areas. For small areas at approximately the same distance from the Base Station (BS), the distribution observed for U_i about its mean value $E\{U\}$ is found to be close to the Gaussian distribution [5]

$$p(U_i - E\{U\}) = \frac{1}{\sigma_{SF}\sqrt{2\pi}} e^{-\frac{(U_i - E\{U\})^2}{2\sigma_{SF}^2}} \tag{8}$$

where σ_{SF} is the standard deviation or local variability of the shadow fading.

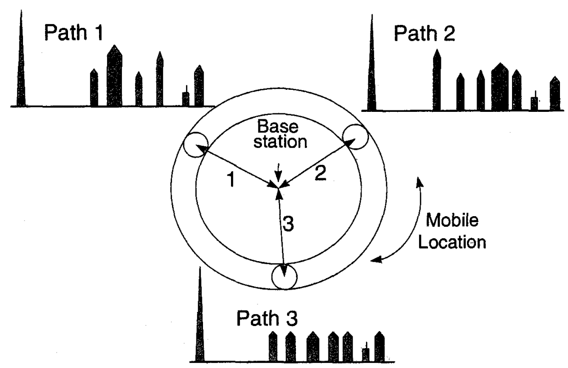

Fig. 7. Variation of path profiles encountered at a fixed range from the Base Station (BS).

VII. Correlated shadowing

Investigations of the shadowing effect, and in particular of the correlation, have suggested that the correlated shadowing could be broken into two classes such as serial and site-to-site correlation [5].

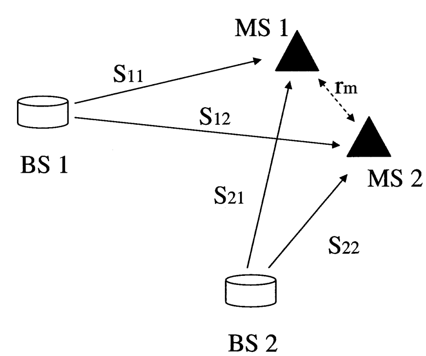

Fig. 8. Representation of four correlated shadowing processes.

The serial correlation defines an auto-correlations between two mobile locations, receiving signals from a single Base Station, such as between S_{11} and S_{12} or between S_{21} and S_{22}, see Fig. 8. This correlation can be expressed as

$$\rho(r_m) = \frac{E\{S_{11}S_{12}\}}{\sigma_1\sigma_2}. \tag{9}$$

where σ_1 and σ_2 represent the local variabilities of the two processes. Notice that if r_m is small and the MS has not move much, then we can consider that the two processes have the same local variability such as $\sigma_1\sigma_2 \simeq \sigma^2$.

On the other hand, the site-to-site correlation identifies the cross-correlations between two base station locations as received at a single mobile location, such as between S_{11} and S_{21} or between S_{12} and S_{22}, see Fig.8. Mathematically it can be expressed as

$$\rho(r_m) = \frac{E\{S_{11}S_{21}\}}{\sigma_1\sigma_2}. \tag{10}$$

Notice that in this case we can not consider $\sigma_1 \simeq \sigma_2$ since the two MS's could be located far from each other.

A. Serial and Site-to-Site correlation

Currently, there is no well-agreed model for describing the correlations. A model illustrating both correlation os depicted in Fig.9.

A model for serial and site-to-site correlations should includes two key variables [5]. First the angle ϕ between the two paths from the BS's and MS. In fact, the correlation should decrease with increasing angle-of-arrival difference. Second the relative values of the two path lengths. If $\phi = 0$, the correlation is expected to be one when the path length are the same. If one of the path length increases, then the correlation should decrease.

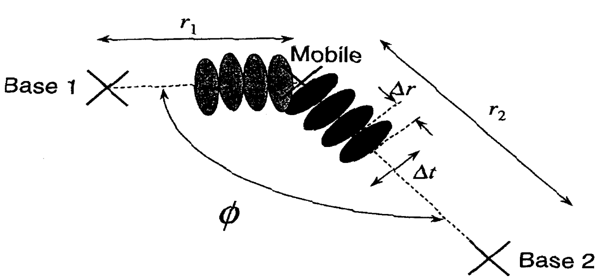

Fig. 9. Physical model for shadowing cross-correlation.

A model that satisfies all the requirements can be found in [**?**] and it is defined as

$$\rho = \begin{cases} \sqrt{\frac{r_1}{r_2}} & \text{for} \quad 0 \le \phi \le \phi_T \\ \left(\frac{\phi_T}{\phi}\right)^{\gamma}\sqrt{\frac{r_1}{r_2}} & \text{for} \quad \phi_T \le \phi \le \pi \ . \end{cases} \tag{11}$$

Here $\phi_T = 2\sin^{-1}\left(\frac{r_c}{2r_1}\right)$ where r_c is the serial correlation distance and γ is smooth correlation function. The serial correlation distance or shadowing correlation distance is the distance taken for the normalized autocorrelation to fall to 0.37 (1/e). The smooth function takes into account the size and heights of the terrain and clutter, and according to the heights of the Base Station antennas relative to them.

In Fig.10 we compare the performance of the model introduced in eq.(11) to the correlation computed from measurements. Unfortunately, even though the model has a low computational cost, it is not accurate.

VIII. Conclusions

In this paper we have described fading models. In particular we have divided the models into three classes by separating the received signal in three scale of spatial variation such as fast fading, slow fading (shadowing) and path loss. Moreover, several models for small scale fading have been considered such as Rayleigh, Ricean, Nakagami and Weibull distributions. Slow fading has also been investigated as well as serial and site-to-site correlations have been compared.

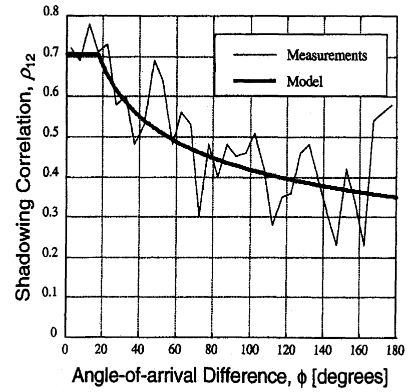

Fig. 10. Comparison of measurements and model for shadowing cross-correlation.

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