MIMO-OFDM

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I. INTRODUCTION

The growing demand of multimedia services and the growth of Internet related contents lead to increasing interest to high speed communications. The requirement for wide bandwidth and flexibility imposes the use of efficient transmission methods that would fit to the characteristics of wideband channels especially in wireless environment where the channel is very challenging. In wireless environment the signal is propagating from the transmitter to the receiver along number of different paths, collectively referred as multipath. While propagating the signal power drops of due to three effects: path loss, macroscopic fading and microscopic fading. Fading of the signal can be mitigated by different diversity techniques. To obtain diversity, the signal is transmitted through multiple (ideally) independent fading paths e.g. in time, frequency or space and combined constructively at the receiver. Multiple-input-multiple-output (MIMO) exploits spatial diversity by having several transmit and receive antennas. However the paper "MIMO principles" assumed frequency flat fading MIMO channels.

OFDM is modulation method known for its capability to mitigate multipath. In OFDM the high speed data stream is divided into N_c narrowband data streams, N_c corresponding to the subcarriers or subchannels i.e. one OFDM symbol consists of N symbols modulated for example by QAM or PSK. As a result the symbol duration is N times longer than in a single carrier system with the same symbol rate. The symbol duration is made even longer by adding a cyclic prefix to each symbol. As long as the cyclic prefix is longer than the channel delay spread OFDM offers inter-symbol interference (ISI) free transmission.

Another key advantage of OFDM is that it dramatically reduces equalization complexity by enabling equalization in the frequency domain. OFDM, implemented with IFFT at the transmitter and FFT at the receiver, converts the wideband signal, affected by frequency selective fading, into N narrowband flat fading signals [1] thus the equalization can be performed in the frequency domain by a scalar division carrier-wise with the subcarrier related channel coefficients. The channel should be known or learned at the receiver. The combination MIMO-OFDM is very natural and beneficial since OFDM enables support of more antennas and larger bandwidths since it simplifies equalization dramatically in MIMO systems.

MIMO-OFDM is under intensive investigation by researchers. This paper provides a general overview of this promising transmission technique.

II. BASIC DEFINITIONS

A. Notation

In this paper a capital letter denotes a frequency domain symbol. $\overline{A}$ is a matrix and $\vec{A}$ is a vector.

B. MIMO-OFDM

The general transceiver structure of MIMO-OFDM is presented in Fig. 1. The system consists of N transmit antennas and M receive antennas. In this paper the cyclic prefix is assumed to be longer than the channel delay spread. The OFDM signal for each antenna is obtained by using inverse fast Fourier transform (IFFT) and can be detected by fast Fourier transform (FFT). The received MIMO-OFDM symbol of the n:th subcarrier and the m:th OFDM symbol of the i:th receive antenna after FFT can be written as

$$R_i[n,m]=\sum_{j=1}^{N}H_{i,j}[n,m]A_j[n,m]+W_i[n,m], \quad i=1,2,...,M \tag{1}$$

where Aj[n,m] is the transmitted data symbol on n:th carrier and m:th OFDM symbol, Wi[n,m] is the additive noise contribution at i:th receive antenna for the corresponding symbol in frequency domain and Hi,j[n,m] is the channel coefficient in the frequency domain between the j:th transmit antenna and the i:th receive antenna. The channel coefficients in frequency domain are obtained as linear combinations of the dispersive channel taps

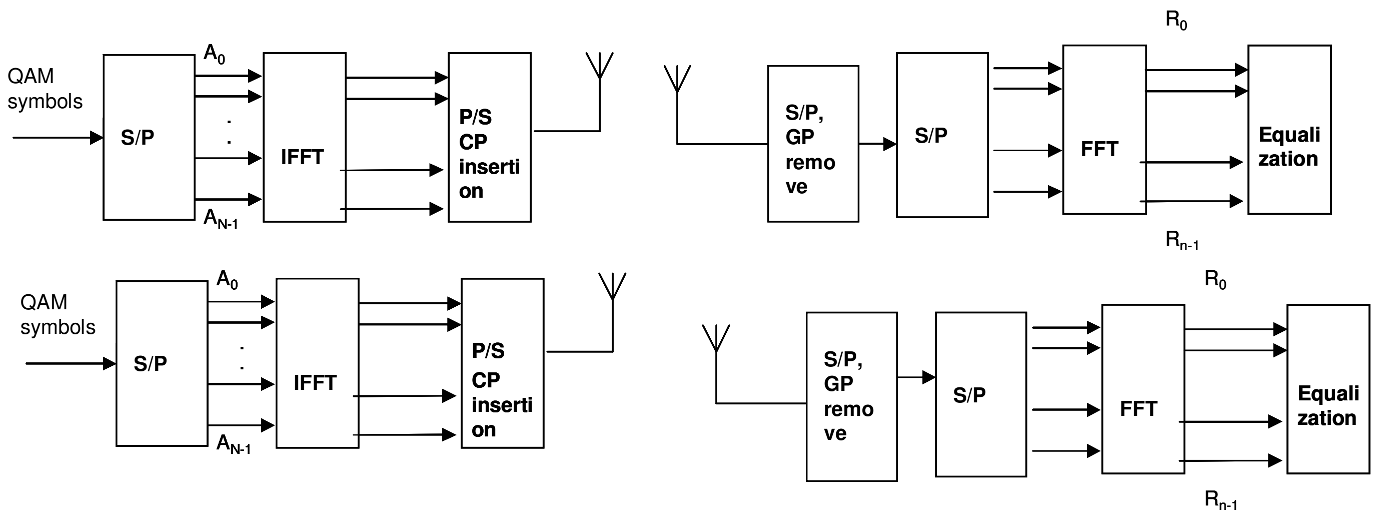

Figure 1 MIMO-OFDM transceiver

$$H[n,m] = \sum_{i=0}^{I-1} h_i\,[m] e^{-j2\pi\tau_i n/T}, \quad n = 0,...,N-1 \tag{2}$$

where I is the number of channel taps in time domain and h^m is modeled as an independent zero-mean random Gaussian process. The impulse response of the Rayleigh fading channel can be expressed as

$$h(t,\tau) = \sum_{i=0}^{I-1} h_i(t)\delta(\tau - \tau_i(t)) \tag{3}$$

where h_i is the tap gain and τ_i is the delay associated to the i:th tap. This delay can be considered to be time invariant. The channel impulse response is assumed to be static over one OFDM channel symbol duration $T_{channel}$ =T+T', where T is the OFDM symbol duration and T' is the cyclic prefix duration. This corresponds to a slowly varying channel where the coherence time is longer than the channel symbol duration. This assumption prevents from experiencing inter-carrier interference (ICI).

The channel matrix $\bar{H}$ is an NxM matix corresponding to the n:th subcarrier and m:th OFDM symbol.

$$\bar{H}[n,m] = \begin{bmatrix} H_{1,1}[n,m] & H_{1,2}[n,m] & \cdots & H_{1,N}[n,m] \\ H_{2,1}[n,m] & H_{2,2}[n,m] & \cdots & H_{2,N}[n,m] \\ \vdots & \vdots & \ddots & \vdots \\ H_{M,1}[n,m] & H_{M,2}[n,m] & \cdots & H_{M,N}[n,m] \end{bmatrix} \tag{4}$$

Taking the received data symbols of all antennas into account, the expression of the received data symbol can be presented in the matrix form as follows

$$\vec{R}[n,m] = \bar{H}[n,m]\vec{A}[n,m] + \vec{W}[n,m] \tag{5}$$

where

$$\vec{A}[n,m] = \begin{bmatrix} A_1[n,m] & A_2[n,m] & \cdots & A_N[n,m] \end{bmatrix}^T \tag{6}$$

and

$$\vec{R}[n,m] = \begin{bmatrix} R_1[n,m] & R_2[n,m] & \cdots & R_M[n,m] \end{bmatrix}^T \tag{7}$$

are the Nx1 and Mx1 vectors of the transmitted and received data symbols. To obtain the transmitted data symbols equation (5) should be solved which is called MIMO-OFDM equalization.

$$\vec{A}[n,m] = \bar{H}[n,m]^{-1}(\vec{R}[n,m] + \vec{W}[n,m]) \tag{8}$$

This equalization works well in case of small noise and no ISI or ICI. In the presence of ICI and ISI the received signal can be written as in [2]

$$\vec{R}_i[n,m] = \sum_{j=1}^{N} R_{j,i}^{U}[n,m] + \sum_{j=1}^{N} R_{j,i}^{ICI}[n,m] + \sum_{j=1}^{N} R_{j,i}^{ISI}[n,m] + W_i[n,m] \quad (9)$$

where $\sum_{j=1}^{N} R_{j,i}^{U}[n,m]$, $\sum_{j=1}^{N} R_{j,i}^{ICI}[n,m]$ and $\sum_{j=1}^{N} R_{j,i}^{ISI}[n,m]$ are the useful term. In order to be able to cancel the interference the ISI and ICI terms should be calculated and then subtracted from the received signal. One such interference cancellation scheme is presented in [2].

III. CAPACITY

In [3] the capacity of conventional MIMO, MIMO-OFDM and spread MIMO-OFDM in presence of multipath is studied. Spread MIMO-OFDM is MIMO with OFDM and CDMA i.e. above MIMO-OFDM a spreading code is used in the signal. In the single user case the results showed that capacity for the conventional MIMO without ISI is the highest and they state that it is the upper bound of capacity limit. MIMO-OFDM and spread MIMO-OFDM give more capacity than conventional MIMO in presence of multipath and based on their results MIMO-OFDM and spread MIMO-OFDM would be similarly impacted by multipath. This seems reasonable since OFDM with long enough cyclic prefix is a powerful mean to mitigate multipath.

In multiuser channel spread MIMO-OFDM provides more capacity than the other schemes. Figures 2-5 from [3] present the results of that paper.

IV. PAPR

A major problem of multicarrier systems is that they show great sensitivity to nonlinear distortions. In-band and out-of-band interferences caused by nonlinear distortions degrade BER performance of the system and give rise to interference to adjacent frequency bands, respectively. At the transmitter, the high power amplifier (PA) is the main source of nonlinear distortions. Due to the fact that amplifier nonlinearity is amplitude dependent, the amplitude fluctuations of the input signal are of a concern. The peak-to-average power ratio (PAPR), which is defined as the ratio of the peak power of the signal to its average power, is a measure of the amplitude fluctuations of the signal. Any multicarrier signal with a large number of subcarriers may have a high PAPR due to occasional constructive addition of subcarriers.

In OFDM, when the number of carriers is large, the central limit theorem holds and the time domain samples of the OFDM signal, sampled at Nyquist rate, are approximately zero-mean complex Gaussian random variables. Then the probability that the PAPR of the OFDM symbol exceeds a given threshold $PAPR_0$ can be expressed as

$$\Pr(PAPR > PAPR_0) = 1 - F(PAPR_0)^{N_c} = 1 - (1 - e^{-PAPR_0})^{N_c} \quad (10)$$

The problem of this PAPR approximation is that it is derived for the Nyquist rate sampled version of a continuous signal. The continuous signal may have higher amplitude peaks than our maximum sample would imply and this analysis underestimates the distribution of the PAPR. It can also be noted that the Gaussian distribution has infinite values but the largest amplitude value of an OFDM signal is only N times the average amplitude of the carriers thus the approximation does not hold very accurately on large amplitudes i.e. the shape of the PAPR distribution is does not follow Gaussian in the tails of the distribution. In Fig. 6 the Gaussian approximation is compared to a CCDF of a Nyquist rate sampled signal and to CCDF of an oversampled signal with oversampling factor 16.

A. *Example of PAPR reduction in MIMO-OFDM*

A number of techniques have been proposed to reduce PAPR and they can be divided in two kinds of approaches. In the first approach, PAPR reduction can be obtained with help of redundancy and the second one is to apply a correcting function to the signal to eliminate the high amplitude peaks. This is very simple approach but it causes interference. Adding redundancy does not cause any interference but it adds complexity of the transmitter and lowers the net transmission rate.

Selective mapping (SLM) belongs to the first approach. In SLM, V statistically independent sequences are generated from the same information by multiplying with a certain vector and that sequence with the lowest PAPR is selected. The information of the vector used to generate the selected sequence has to be sent to the receiver. Detection of the signal depends also on the errors on the side information transmission.

In MIMO-OFDM SLM can be applied to individual antennas in a way that every antenna selects

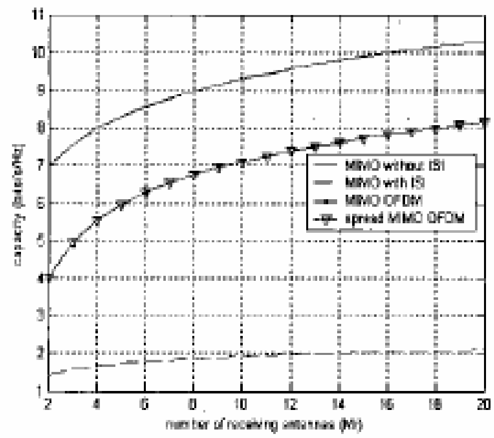

Fig. 2 Channel capacity versus number of receive antennas at Mt=2, SNR=15 dB, L=5

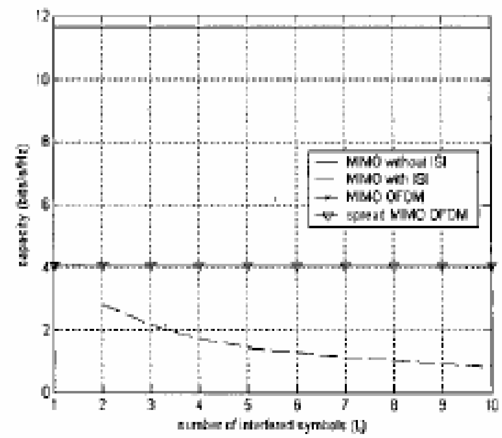

Fig.4 Channel capacity versus number of interfered symbols at Mt=Mr=10, SNR=15 dB

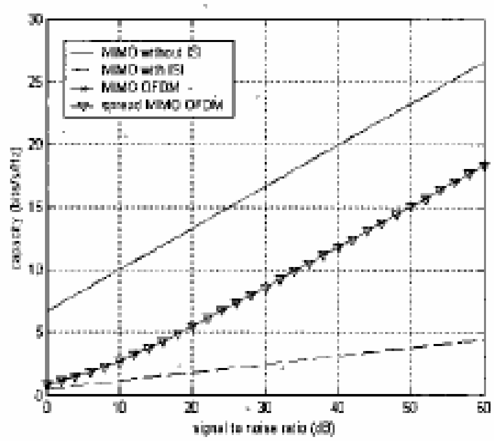

Fig. 3 Channel capacity versus signal to noise ratio at Mt=Mr=10, L=5

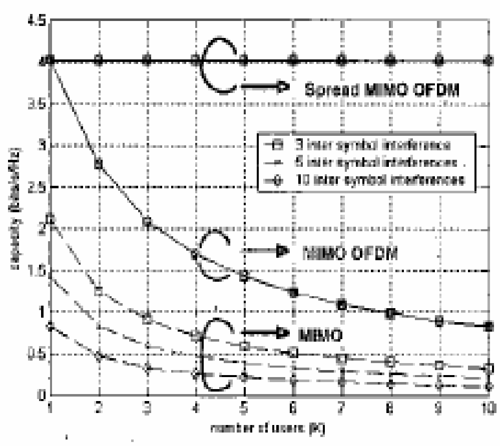

Fig.5 Average capacity rate per user versus number of users at Mt=Mr=10, SNR=15 dB

independently one of V sequences to be transmitted. In this way each antenna are sending different side information and the complementary cumulative distribution (CCDF) of the best sequence is

$$\Pr(PAPR > PAPR_0)^V = (1-(1-e^{-PAPR_0})^{N_c})^V \qquad (11)$$

In [4] a concurrent SLM approach is proposed. In this approach a common vector to all transmit antennas among the V vectors is selected. The selection is made based on the lowest average PAPR over the N transmit antennas. In this way the same information about the selected vector is sent over all transmit antennas and thus diversity gain is obtained and the errors are reduced. Correspondingly the amount of redundancy could be lowered. As the selection is made according to the average PAPR there will be a sligth degradation in PAPR performance compared to the individual SLM approach. The CCDF of the best sequence is

$$\Pr(PAPR > PAPR_0)^V = (1-(1-e^{-PAPR_0})^{N_c N})^V \qquad (12)$$

V. Spatial Diversity Coding for MIMO-OFDM

In MIMO systems the Alamouti scheme realizes full spatial diversity gain in the absence of channel knowledge at the transmitter. This requires that the channel remains constant over at least two consecutive symbol periods. In MIMO-OFDM the coding is performed in the frequency rather than in time. Symbols s_1 and s_2 are transmitted over antennas 1 and 2 on tone n and symbols $-s_2^*$ and s_1^* are transmitted over antennas 1 and 2 on tone n+1. At the receiver the symbols received from these two tones are detected using the Alamouti detection technique. Any pair of tones could be used as long as the associated channels are equal i.e. the channel requirement is different from the MIMO case.

An alternative technique is to use diversity coding on a per-tone basis across OFDM symbols in time but then the channel should be constant during two consecutive OFDM symbols. This is not usually true due to the long duration of OFDM symbols. [5]

VI. Space-Frequency coded MIMO-OFDM

The above mentioned spatial diversity coding realizes spatial diversity gain in MIMO-OFDM system. However, also frequency diversity is available in tones with spacing larger than the coherence bandwidth of the channel. The total diversity gain that can be realized in a MIMO-OFDM system has been shown in a reference of [5]. The total diversity gain equals to NMD, where D is the number of coherence bandwidths. [5]

Conclusion

MIMO and MIMO-OFDM are very hot topics of current research. The information-theoretic performance limits, particularly in the multiuser context and space time code and receiver design have attracted significant research interest.

References

[1] A. Paulraj, R. Nabar, D. Gore: Introduction to Space-Time Wireless Communications, Published May 2003, ISBN: 0521826152

[2] V.D.Nguyen, M.Pätzhold, "Frequency domain interference cancellation for MIMO-OFDM systems" Proc. 9th International OFDM-Workshop, Dresden, Germany, 15. – 16. Sept. 2004, p. 114–117.

[3] P. Uthansaku M.E. Bialkows "Multipath signal effect on the capacity of MIMO, MIMO-OFDM and spread MIMO-OFDM" Microwaves, Radar and Wireless Communications, 2004. MIKON-2004. 15th International Conference on , Volume: 3 , 17-19 May 2004 Pages:989 - 992

[4] Y-L. Lee, Y-H. You, W-G. Jeon, J-H. Paik, H-K. Song, " Peak-to-average power ratio in MIMO-OFDM systems using selective mapping" IEE Communications letters, Vol. 7, No. 12, December 2003

[5] A.J. Paulraj, D.A. Gore, R.U. Nabar, H. Bölcskei, "An overview of MIMO communications - a key to gigabit wireless" Proceedings of the IEEE ,Volume: 92 , Issue: 2 , Feb. 2004 Pages:198 – 218

VII. Homework

Comment on the advantages and disadvantages of combining MIMO with OFDM and CDMA. For example you can comment on the results of [3] or try to find another paper that compares these or some of these schemes.

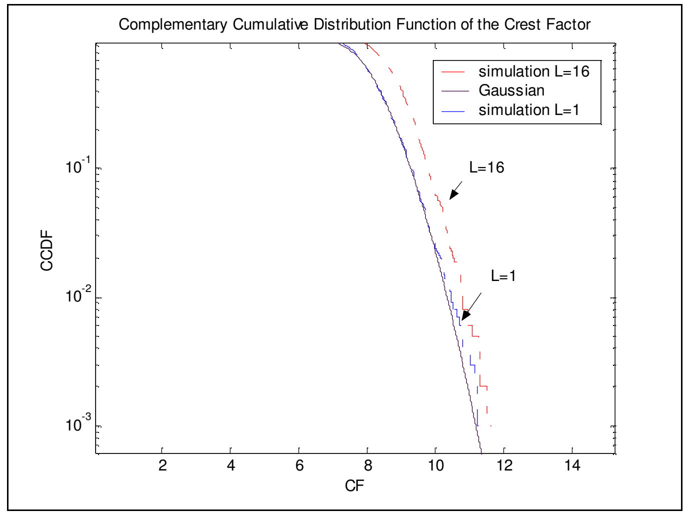

Figure 6 CCDF of oversampled and Nyquist rate sampled signals compared to Gaussian approximation, N=512, 16-QAM