MIMO principles

Helka Määttänen

I. INTRODUCTION

The growing demand of multimedia services and the growth of Internet related contents lead to increasing interest to high speed communications. The requirement for wide bandwidth and flexibility imposes the use of efficient transmission methods that would fit to the characteristics of wideband channels especially in wireless environment where the channel is very challenging. In wireless environment the signal is propagating from the transmitter to the receiver along number of different paths, collectively referred as multipath. While propagating the signal power drops of due to three effects: path loss, macroscopic fading and microscopic fading. Fading of the signal can be mitigated by different diversity techniques. To obtain diversity, the signal is transmitted through multiple (ideally) independent fading paths e.g. in time, frequency or space and combined constructively at the receiver. Multiple-input-multiple-output (MIMO) exploits spatial diversity by having several transmit and receive antennas as depicted in Fig. 1. For example in receive antenna diversity, in rich scattering environment, each receive antenna sees different versions of the transmitted signal and when these versions are combined in a proper manner the outcome has better quality (lower bit-error-rate (BER)) or higher data rate than a single version of the signal. Specifically, if the number of multipath components exceeds a certain value the channel capacity increase can be proportional to the number of transmit and receive antennas and no additional power or bandwidth is required.

MIMO effectively takes advantage of random fading and when available, multipath delay spread. Actually, the ability to turn multipath propagation, which conventionally is considered as a drawback of wireless transmission, into a benefit for the user is the key feature of MIMO systems.

MIMO has emerged as one of the most significant technical breakthroughs in modern communications. This paper provides a general overview of this promising transmission technique.

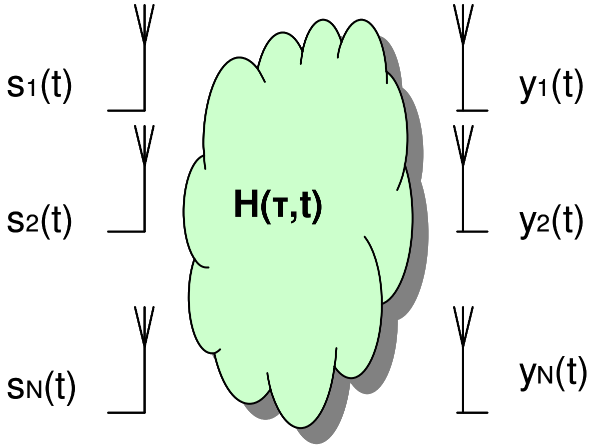

Figure 1 MIMO system

II. BASIC DEFINITIONS

A. MIMO

Consider a MIMO channel with N transmit antennas and M receive antennas as depicted in Fig. 1. The signal transmitted from the jth transmit antenna is $s_j(t)$. The time-varying channel impulse response between the jth transmit antenna and the ith receive antenna is denoted as $h_{i,j}(\tau,t)$. The MIMO channel response can be expressed as an NxM matrix:

$$H(\tau,t)=\begin{bmatrix} h_{1,1}(\tau,t) & h_{1,2}(\tau,t) & \cdots & h_{1,N}(\tau,t) \\ h_{2,1}(\tau,t) & h_{2,2}(\tau,t) & \cdots & h_{2,N}(\tau,t) \\ \vdots & \vdots & \ddots & \vdots \\ h_{M,1}(\tau,t) & h_{M,2}(\tau,t) & \cdots & h_{M,N}(\tau,t) \end{bmatrix} \tag{1}$$

The signal received at the ith receive antenna is given by

$$y_i(t)=\sum_{j=1}^{N} h_{i,j}(\tau,t) * s_j(t) + n_i(t), \qquad i=1,2,\ldots,M \tag{2}$$

where * denotes the convolution and $n_i(t)$ is the noise added in the receiver. The spatio-temporal signature induced by the jth transmit antenna across the receive antenna array is given by the vector $\lfloor h_{1,j}(\tau,t) \quad h_{2,j}(\tau,t) \quad \cdots \quad h_{M,j}(\tau,t) \rfloor$.

B. MISO

If the number of receive antennas is reduced to 1, a multiple-input-single-output (MISO) system is attained. MISO channel can be decomposed into N single-input-single-output (SISO) channels. The MISO channel impulse response between the N transmit antennas and the receive antenna can presented by a 1xN vector

$$\bar{h}(\tau,t) = [h_1(\tau,t) \quad h_2(\tau,t) \quad \cdots \quad h_N(\tau,t)]. \qquad (3)$$

The received signal can be represented as

$$y(t) = \sum_{j=1}^{N} h_j(\tau,t) * s_j(t) + n(t) \qquad (4)$$

or in vector notation as

$$y(t) = \bar{h}(\tau,t) * \bar{s}(t) + n(t) \qquad (5)$$

where the input signal vector is a Nx1 vector $\bar{s}(t) = [s_1(t) \quad s_2(t) \quad \cdots \quad s_N(t)]^T$

C. SIMO

If the number of transmit antennas is 1 and the number of receive antennas is M the system is called single-input-multiple-output (SIMO) system. It comprises N SISO channels like the MISO case. The SIMO channel can be represented as an Mx1 vector

$$\bar{h}(\tau,t) = [h_1(\tau,t) \quad h_2(\tau,t) \quad \cdots \quad h_M(\tau,t)]^T \qquad (6)$$

The signal received at ith receive antenna is given by

$$y_i(t) = h_i(\tau,t) * s(t) + n_i(t) \qquad (7)$$

or in a vector form, $\bar{y}(t) = [y_1(t) \quad y_2(t) \quad \cdots \quad y_M(t)]^T$, by

$$\bar{y}(t) = \bar{h}(\tau,t) * s(t) + n(t) \qquad (8)$$

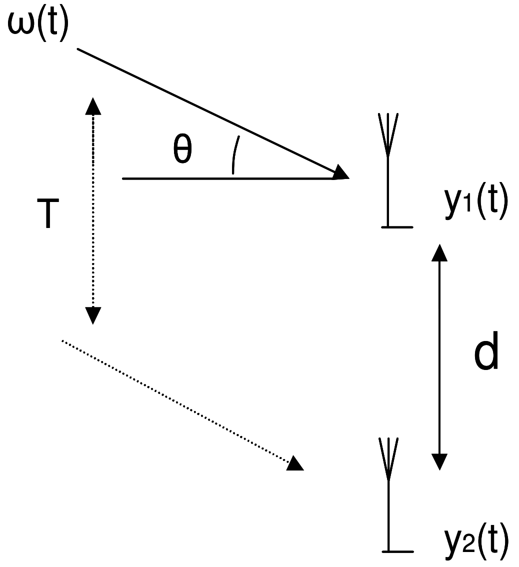

Figure 2 Schematic of wavefront impinging on an antenna array

III. CHANNEL MODEL

Equations (1)-(8) incorporate time variation nature of the wireless environment but for convenience we now suppress the time-varying nature of the channel. Also a "narrowband array" assumption described briefly below is made when deriving a MIMO wireless channel model from a simplistic physical scattering description.

Fig. 2 shows the schematic of wavefront impinging on an antenna array. The impinging wavefront having a bandwidth B is denoted as ω(t), the signal received by the first and second antennas is denoted by $y_1(t)$ and $y_2(t)$ and the antenna separation is d. ω(t) is impinging at angle θ is represented as

$$\omega(t) = \beta(t) e^{j2\pi \upsilon_c t} \qquad (9)$$

where β is the complex envelope representation of the signal and υ_c is the carrier frequency. Under the narrowband assumption we take the bandwidth B to much smaller than the reciprocal of the transit time of the wavefront across the antenna array, i.e. B≈1/T. Now $y_2(t)$ can be represented with $y_1(t)$, since the signals are identical except for a phase shift that depends on the array geometry and the angle of arrival of the wavefront.

$$y_2(t) = y_1(t) e^{-j2\pi \sin(\theta)(d/\lambda)} \qquad (10)$$

where λ is the wavelength of the signal wavefront. Note that the narrowband assumption does not imply frequency flat channel.

Fig. 3 shows the MIMO schematic with a channel of finite number of iso-delay scatterers. Under this assumption the signal scattered from different scatterers arrives at the receiver at the same time i.e. the multipath characteristics of the channel are not modeled. In Fig. 3 a scatterer k is located at angle $\varphi_{k,}$ with respect to the transmitter and at angle θ_k and delay τ with respect to the receiver. Any two of these variables define the third one by virtue of the geometries in Fig.3.

With these assumption a steering vectors at receiver and transmitter arrays may be defined, respectively as

$$\bar{a}(\theta_k) = \begin{bmatrix}1 & e^{-j\pi\sin(\theta_k)} & \cdots & e^{-j\pi(N-1)\sin(\theta_k)}\end{bmatrix}^T \tag{11}$$

$$\bar{b}(\varphi_k) = \begin{bmatrix}1 & e^{-j\pi\sin(\varphi_k)} & \cdots & e^{-j\pi(N-1)\sin(\varphi_k)}\end{bmatrix}^T \tag{12}$$

The antenna separation d is assumed to be $\lambda/2$. The transmitted signal vectors can be denoted as $\bar{s}(t) = \begin{bmatrix}s_1(t) & s_2(t) & \cdots & s_N(t)\end{bmatrix}^T$ and the received contribution from the kth scatterer as $\bar{y}^{(k)}(t) = \begin{bmatrix}y_1^{(k)}(t) & y_2^{(k)}(t) & \cdots & y_M^{(k)}(t)\end{bmatrix}^T$, which can be expressed as

$$\bar{y}^{(k)} = \gamma_k \bar{a}(\theta_k)\bar{b}(\varphi_k)^T \bar{s} \tag{13}$$

where γ_k is the complex valued scatterer amplitude. The received signal is the sum of all contributions from all K scatterers

$$\bar{y} = \sum_k^K \bar{y}^{(k)} = \left(\sum_k^K \gamma_k \bar{a}(\theta_k)\bar{b}(\varphi_k)^T\right)\bar{s} = H\bar{s} \tag{14}$$

Rank of matrix H tells the number of independent contribution at the receiver side. H is of size NxM i.e. it has rank smaller or equal to min(N,M). Rank of H depends also on the number of independent scatteres since it is built as a sum of rank one matrices (in case of only one scatterer the rank of H is obviously 1). As a conclusion the rank of the channel matrix is smaller or equal to min(N,K,M).

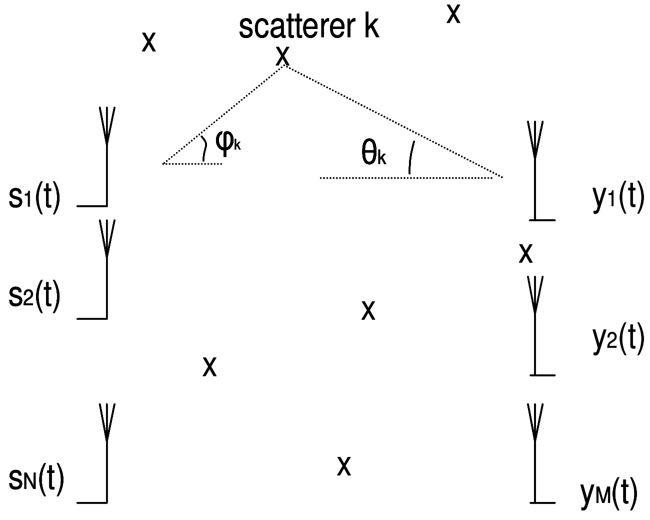

Figure 3 MIMO schematic with a channel of finite number of iso-delay scatterers

IV. MIMO CAPACITY

MIMO channels exhibit fading and encompass spatial dimensions. The channel H (1) is random and therefore the capacity is also a random variable. The ergodic capacity of the MIMO channel is

$$C = \max_{Tr(R_{ss})=N} E_H\left[\log_2\left[\det\left(I_M + \frac{1}{N}\frac{E_s}{N_0} H R_{ss} H^H\right)\right]\right] \tag{15}$$

where E_s is the total available average energy over a symbol period available at the transmitter and $R_{ss}=E[ss^H]$ is the covariance matrix of s. s is assumed to have zero mean and the trace of R_{ss} must be N in order to constrain the total average energy transmitted over a symbol period. Channel knowledge at the receiver is assumed.

Acquiring channel knowledge at the transmitter is in general very difficult in practical systems. If the channel is unknown to the transmitter and the channel has no preferred direction s may be chosen to be statistically non-preferential which implies that the signals are independent and equi-powered at the transmit antennas. I.e. $Rss=I_N$ and the capacity in case of no channel knowledge at the transmitter reads as

$$C = E_H\left[\log_2\left[\det\left(I_M + \frac{1}{N}\frac{E_s}{N_0} HH^H\right)\right]\right] \tag{16}$$

V. PERFORMANCE IMPROVEMENT OF MIMO SYSTEM

Array gain, diversity gain, spatial multiplexing gain and interference reduction are the key features that constitute the performance improvements of MIMO system.

A. Array gain

Array gain refers to the average increase in the signal-to-noise ratio (SNR) at the receiver when the received signals have been coherently combined. The transmit/receive array gain requires channel knowledge in the transmitter and receiver, respectively, and depends on N and M. For example, consider a MISO channel with N transmit antennas. Total transmit power is $E[|s(t)|^2]=P$ i.e. s(t)/sqrt(N) is transmitted from each antenna. Assume the noise power to be N_0 and all channel responses to be 1 (non-physical channel model). The received signal is (see eq. (4)) y(t)=N*s(t)/sqrt(N)+n(t). The received useful signal power is $E[|y(t)-n(t)|^2]=N*P$ i.e. the SNR is $N*P/N_0$. This is N times the SNR of a SISO link hence the array gain in this case is N. If one of the channel responses is for example -1, the array gain depends on whether the transmitter knows that or not. If the transmitter has channels state information (CSI) it will send –s(t)/sqrt(N) from the corresponding antenna in order to exploit the same array gain. [4]

B. Diversity gain

As mentioned in the introduction diversity is a powerful way to mitigate fading in wireless links. The idea of diversity is based on the fact that if we have several uncorrelated channels they are not likely to have deep fades simultaneously and thus when receiving the same information through all these uncorrelated channels the probability that the signal experiences a deep fade is smaller than in single channel case. MIMO exploits spatial diversity that does not sacrifice time or frequency unlike time and frequency diversities. Only complexity is added. The diversity order is the number of uncorrelated channels in the system and it depends on the available channel and the antenna configuration. For example consider a SIMO channel and assume that the antennas are separated by the coherence distance i.e. the channels are independent (assume also independent scattering). In SISO channel the probability that the signal level goes below a threshold is p. In MISO channel the probability that the signal level in every channels goes below the same threshold is p^M. Hence, in this example, a diversity order M is achieved compared to the SISO case. If the channels are correlated the diversity order decreases. [4]

The preceding example had only one transmit antenna. In order to capitalize of several transmit antennas without channel knowledge at the transmitter the signal must be pre-processed or pre-coded before the transmission. In case the channel is known to the transmitter full diversity can be exploited, for example in MISO case, by weighting the signal appropriately, so that the signals arrive in phase at the receive antenna and add coherently. In MIMO case spatial diversity can be extracted through a technique known as dominant eigenmode transmission. As with MISO case the same signal is transmitted from all antennas in the transmit array with an appropriate weight vector.

C. Spatial multiplexing gain

Spatial multiplexing gain is the increase of capacity. It can be described with the following formula, where r is the spatial multiplexing gain

$$C = r\log_2(SNR)\,. \tag{17}$$

The gain is achieved when more than one independent symbols can be transmitted during the same symbol duration. In other words the achieved spatial multiplexing gain is dependent on the number of independent data streams that can be supported reliably i.e. the rank of H.

D. Interference reduction

As mentioned in Sec. II the signal send from antenna j has a spatial signature that can be used in recognizing the signal. Obviously, the knowledge of the desired signal’s channel is required. The difference between spatial signatures of the desired signal and the cochannel signal can be exploited in to reduce cochannel interference that arises due to frequency reuse. Efficient cochannel interference cancellation gives prerequisites for aggressive frequency reuse.

VI. CONCLUSION

It has been seen that MIMO is a very promising transmission technique but it should be noted that not all features mentioned in Sec. IV can not be exploited simultaneously due to different demands on the spatial degrees of freedom (or number of antennas).

REFERENCES

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VII. HOMEWORK

We assumed a very simplistic channel. What is your opinion, how would MIMO be working in a real wireless communications channel? Could MIMO be a practical application or only a nice research topic? Give your opinion with reasoning.