Helsinki University of Technology
Communications laboratory

S-72.333 Postgraduate Course in Radio Communications

1.2.2005 MIMO-OFDM

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- 5. PAPR
- 6. Example of PAPR reduction in MIMO-OFDM
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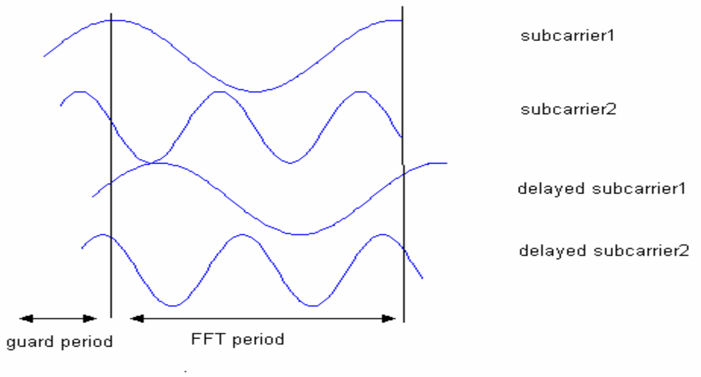
1. Introduction

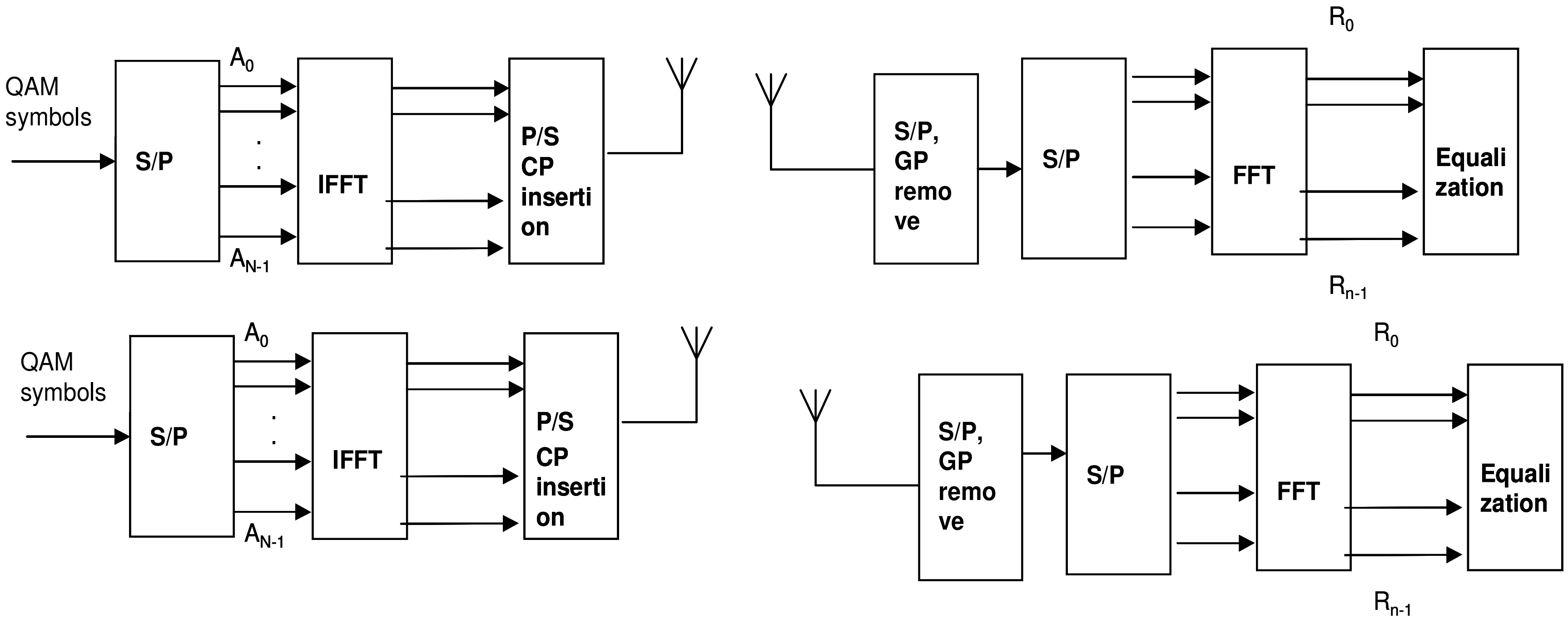
- MIMO combats fading with diversity
- OFDM is modulation method known for its capability to mitigate multipath
- The symbol duration of OFDM is N times longer than in a single carrier system with the same symbol rate.
- Symbols made even longer by adding a cyclic prefix to each symbol
- As long as the cyclic prefix is longer than the channel delay spread, OFDM offers inter-symbol interference (ISI) free transmission



1.Introduction

- Another key advantage of OFDM is that it dramatically reduces equalization complexity by enabling equalization in the frequency domain
- OFDM converts the wideband signal, affected by frequency selective fading, into N narrowband flat fading signals
- In other words MIMO-OFDM transmits N parallel MIMO channels from each transmit antenna
- The equalization can be performed in the frequency domain by a scalar division carrier-wise with the subcarrier related channel coefficients

2. MIMO-OFDM


QAM symbols
S/P
A_0
A_{N-1}
IFFT
P/S CP insertion
S/P, GP remove
S/P
FFT
R_0
R_{n-1}
Equalization
QAM symbols
S/P
A_0
A_{N-1}
IFFT
P/S CP insertion
S/P, GP remove
S/P
FFT
R_0
R_{n-1}
Equalization

2. MIMO-OFDM

- The received MIMO-OFDM symbol of the n:th subcarrier and the m:th OFDM symbol of the i:th receive antenna after FFT can be written as

$$R_i[n,m] = \sum_{j=1}^{N} H_{i,j}[n,m] A_j[n,m] + W_i[n,m], \qquad i = 1, 2, \ldots, M$$

- where Aj[n,m] is the transmitted data symbol on n:th carrier and m:th OFDM symbol, Wi[n,m] is the additive noise contribution at i:th receive antenna for the corresponding symbol in frequency domain and Hi,j[n,m] is the channel coefficient in the frequency domain between the j:th transmit antenna and the i:th receive antenna

2.MIMO-OFDM

- The channel coefficients in frequency domain are obtained as linear combinations of the dispersive channel taps

$$H[n,m] = \sum_{i=0}^{I-1} h_i[m] e^{-j2\pi\tau_i n/T}, \quad n = 0, \ldots, N-1$$

- where I is the number of channel taps in time domain and hm is modeled as an independent zero-mean random Gaussian process.
- The channel matrix:

$$\bar{H}[n,m] = \begin{bmatrix} H_{1,1}[n,m] & H_{1,2}[n,m] & \cdots & H_{1,N}[n,m] \\ H_{2,1}[n,m] & H_{2,2}[n,m] & \cdots & H_{2,N}[n,m] \\ \vdots & \vdots & \ddots & \vdots \\ H_{M,1}[n,m] & H_{M,2}[n,m] & \cdots & H_{M,N}[n,m] \end{bmatrix}$$

2. MIMO-OFDM

- Taking the received data symbols of all antennas into account, the expression of the received data symbol can be presented in the matrix form as follows

$$\vec{R}[n,m] = \bar{H}[n,m]\vec{A}[n,m] + \vec{W}[n,m]$$

- where

$$\vec{A}[n,m] = \begin{bmatrix} A_1[n,m] & A_2[n,m] & \cdots & A_N[n,m] \end{bmatrix}^T$$

$$\vec{R}[n,m] = \begin{bmatrix} R_1[n,m] & R_2[n,m] & \cdots & R_M[n,m] \end{bmatrix}^T$$

3. Equalization

- To obtain the transmitted data symbols equation

$$\vec{A}[n,m]=\bar{H}[n,m]^{-1}(\vec{R}[n,m]+\vec{W}[n,m])$$

- This equalization works well in case of small noise and no ISI or ICI
- In the presence of ICI and ISI the received signal can be written as

$$\vec{R}_i[n,m]=\sum_{j=1}^{N}R_{j,i}^{U}[n,m]+\sum_{j=1}^{N}R_{j,i}^{ICI}[n,m]+\sum_{j=1}^{N}R_{j,i}^{ISI}[n,m]+W_i[n,m]$$

- In order to be able to cancel the interference the ISI and ICI terms should be calculated and then subtracted from the received signal

4. Capacity

- In [3] the capacity of conventional MIMO, MIMO-OFDM and spread MIMO-OFDM in presence of multipath is studied.
- In the single user case the capacity for the conventional MIMO without ISI is the highest and they state that it is the upper bound of capacity limit
- MIMO-OFDM and spread MIMO-OFDM give more capacity than conventional MIMO in presence of multipath and based on their results MIMO-OFDM and spread MIMO-OFDM would be similarly impacted by multipath
- This seems reasonable since OFDM with long enough cyclic prefix is a powerful mean to mitigate multipath
- In multiuser channel spread MIMO-OFDM provides more capacity than the other schemes

4. Capacity

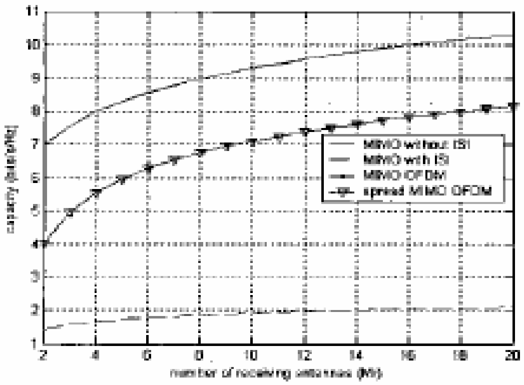


Fig. 2 Channel capacity versus number of receive antennas at Mt=2, SNR=15 dB, L=5

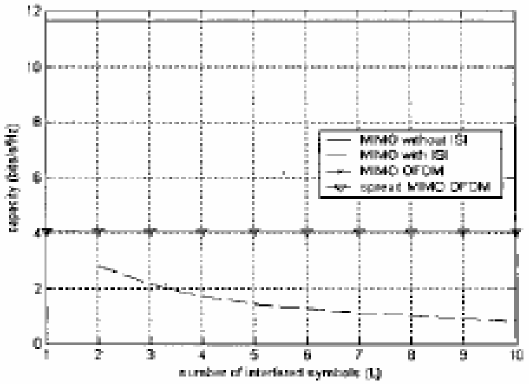


Fig.4 Channel capacity versus number of interfered symbols at Mt=Mr=10, SNR=15 dB

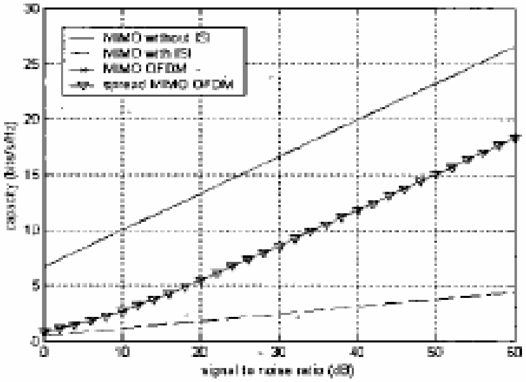


Fig. 3 Channel capacity versus signal to noise ratio at Mt=Mr=10, L=5

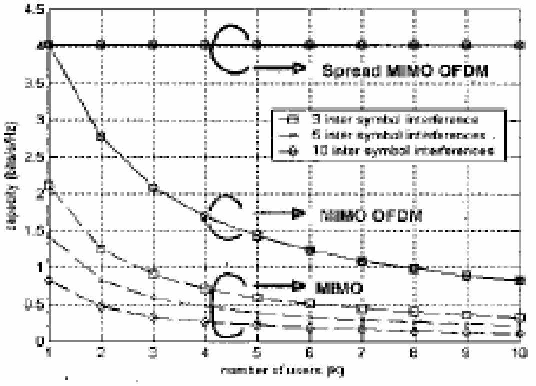


Fig.5 Average capacity rate per user versus number of users at Mt=Mr=10, SNR=15 dB

PAPR

- The peak-to-average power ratio (PAPR), is a measure of the amplitude fluctuations of the signal
- Any multicarrier signal with a large number of subcarriers may have a high PAPR due to occasional constructive addition of subcarriers
- At the transmitter, the power amplifier (PA) is the main source of nonlinear distortions
- As amplifier nonlinearity is amplitude dependent, the amplitude fluctuations of the input signal are of a concern

5. PAPR

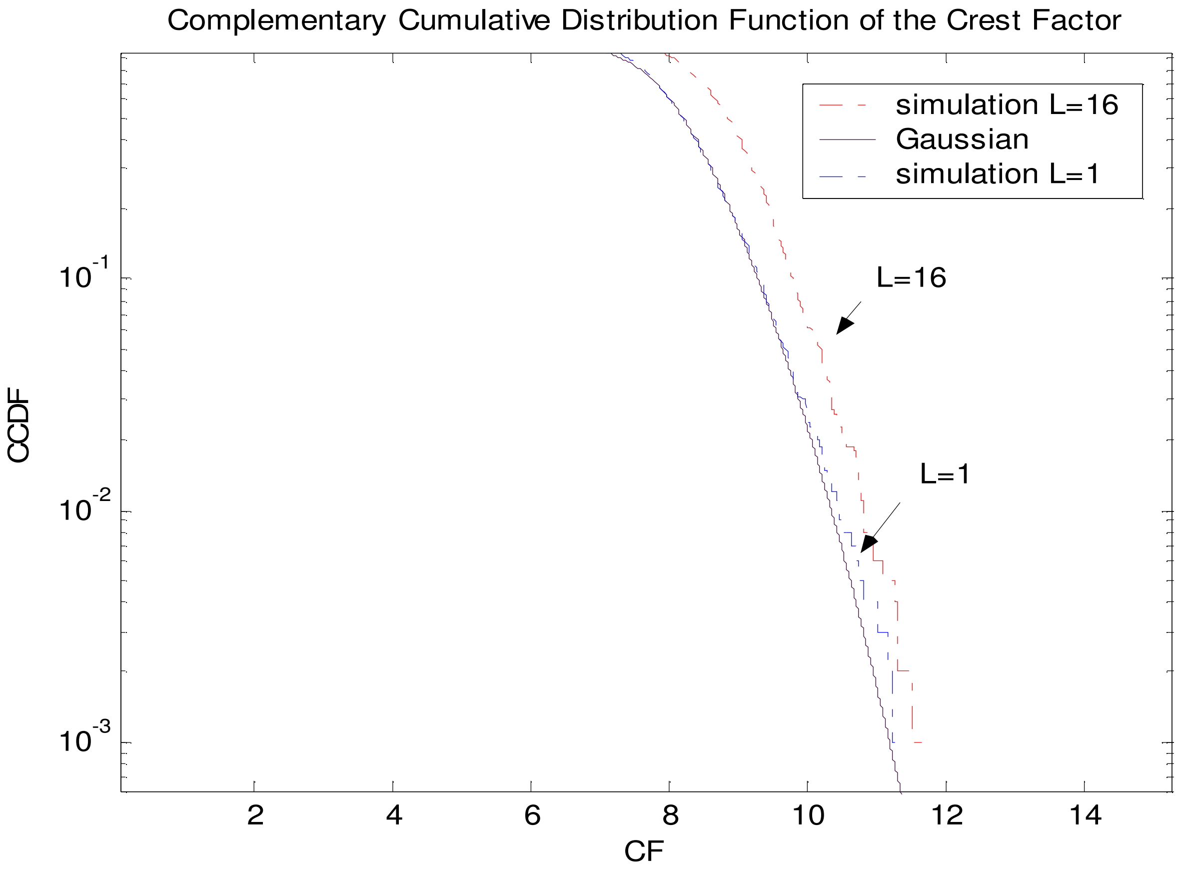
- In OFDM, when the number of carriers is large, the central limit theorem holds and the time domain samples of the OFDM signal, sampled at Nyquist rate, are approximately zero-mean complex Gaussian random variables
- Then the probability that the PAPR of the OFDM symbol exceeds a given threshold PAPR0 can be expressed as

$$\Pr(PAPR > PAPR_0) = 1 - F(PAPR_0)^{N_c} = 1 - (1 - e^{-PAPR_0})^{N_c}$$

- This analysis underestimates the distribution of the PAPR
- It can also be noted that the Gaussian distribution has infinite values but the largest amplitude value of an OFDM signal is only N times the average amplitude of the carriers thus the approximation does not hold very accurately on large amplitudes i.e. the shape of the PAPR distribution is does not follow Gaussian in the tails of the distribution

5. PAPR

- The Gaussian approximation is compared to a CCDF of a Nyquist rate sampled 16-QAM OFDM signal and to CCDF of an oversampled signal with oversampling factor 16



6. Example of PAPR reduction in MIMO-OFDM

- PAPR reduction techniques can be divided in two approaches
- With help of redundancy
  - Adding redundancy does not cause any interference but it adds complexity of the transmitter and lowers the net transmission rate
  - For example selective mapping or partial transmit sequence
- To apply a correcting function to the signal to eliminate the high amplitude peaks
  - very simple approach but it causes interference
  - For example clipping or windowing

6. Example of PAPR reduction in MIMO-OFDM

- In selective mapping, V statistically independent sequences are generated from the same information by multiplying with a certain vector
- The sequence with the lowest PAPR is selected
- The information of the vector used to generate the selected sequence has to be sent to the receiver
- Detection of the signal depends also on the errors on the side information transmission.

6. Example of PAPR reduction in MIMO-OFDM

- In MIMO-OFDM SLM can be applied to individual antennas in a way that every antenna selects independently one of V sequences to be transmitted
- In this way each antenna are sending different side information and the complementary cumulative distribution (CCDF) of the best sequence is

$$\Pr(PAPR > PAPR_0)^V = (1-(1-e^{-PAPR_0})^{N_c})^V$$

6. Example of PAPR reduction in MIMO-OFDM

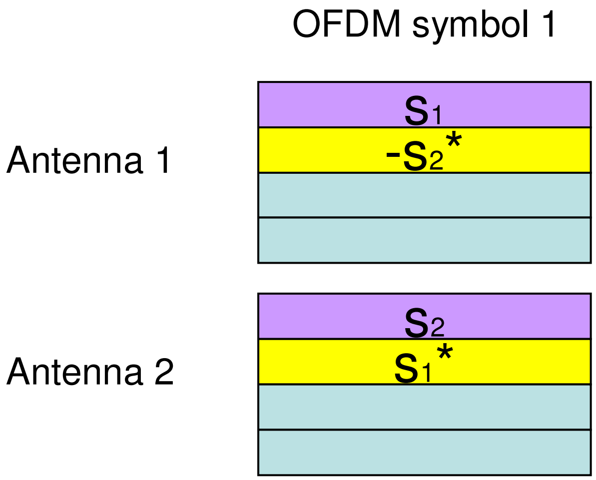
- In [4] a concurrent SLM approach is proposed
- In this approach a common vector to all transmit antennas among the V vectors is selected
- The selection is made based on the lowest average PAPR over the N transmit antennas
- The selected vector is sent over all transmit antennas and thus diversity gain is obtained
- Correspondingly the amount of redundancy could be lowered
- The CCDF of the best sequence is

$$\Pr(PAPR > PAPR_0)^V = (1-(1-e^{-PAPR_0})^{N_c N})^V$$

- As the selection is made according to the average PAPR there will be a sligth degradation in PAPR performance compared to the individual SLM approach

7. Spatial diversity coding for MIMO-OFDM

- In MIMO system the Alamouti scheme realizes full spatial diversity gain in the absence of channel knowledge at the transmitter.
- This requires that the channel remains constant over at least two consecutive symbol periods
- In MIMO-OFDM the coding is performed in the frequency rather than in time
- Symbols s1 and s2 are transmitted over antennas 1 and 2 on tone n and symbols –s2* and s1* are transmitted over antennas 1 and 2 on tone n+1


OFDM symbol 1

Antenna 1

s_1

$-s_2^*$

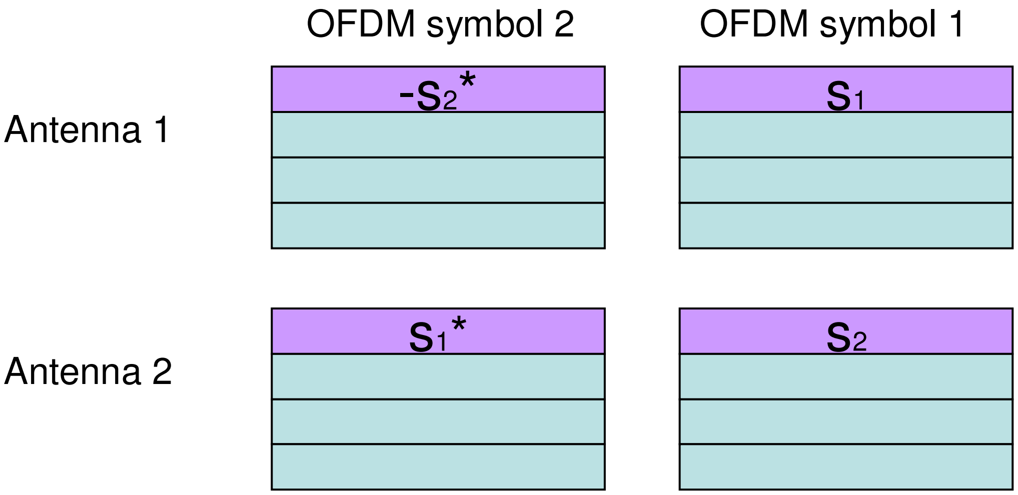
Antenna 2

s_2

s_1^*

7. Spatial diversity coding for MIMO-OFDM

- Any pair of tones could be used as long as the associated channels are equal i.e. the channel requirement is different from the MIMO case
- An alternative technique is to use diversity coding on a per-tone basis across OFDM symbols in time but then the channel should be constant during two consecutive OFDM symbols
- This is not usually true due to the long duration of OFDM symbols



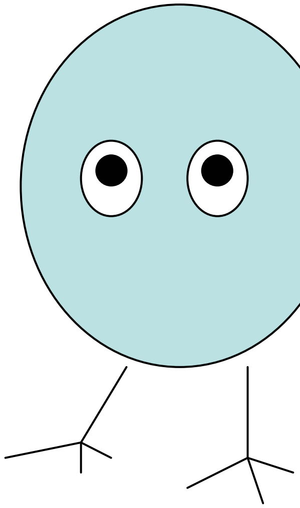
	OFDM symbol 2	OFDM symbol 1
Antenna 1	$-s_2^*$	s_1
Antenna 2	s_1^*	s_2

8. Space frequency coded MIMO-OFDM

- The spatial diversity coding mentioned in the last slide realizes spatial diversity gain in MIMO-OFDM system
- However, also frequency diversity is available in tones with spacing larger than the coherence bandwidth of the channel
- The total diversity gain that can be realized in a MIMO-OFDM system has been shown in a reference of [5]
- The total diversity gain equals to NMD, where D is the number of coherence bandwidths

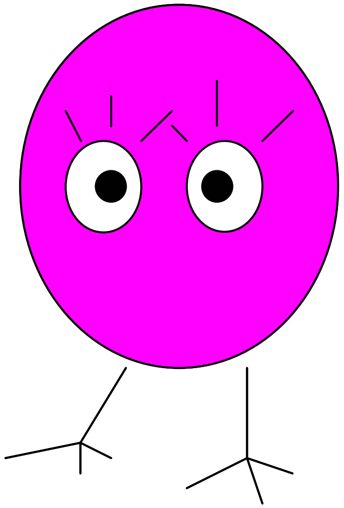
9. Conclusions

- MIMO and MIMO-OFDM are very hot topics of current research
- The information-theoretic performance limits, particularly in the multiuser context and space time code and receiver design have attracted significant research interest



Homework

- Comment on the advantages and disadvantages of combining MIMO with OFDM and CDMA. For example you can comment on the results of [3] or try to find another paper that compares these schemes.



References

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[2] V.D.Nguyen, M.Pätzhold, “Frequency domain interference cancellation for MIMO-OFDM systems” Proc. 9th International OFDM-Workshop, Dresden, Germany, 15. – 16. Sept. 2004, p. 114–117.

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