S-72.333 Postgraduate Course in Radio Communications

9.11.2004 Multipath propagation

Aki Silvennoinen
aki.silvennoinen@hut.fi

Agenda

- What is multipath propagation?
- Multipath propagation modeling
  - System functions of a linear time-variant channel
  - Practical LTV-channels: WSSUS
- Multipath channel characteristics
- Tapped delay line channel model

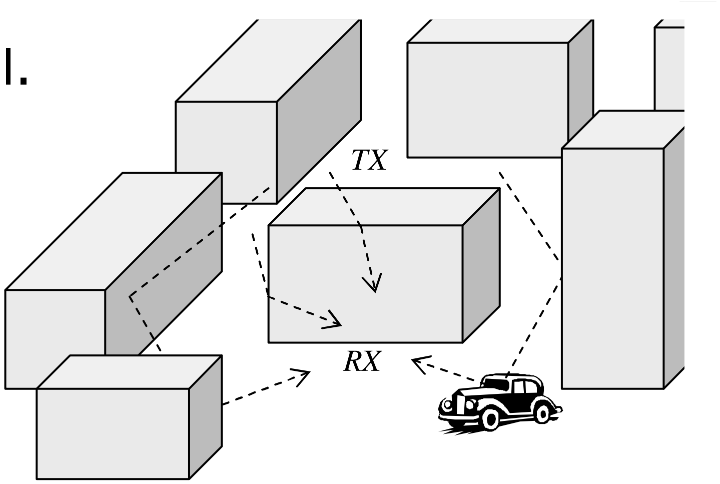
Multipath propagation

Multipath propagation is caused by multiple receptions of the same signal.

In city environment or indoors signal travels along different path from TX to RX

-signal components received at slightly different times (*delay*)

-these components are combined at RX



➔results as a signal that varies widely in *amplitude*, *phase* or *polarisation*

➔MULTIPATH FADING:
- -when the components add destructively due to phase differences
  - -amplitude of the received signal is very small
- -at other times the components add constructively
  - -large amplitude of the received signal
- -amplitude variations in the received signal, called signal fading, are due to the time-variant characteristics of the channel

Multipath propagation

-Relative motion between TX and RX (or surrounding objects causing e.g. reflection) causes random frequency modulation

-each multipath component has a different Doppler shift

- The Doppler shift can be calculated by using: $f_d = \frac{V}{\lambda}\cos\alpha$

V is the velocity of the terminal
α is the spatial angle between the direction of motion and the direction of the wave
λ is the wavelength

The three most important effects of multipath fading and moving scatters are:

-rapid changes in signal strength over a small travelled distance or time interval
-random frequency modulation due to varying Doppler shifts on different multipath signals
-time dispersion (echoes) caused by multipath propagation delays.

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Multipath propagation modeling

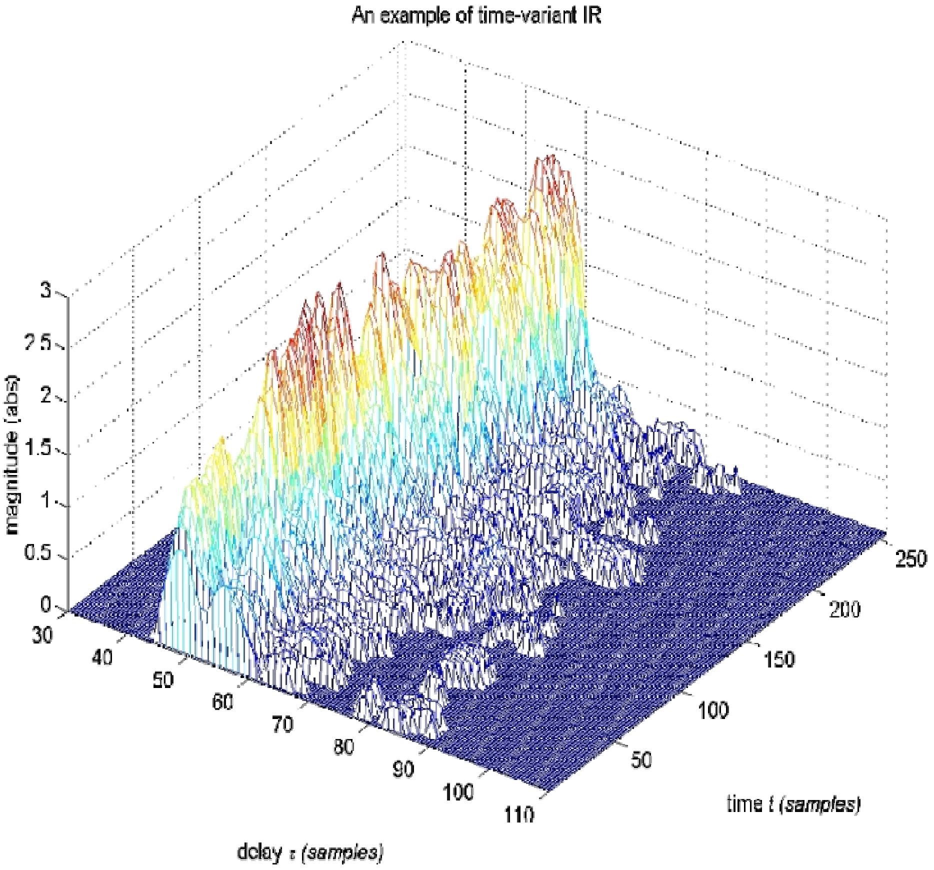
- Radio propagation channel can be visualised by a *system element that transforms the input signal into an output signal.*
- It is similar to a *linear filter with the extension that the radio propagation channel is **time-variant***.
- The radio channel can be modelled as a linear time-variant (LTV) channel that can be characterised by four functions
  - the time-variant impulse response (also known as the channel delay spread function)
  - the time-variant transfer function
  - the channel output Doppler-spread function
  - the delay/Doppler-spread function.

System functions of the LTV-channel

- *The time-variant impulse response*

$$h(\tau,t)=\sum_{n=1}^{N}\alpha_n(t)e^{-j2\pi f_c\tau_n(t)}\delta(\tau-\tau_n(t))$$

  - $\tau_n(t)$ is the propagation delay of the nth propagation path as function of time
  - $\alpha_n(t)$ is the amplitude of the nth propagation path as function of time
  - f_c is the carrier frequency



System functions of the LTV-channel

- If an unmodulated carrier at frequency f_c is transmitted using the above mentioned channel, the received signal is in the form of

$$r(t) = \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} = \sum_n \alpha_n(t) e^{-j\theta_n(t)}$$

- received signal consists of a sum of time-variant phasors having amplitudes $\alpha_n(t)$ and phases $\theta_n(t)$.
- For large dynamic changes in the channel, $\alpha_n(t)$ are required to change sufficiently to cause significant changes in the received signal.
- However, $\theta_n(t)$ will change by 2π whenever $\tau_n(t)$ changes by $1/f_c$ which is a relatively small number
  - hence, $\theta_n(t)$ can change by 2π with relative small motions in the medium.
  - ➔ The changes in $\theta_n(t)$ results as channel fading as explained earlier

System functions of the LTV-channel

- Delays $\tau_n(t)$ change:
  - at different rates
  - in an unpredictable way
  - ➔ Received signal can be modelled as a random process

- Large number of independent random components
  - ➔ the central limit theorem can be applied:

Given n independent random variables x_i, their sum is formed: $x = x_1 + x_2 + ... + x_n$. This is a random variable with mean $\eta = \eta_1 + \eta_2 + ... + \eta_n$ and variance $\sigma^2 = \sigma_1^2 + \sigma_2^2 + ... + \sigma_n^2$. The central limit theorem states that under certain conditions, the distribution $F(x)$ of x approaches a normal distribution with the same mean and variance.

System functions of the LTV-channel

- ➔ Received signal may be modelled as a complex-valued Gaussian process
  - ➔ This means that the time-variant impulse response $h(\tau,t)$ is a complex-valued Gaussian random process in the variable t
- \- When the envelope of the impulse response is modelled as a zero-mean complex valued Gaussian process, it can be shown that the envelope $|h(\tau,t)|$ at any instant t is Rayleigh-distributed.
    - ➔ channel is said to be *Rayleigh fading*
- \- In the case that there are fixed scatterers in addition to randomly moving scatterers in the channel
  - \- Impulse response can no longer be modelled as having zero-mean
    - ➔ the envelope has a Rice distribution and the channel is said to be *Rice fading*

System functions of the LTV-channel

- *Time-variant transfer function*
  - Fourier-transform the impulse response with respect to the delay variable

$$H\left(f,t\right)=F_{\tau}\left\{h\left(\tau,t\right)\right\}=\int_{-\infty}^{\infty}h\left(\tau,t\right)e^{-j2\pi f\tau}d\tau$$

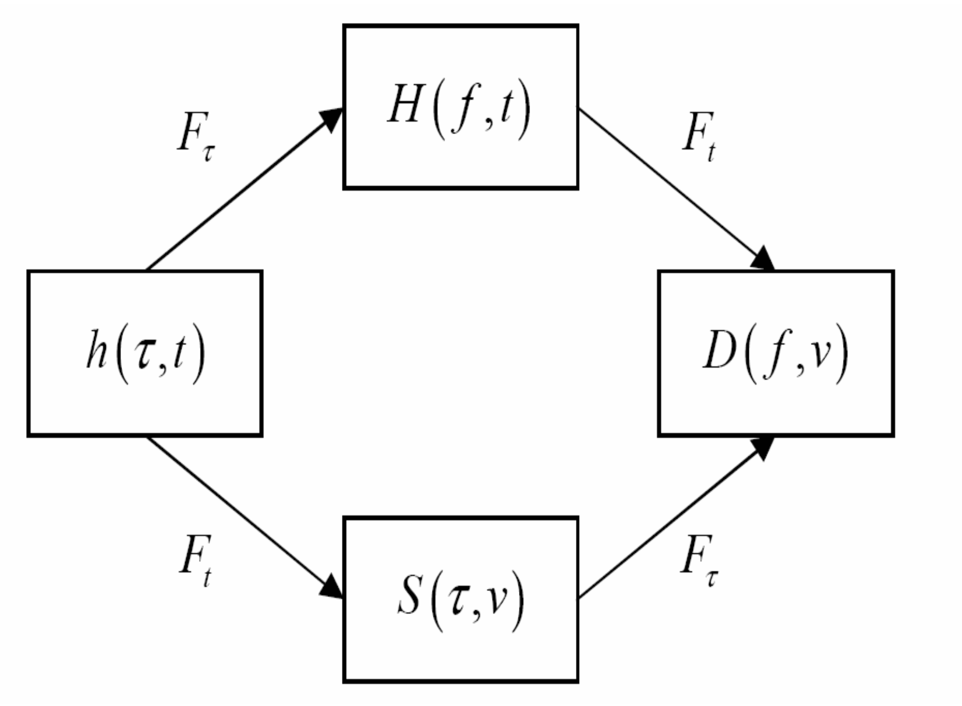
- *The output Doppler-spread function*
  - Fourier-transforming the time-variant transfer function with respect to the time variable

$$D\left(f,v\right)=F_{t}\left\{H\left(f,t\right)\right\}=\int_{-\infty}^{\infty}H\left(f,t\right)e^{-j2\pi vt}dt$$

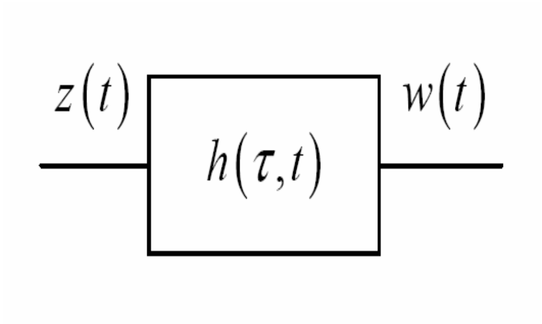
- *The delay/Doppler-spread function*
  - Fourier-transforming the time-variant impulse response with respect to the time variable

$$S\left(\tau,v\right)=F_{t}\left\{h\left(\tau,t\right)\right\}=\int_{-\infty}^{\infty}h\left(\tau,t\right)e^{-j2\pi vt}dt$$

Relationship of the system functions



$$\begin{tikzcd}
 & H(f,t) \arrow[dr, "F_t"] & \\
h(\tau,t) \arrow[ur, "F_\tau"] \arrow[dr, "F_t"'] & & D(f,v) \\
 & S(\tau,v) \arrow[ur, "F_\tau"'] &
\end{tikzcd}$$



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- Tapped delay line channel model

Practical LTV-channels: WSSUS channels (1)

- Wide-sense stationary uncorrelated scattering (WSSUS) channels are an important class of practical channels
  - System functions can be described using 4-dimensional autocorrelation functions
    - However, use of such 4-dimensional autocorrelation functions is rather unpractical

    ➔ two assumptions are made in order to get 2-dimensional autocorrelation functions

1) The LTV-channel under consideration is wide-sense stationary

   - e.g. autocorrelation functions depends only on the time difference between two observation instants

2) The LTV-channel under consideration is a multipath channel where the propagation paths are statistically independent or at least uncorrelated.

Practical LTV-channels: WSSUS channels (2)

- This LTV-channel is called a WSSUS-channel
- The autocorrelation functions for a WSSUS-channel can be presented as $P_h(\tau,\Delta t), P_H(\Delta f,\Delta t), P_D(\Delta f, v)$ and $P_S(\tau, v)$ similarly as the previous system functions

- By putting one of the variables to zero, the following one-dimensional functions are obtained
  - The multipath intensity profile, also called power delay profile $P_h(\tau,0)$
  - The frequency correlation function $P_H(\Delta f,0)$
  - The time correlation function $P_H(0,\Delta t)$
  - The Doppler power spectrum $P_D(0,v)$

Practical LTV-channels: WSSUS channels (3)

- Through these functions the characteristic parameters of the WSSUS channel, i.e. delay spread T_m, coherence bandwidth B_m, coherence time T_D, and Doppler-spread B_D, can be obtained.

$$\begin{tikzcd}
 & & \begin{array}{l} P_H(\Delta f,0)\Rightarrow B_m \ \text{coherence bandwidth} \\ P_H(0,\Delta t)\Rightarrow T_D \ \text{coherence time} \end{array} & & \\
 & & \boxed{P_H(\Delta f,\Delta t)} \arrow[u, Rightarrow] \arrow[dr, "F_t"] & & \\
\begin{array}{c} P_h(\tau,0)\Rightarrow T_m \\ \text{delay spread} \end{array} & \boxed{P_h(\tau,\Delta t)} \arrow[l, Rightarrow] \arrow[ur, "F_\tau"] \arrow[dr, "F_t"'] & & \boxed{P_D(\Delta f,\nu)} \arrow[r, Rightarrow] & \begin{array}{c} P_D(0,\nu)\Rightarrow B_D \\ \text{Doppler-spread} \end{array} \\
 & & \boxed{P_S(\tau,\nu)} \arrow[ur, "F_\tau"'] & &
\end{tikzcd}$$

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Multipath channel characteristics (1)

Multipath channel is usually characterised by following parameters

-Delay spread

  -quantifies the time dispersive properties of the channel

-Doppler spread

  -measure of the spectral broadening caused by the time rate of change of the mobile radio channel

-Coherence time

  -the channel affects differently to two signals arriving with a time separation greater than coherence time

$$T_D \approx \frac{1}{B_D}$$

-Coherence bandwidth

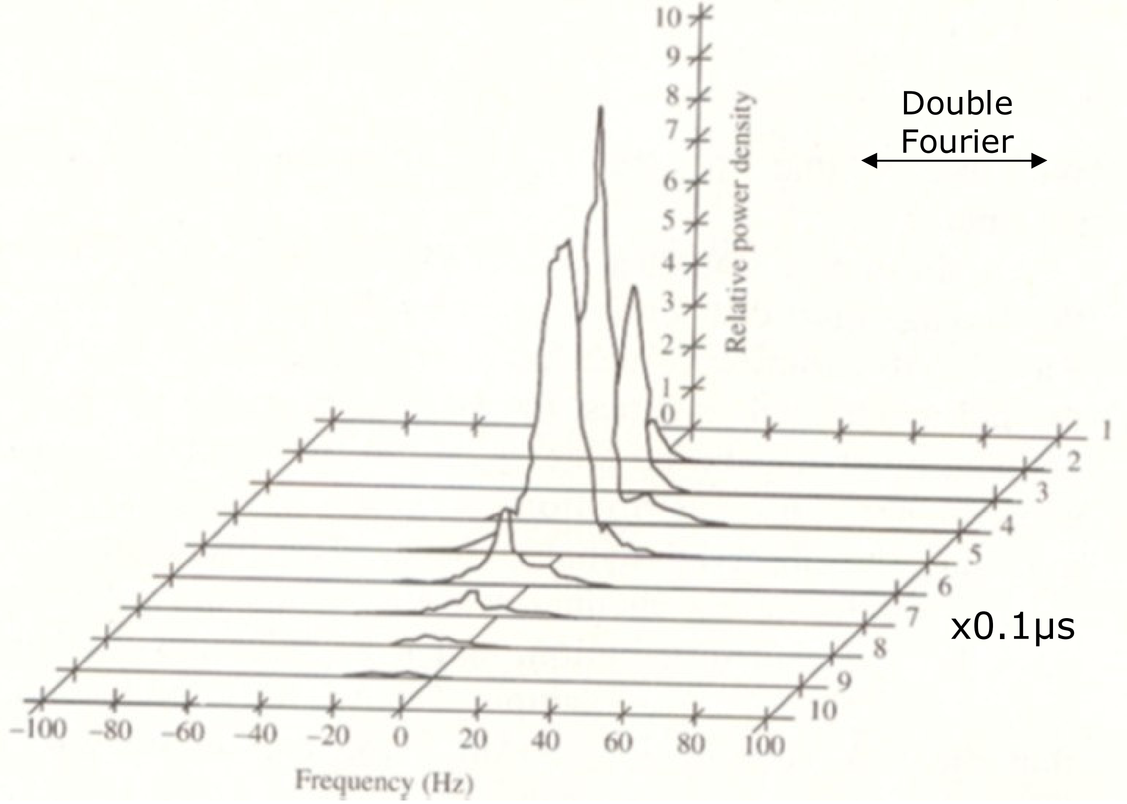
  -statistical measure of the range of frequencies over which the channel can be considered flat

$$B_m \approx \frac{1}{T_m}$$

Multipath channel characteristics (2)

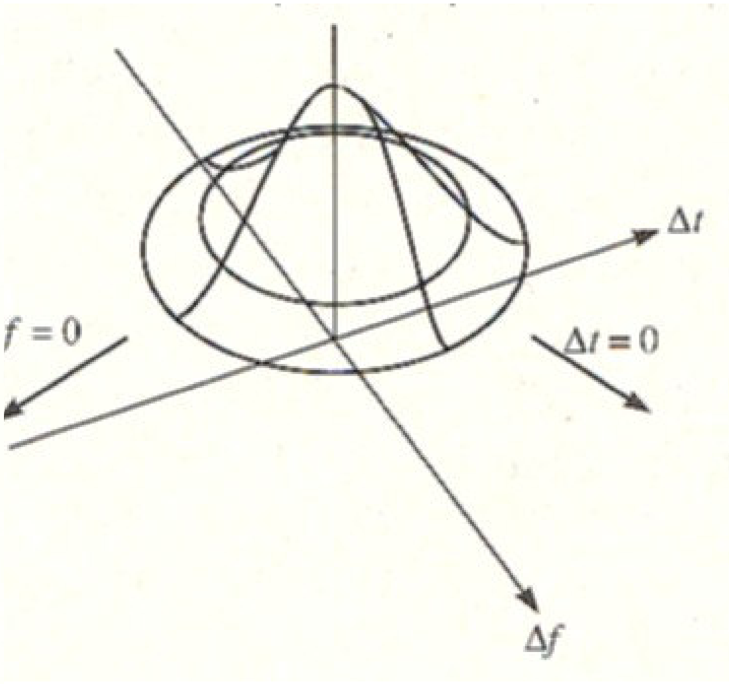
Scattering function $P_S(\tau, v)$

Spaced-frequency, spaced-time correlation function $P_H(\Delta f, \Delta t)$



Double Fourier ↔

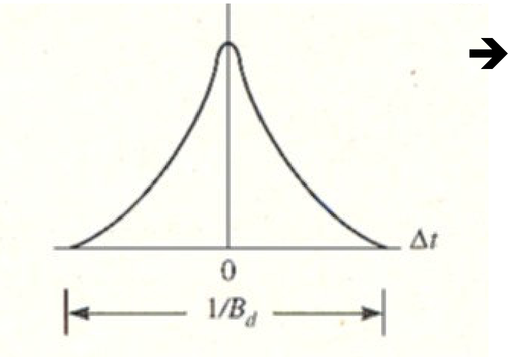
x0.1μs



Source: Proakis

Multipath channel characteristics (3)

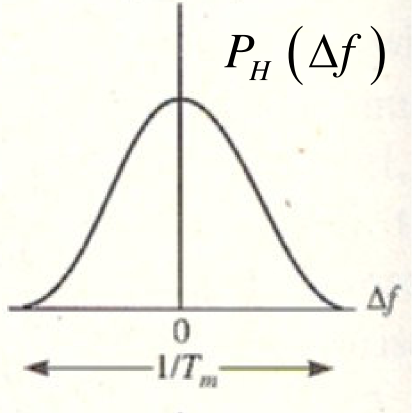
$P_H(\Delta t)$



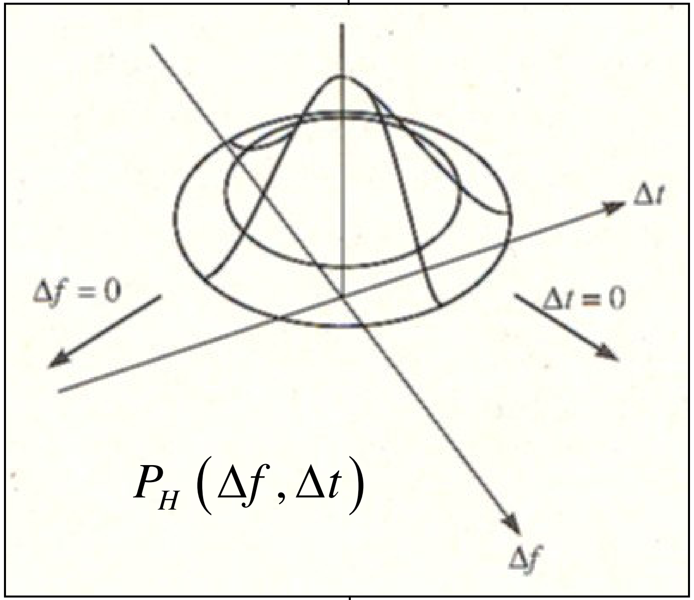
→ Coherence time

Coherence bandwidth ←

$P_H(\Delta f)$



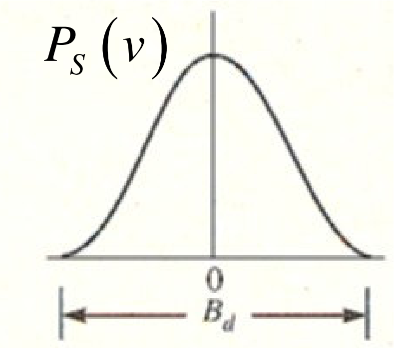
Fourier transform



$P_H(\Delta f, \Delta t)$

Fourier transform

$P_S(v)$

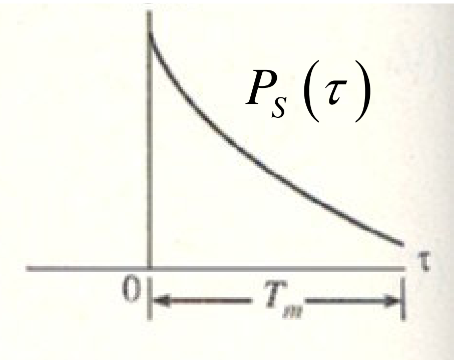


v

→ Doppler spread

Delay spread ←

$P_S(\tau)$



Source: Proakis

Multipath channel characteristics

- The RMS delay spread S is the square root of the second central moment of the power delay profile $P_h(\tau,0)$

$$S = \sqrt{\frac{\int_0^\infty (\tau - D)^2 P_h(\tau) d\tau}{\int_0^\infty P_h(\tau) d\tau}}$$ (continuous case)

- where D is the mean excess delay of the channel. It is the first moment of the power delay profile

$$D = \sqrt{\frac{\int_0^\infty \tau P_h(\tau) d\tau}{\int_0^\infty P_h(\tau) d\tau}}$$

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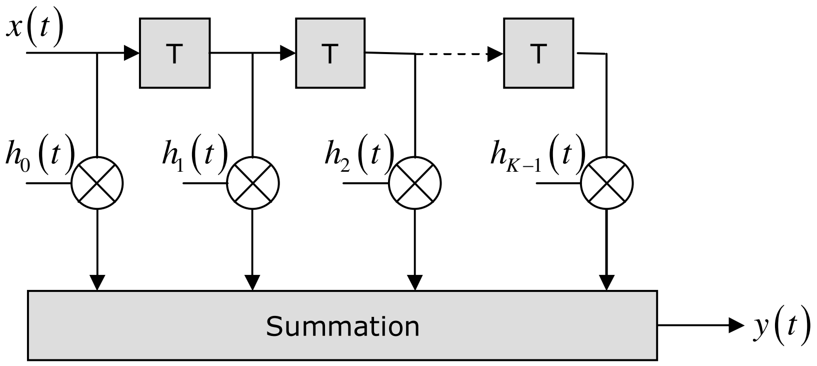
Tapped delay line channel model

- When the bandwidth of the channel increases, the Rayleigh and Rice distributions may not be suitable for modelling the behaviour of a channel.
- When a bandwidth $W >> B_m$ is available to the user, the wideband channel is generally characterised by *the tapped delay line channel model*.
  - Fixed number of taps characterize channel, with information of:
    - Excess delay, normalized amplitudes, amplitude distributions

Tapped delay line channel model

- The tapped delay line channel is represented by a time-variant FIR-filter in complex equivalent low-pass signal domain. The tapped delay line model with K tap coefficients $h_k(t), \quad k = 0,1,\ldots,K-1$



$$y(t) = \sum_{k=1}^{K} x\left(t - \frac{k}{W}\right) h_k(t)$$

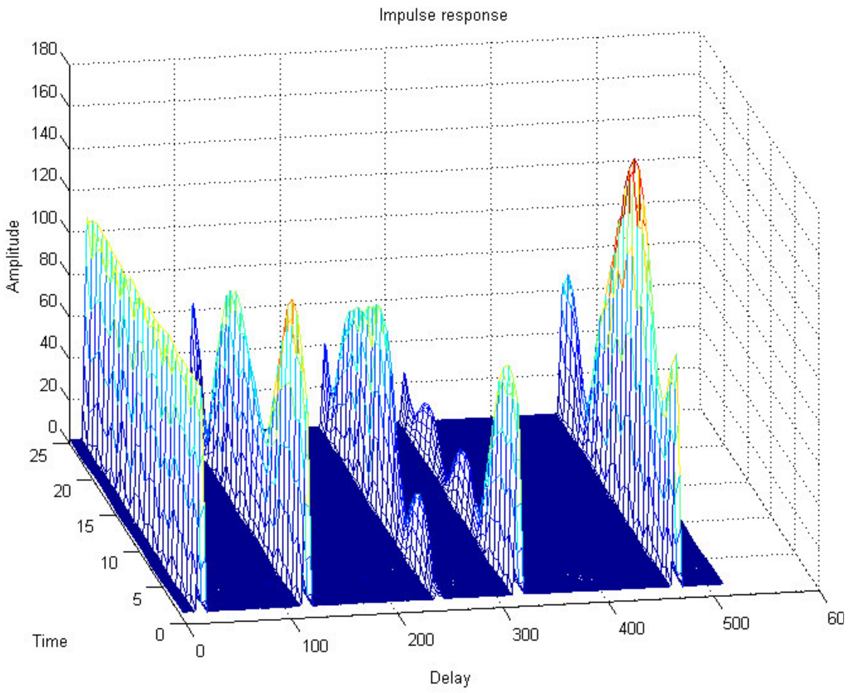
$$T = 1/W$$

W is the bandwidth occupied by the real bandpass signal

Tapped delay line model

- Since the total multipath spread is T_m, the tapped delay line model for the channel can be truncated at taps according to $K = (T_m W) + 1$
- The time-variant tap coefficients $h_k(t)$ are complex-valued stationary random processes.
- Since $h_k(t)$ represent the tap weights corresponding to the different delays
  $\tau = k/W, \ \mathrm{k} = 0, 1, 2, \ldots, K-1$,
  the uncorrelated scattering assumption implies that $h_k(t)$ are mutually uncorrelated

Impulse response of 5-tap channel



Tapped delay line channel model

- The tapped delay line model with only one tap represents the narrowband channel case
- As the bandwidth of the transmitted signal increases the number of taps increases also
- In the special case of Rayleigh fading, the magnitudes of the taps are Rayleigh distributed and the phases are uniformly distributed
- However, as the bandwidth increases even more, the Rayleigh or the Rice distributions are no longer valid.
- In such a case also the number of taps can increase to give very impractical models
  - One possibility is to use sparse channel models, in which a large part of the tap coefficients are set to zero.
  - This leads to the situation where the tap delays do not necessarily have to be uniformly spaced.

Example:UTMS model (ITU-R M.1225)

UTMS model for 2 GHz, 5 MHz bandwidth

- Covers indoor, outdoor-indoor and vehicular environment. Here, *vehicular environment* model is presented

Tap	Channel A		Channel B		Doppler spectrum
	Relative delay [ns]	Avg. power [dB]	Relative delay [ns]	Avg. power [dB]	
1	0	0	0	-2.5	Classic
2	310	-1.0	300	-0.0	Classic
3	710	-9.0	8900	-12.8	Classic
4	1090	-10.0	12900	-10.0	Classic
5	1730	-15.0	17100	-25.2	Classic
6	2510	-20.0	20000	-16.0	Classic

Channel A
-low delay spread
-assumed 40% of time

Channel B
-large delay spread
-assumed 55% of time

Each tap Rayleigh distributed

References

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Multipath propagation

Questions...?

Thank you!!