Helsinki University of Technology
Communications laboratory

S-72.333 Postgraduate Course in Radio Communications

18.1.2005 MIMO principles

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Agenda

- 1. Introduction
- 2. Basic definitions and notation
- 3. Channel model
  - ”narrowband array” assumption
- 4. MIMO capacity
- 5. Performance improvment of MIMO system
  - Array gain, diversity, spatial multiplexing gain, interference cancellation
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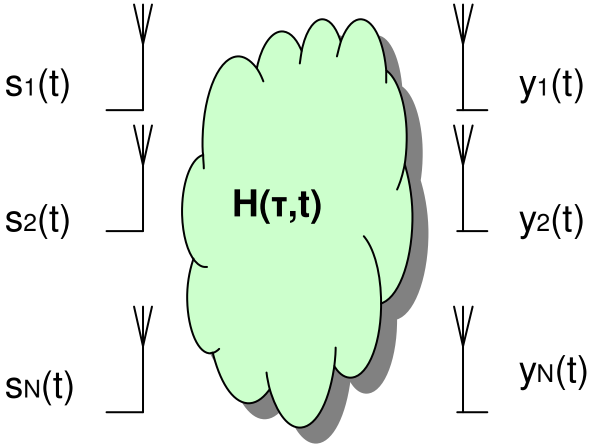
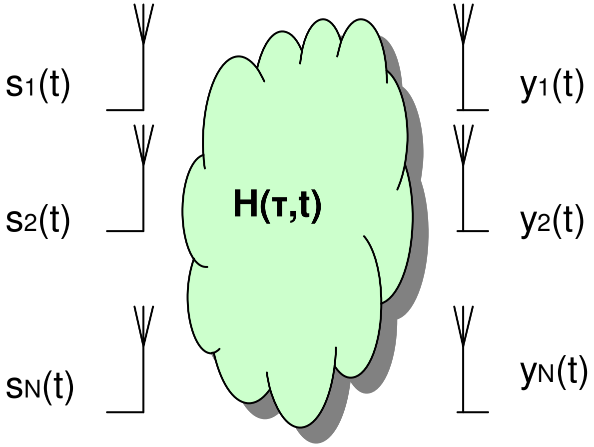
1. Introduction



- Fading, the well known pitfall of a wireless channel, can be combat by diversity
- MIMO exploits spatial diversity by having several transmit and receive antennas
- Actually, MIMO effectively takes advantage of random fading and when available, multipath delay spread
- The ability to turn multipath propagation into a benefit for the user is the key feature of MIMO systems.

2. Basic definitions and notation, MIMO

- N transmitter antennas
- M receiver antennas
- $s_j(t)$ signal transmitted from the jth antenna
- The signal $y_i(t)$ received at the ith receive antenna is given by

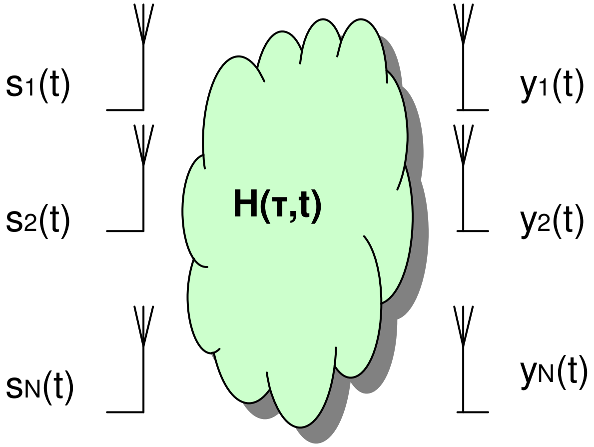


$$y_i(t) = \sum_{j=1}^{N} h_{i,j}(\tau,t) * s_j(t) + n_i(t), \qquad i = 1,2,\ldots,M$$

- where * denotes the convolution and $n_i(t)$ is the noise added at the receiver

2. Basic definitions and notation, MIMO

- The channel impulse response between the jth transmit and the ith receive antenna is denoted as $h_{i,j}(\tau,t)$
- Channel response $H(\tau,t)$ is the following MxN matrix

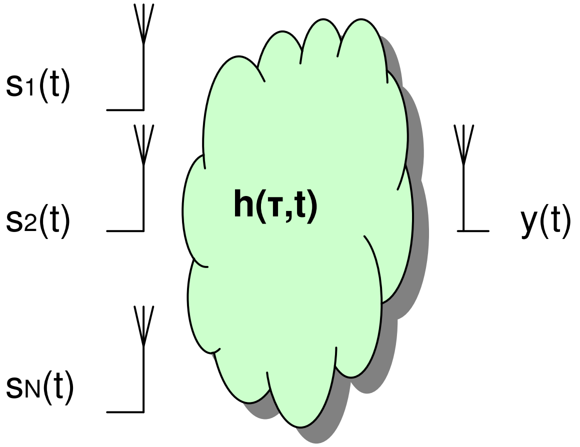


$$H(\tau,t)=\begin{bmatrix} h_{1,1}(\tau,t) & h_{1,2}(\tau,t) & \cdots & h_{1,N}(\tau,t) \\ h_{2,1}(\tau,t) & h_{2,2}(\tau,t) & \cdots & h_{2,N}(\tau,t) \\ \vdots & \vdots & \ddots & \vdots \\ h_{M,1}(\tau,t) & h_{M,2}(\tau,t) & \cdots & h_{M,N}(\tau,t) \end{bmatrix}$$

spatio-temporal signature induced by jth transmitter antenna

2. Basic definitions and notation, MISO

- N transmit antennas
- One receiver antenna
- $s_j(t)$ signal transmitted from the jth antenna



$$\bar{s}(t) = \begin{bmatrix} s_1(t) & s_2(t) & \cdots & s_N(t) \end{bmatrix}^T$$

- The received signal y(t) is given by

$$y(t) = \sum_{j=1}^{N} h_j(\tau, t) * s_j(t) + n(t)$$

- or

$$y(t) = \bar{h}(\tau, t) * \bar{s}(t) + n(t)$$

- The MISO channel impulse response is presented by a 1xN vector

$$\bar{h}(\tau, t) = \begin{bmatrix} h_1(\tau, t) & h_2(\tau, t) & \cdots & h_N(\tau, t) \end{bmatrix}$$

2. Basic definitions and notation, SIMO

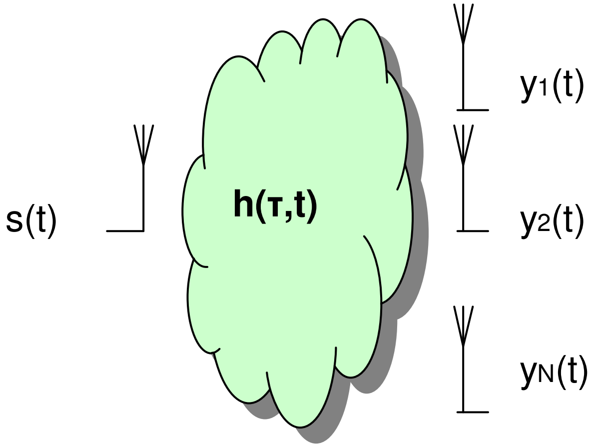
- One transmit antenna
- M receiver antennas
- The received signal vector ỹ(t)

$$\bar{y}(t) = \left[y_1(t) \quad y_2(t) \quad \cdots \quad y_M(t) \right]^T$$

- is given by

$$\bar{y}(t) = \bar{h}(\tau, t) * s(t) + n(t)$$

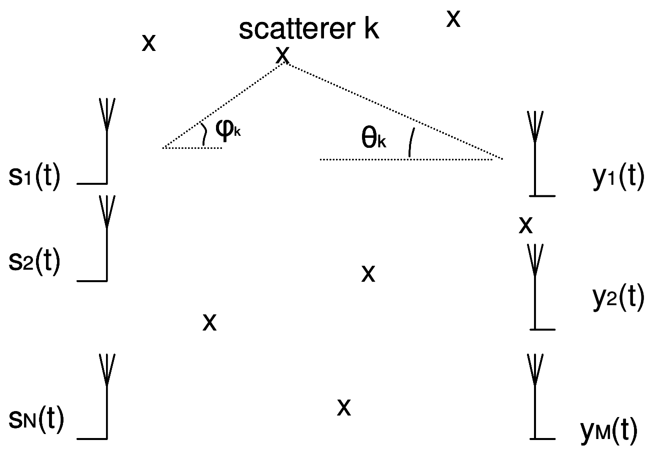
- where ĥ is the Mx1 SIMO channel vector that can be presented as



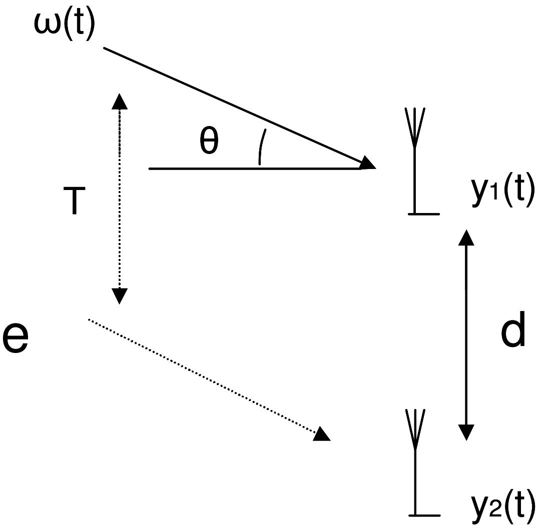
$$\bar{h}(\tau, t) = \left[h_1(\tau, t) \quad h_2(\tau, t) \quad \cdots \quad h_M(\tau, t) \right]^T$$

3. Channel model

- Suppress the time-varying nature of the channel
- Use the “narrowband array” assumption
- Assume a channel of finite number of iso-delay scatterers
- a scatterer k is located at angle φ_k, with respect to the transmitter and at angle θ_k and delay τ with respect to the receiver
- γ_k is the complex valued scatterer amplitude



3. ”Narrowband array” assumption



- The wavefront ω(t) impinging at angle θ has a bandwidth B

$$\omega(t) = \beta(t) e^{j 2\pi \upsilon_c t}$$

- Under the narrowband assumption we take the bandwidth B to much smaller than the reciprocal of the transit time of the wavefront across the antenna array, i.e. B≈1/T
- Now y2(t) can be represented with y1(t), since the signals are identical except for a phase shift

$$y_2(t) = y_1(t) e^{-j 2\pi \sin(\theta)(d/\lambda)}$$

3. Channel model

- With the preceding assumptions a steering vectors at receiver and transmitter arrays may be defined, respectively as

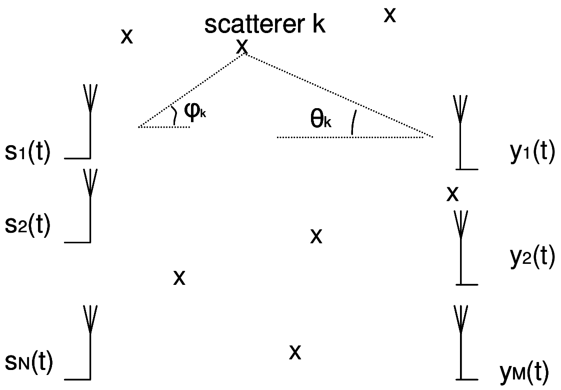
$$\overline{a}(\theta_k) = \left[1 \quad e^{-j\pi\sin(\theta_k)} \quad \cdots \quad e^{-j\pi(N-1)\sin(\theta_k)}\right]^T \qquad \overline{b}(\varphi_k) = \left[1 \quad e^{-j\pi\sin(\varphi_k)} \quad \cdots \quad e^{-j\pi(N-1)\sin(\varphi_k)}\right]^T$$

- The antenna separation d is assumed to be λ/2
- The received contribution from the kth scatterer is a vector

$$\overline{y}^{(k)}(t) = \left[y_1^{(k)}(t) \quad y_2^{(k)}(t) \quad \cdots \quad y_M^{(k)}(t)\right]^T$$

which is obtained as follows

$$\overline{y}^{(k)} = \gamma_k \overline{a}(\theta_k)\overline{b}(\varphi_k)^T \overline{s}$$



3. Channel model

- The received signal is the sum of all contributions from all K scatterers

$$\bar{y} = \sum_{k}^{K} \bar{y}^{(k)} = \left(\sum_{k}^{K} \gamma_k \bar{a}(\theta_k) \bar{b}(\varphi_k)^T \right) \bar{s} = H\bar{s}$$

- Rank of matrix H tells the number of independent contribution at the receiver side
- H is of size NxM i.e. it has rank smaller or equal to min(N,M).
- Rank of H depends also on the number of independent scatteres since it is built as a sum of rank one matrices
- As a conclusion the rank of the channel matrix is smaller or equal to min(N,K,M).

Capacity

- The ergodic capacity of the MIMO channel is

$$C = \max_{Tr(R_{ss})=N} E_H\left[\log_2\left[\det\left(I_M + \frac{1}{N}\frac{E_s}{N_0}HR_{ss}H^H\right)\right]\right]$$

- where Es is the total average energy over a symbol period and $R_{ss} = E\left[ss^H\right]$ is the covariance matrix of s
- If the channel is unknown to the transmitter, the signals are chosen to be independent and equi-powered at the transmit antennas i.e. $R_{ss} = I_N$ and then the capacity is

$$C = E_H\left[\log_2\left[\det\left(I_M + \frac{1}{N}\frac{E_s}{N_0}HH^H\right)\right]\right]$$

5. Array gain

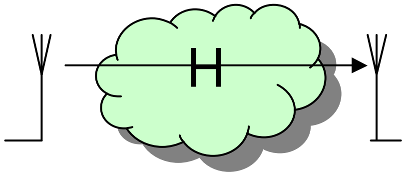
- The average increase in the signal-to-noise ratio (SNR) at the receiver when the received signal have been coherently combined
- The transmit/receive array gain depends on N and M and requires channel knowledge in the transmitter and receiver, respectively
- For example in MISO channel if all channel responses are 1 the received signal is y(t)=N*s(t)/sqrt(N)+n(t) and the signal-to-noise ratio is

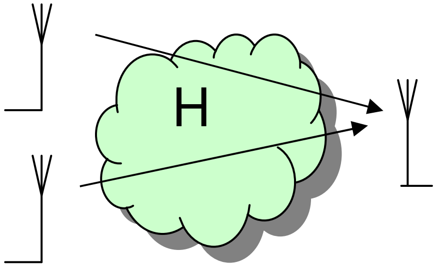
$$SNR_{MISO} = \left(\frac{N}{\sqrt{N}}\right)^2 \frac{E\left[\left\|s(t)\right\|^2\right]}{E\left[\left\|n(t)\right\|^2\right]} = N * SNR_{SISO}$$

- i.e. the array gain in this case is N

5. Diversity

- Assume several uncorrelated channels
- The uncorrelated channels are not likely to have deep fades simultaneously
- For example consider a SIMO channel and assume that the antennas are separated by the coherence distance (assume also independent scattering)
- If the probability that the signal level goes below a threshold is p in a SISO channel, the probability that the signal level in every channels goes below the same threshold in MISO channel is p^M
- Diversity in this case is M



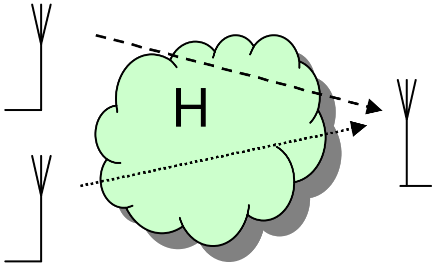


5. Spatial multiplexing gain

- Spatial multiplexing gain r is the increase of capacity as can be described with the following formula

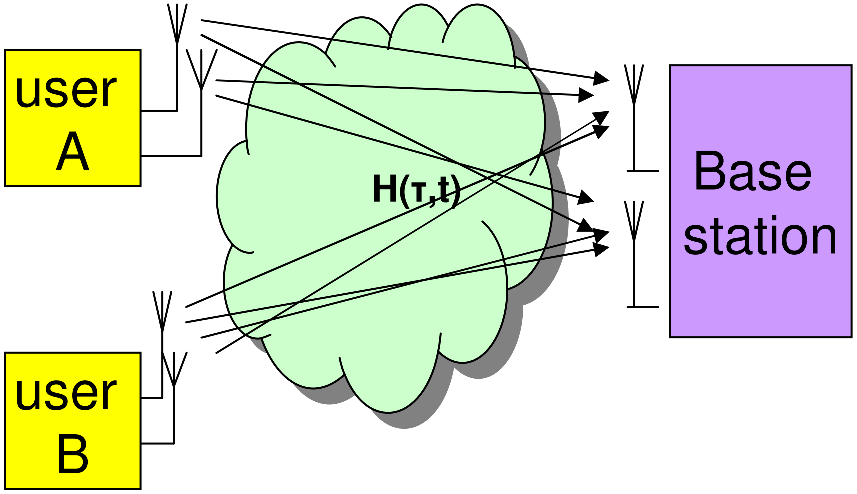
$$C = r \log_2(SNR)$$

- The gain is achieved when more than one independent symbols can be transmitted during the same symbol duration
- In other words the achieved spatial multiplexing gain is dependent on the number of independent data streams that can be supported reliably i.e. the rank of H



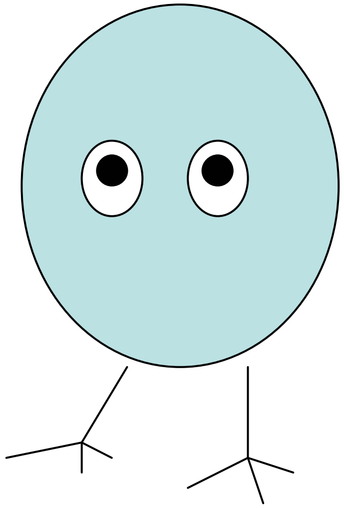
5. Interference cancellation

- The difference between spatial signatures of the desired signal and the cochannel signal can be exploited in to reduce cochannel interference that arises due to frequency reuse
- Obviously, the knowledge of the channel is required
- Efficient cochannel interference cancellation gives prerequisites for aggressive frequency reuse which leads to increase in multicell capacity



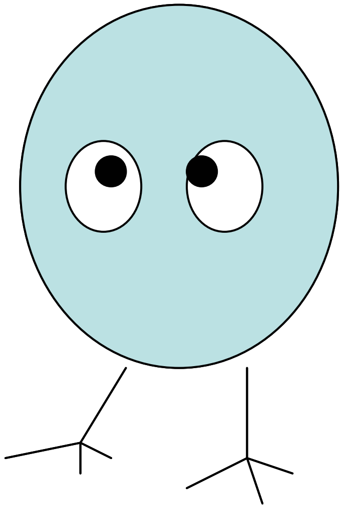
6. Conclusions and remarks

- It has been seen that MIMO is a very promising transmission technique
- Channel considered was a simplified version of the real communications channel
- The performance improvements mentioned can not be all exploited simultaneously due to different requirements



Homework

- We assumed a very simplistic channel. What is your opinion, how would MIMO be working in a real wireless communications channel? Could MIMO be a practical application or only a nice research topic? Give your opinion with reasoning.



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[2] D. Gesbert, M. Shafi, S. Da-shan, P.J. Smith, A. Naguib, **“From theory to practice: an overview of MIMO space-time coded wireless systems”,** Selected Areas in Communications, IEEE Journal on ,Volume: 21, Issue: 3 , April 2003 Pages:281 – 302

[3] A.J. Paulraj, D.A. Gore, R.U. Nabar, H. Bölcskei, **“An overview of MIMO communications - a key to gigabit wireless”** Proceedings of the IEEE ,Volume: 92 , Issue: 2 , Feb. 2004 Pages:198 – 218