Multipath propagation and LTV channel model

Aki Silvennoinen
Communications laboratory
aki.silvennoinen@hut.fi

Abstract — Radio wave propagation plays a significant role in the performance of radio systems. Radio wave propagation could be modeled exactly by using Maxwell’s equations derived from the electromagnetic field theory, but this method would be mathematically restrictively complex. In this paper, phenomenon of multipath propagation is considered. Multipath propagation can be modeled using linear time-variant (LTV) channel model, which is derived here. LTV-channel model is then extended to practical wide-sense stationary uncorrelated scattering (WSSUS) channel models. Multipath channel characteristics are derived and the concept of tapped delay line (TDL) channel model is introduced. UMTS TDL channel model is presented as an example.

Index Terms—Multipath propagation, channel modeling, linear time-variant channel

I. INTRODUCTION

Radio wave propagation plays a significant role in the performance of radio systems. Radio waves propagate in a radio channel that is understood as the radio path between the transmitter and the receiver, including antennas. The radio path consists of a variable environment and various obstacles that affect the way the radio waves propagate. Radio wave propagation could be modeled exactly by using Maxwell’s equations derived from the electromagnetic field theory, but this method would be mathematically restrictively. There are four basic phenomena that result in the basic radio wave propagation mechanisms: free space loss, reflection and penetration, diffraction, and scattering.

Fixed radio systems are planned in such a way that there are no obstacles in the radio path that cause attenuation. This is called line-of-sight (LOS) situation. LOS situation is often impossible to maintain while using mobile radio systems, since these are many times used in an urban environment. In such environments there are usually many objects and reflecting surfaces in the radio path that affect the radio wave propagation. Therefore such a situation is called non-line-of-sight (NLOS).

Radio wave propagation channels can be divided according to the effects of the channel on the transmitted signal. Traditionally, propagation models have focused on giving estimates for average received signal strength at a certain distance from the transmitter. These simple models are used to estimate the path loss of the radio channel, and are called large-scale propagation models. Large-scale models are important for coverage planning of the radio network. Propagation models that characterize rapid changes of the signal strength over very short distances (a few wavelengths) between TX and RX or short durations (in the order of seconds) are called small-scale or fading models. Small-scale models and deeper understanding of the channel characteristics is needed for today’s complex radio systems. For the objectives of this paper, only small-scale channel models are relevant.

In general, small-scale propagation channels are time and environment specific. Construction of such models requires heavy measurement campaigns and the results are correct only for a given measurement location at the measurement instant. These channel models can be divided into narrowband and wideband channel models. Narrowband models are simpler and their behavior can be modeled using Rayleigh or Rice distribution functions. As the bandwidth increases a tapped delay line model must be used where the individual tap amplitudes are modeled with independent Rayleigh or Rice distributions. In very wideband channels new statistics of tap amplitude distribution are needed since Rayleigh and Rice distributions give poor models because of insufficient number of physical components in each tap in the channel model.

II. MULTIPATH PROPAGATION

Multipath fading is caused by multiple receptions of the same signal. The signal travels along different paths, and therefore is received at slightly different times. These multipath components combine at the receiver antenna to give a resulting signal that varies widely in amplitude, phase or polarization. A common multipath environment, and an illustrative example, is a city, where the multipath components can reflect, penetrate, diffract and scatter from various obstacles and surfaces as shown in Figure 1.
The three most important effects of multipath fading and moving scatters are [1]:
- Rapid changes in signal strength over a small traveled distance or time interval,
- Random frequency modulation due to varying Doppler shifts on different multipath signals, and
- Time dispersion (echoes) caused by multipath propagation delays.

Fading is caused by the phase difference of the received components. In a dynamic multipath environment where the channel is time-variant (due to movement of the receiver or obstacles) multiple versions of the transmitted signal reach the receiver via path of different lengths. Due to the time-variant nature of the channel, there is a continuous change in the length of each propagation path, and thus, the phases of the components also change as a function of time. The amplitude of the received signal is very small or practically zero at times when the components add destructively due to phase differences. At other times the components add constructively, resulting in a large amplitude of the received signal. Thus the amplitude variations in the received signal, called signal fading, are due to the time-variant characteristics of the channel.

Relative motion between TX and RX (or surrounding objects causing e.g. reflection) causes random frequency modulation, since each multipath component has a different Doppler shift (phase change per time unit). The Doppler shift can be calculated by using (1).

$$f_d = \frac{V}{\lambda} \cos \alpha,$$

where $V$ is the velocity of the terminal, $\alpha$ is the angle between the direction of motion of the terminal and the direction of arrival of the wave, and $\lambda$ is the RF wavelength.

In city environments mobile radio system antenna heights are usually well below the rooftop level of the surrounding buildings and therefore there might not be a line-of-sight (LOS) path between TX and RX. Even if a LOS situation occurs there are many reflecting surfaces that still produce multipath components that cause time dispersion caused by multipath propagation delays.

A. Countermeasures against multipath fading
Many methods have been developed to decrease the effect of multipath propagation. Usage of different diversity methods, e.g. space, time, frequency diversity etc., is a powerful means for reducing multipath fading. Multipath diversity means that multipath components are received in many different ways at the same time to increase the probability of receiving a strong signal at least in one way. For combining the received components, certain techniques, e.g. selection, equal-gain or maximum-ratio combining have been developed. The RAKE receiver is also a commonly used countermeasure against the adverse effects of multipath propagation in spread spectrum systems like UMTS and WLAN. Also the usage of equalizers or smart antennas is powerful countermeasures against multipath fading.

III. SYSTEM FUNCTIONS OF THE LINEAR TIME-VARIANT CHANNEL

The radio propagation channel can be visualized by a system element that transforms the input signal into an output signal. It is therefore similar to a linear filter with the extension that the radio propagation channel is time-variant. The radio channel can be modeled as a linear time-variant (LTV) channel that can be characterized by four functions, the time-variant impulse response (also known as the channel delay spread function [2]), the time-variant transfer function, the channel output Doppler-spread function, and the delay/Doppler-spread function.

A. System functions of the deterministic LTV-channel

The time-variant impulse response $h(\tau, t)$ is shown in (2) [3].

$$h(\tau, t) = \sum_{n=1}^{N} \alpha_n(t) e^{j 2\pi f_c \tau_n(t)} \delta(\tau - \tau_n(t)), \quad (2)$$

where $\tau_n(t)$ is the propagation delay of the $n^{th}$ propagation path as function of time, $\alpha_n(t)$ is the gain of the $n^{th}$ propagation path as function of time and $f_c$ is the carrier frequency. Figure 2 shows a measured impulse response.
If an unmodulated carrier at frequency $f_c$ is transmitted using the channel in (2), the received signal is in the form of

$$r(t) = \sum_{n} \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} = \sum_{n} \alpha_n(t) e^{-j\theta_n(t)},$$

where $\theta_n(t) = 2\pi f_c \tau_n(t)$. The received signal consists of a sum of time-variant phasors having amplitudes $\alpha_n(t)$ and phases $\theta_n(t)$. It can be seen that large dynamic changes in the channel are required for $\alpha_n(t)$ to change sufficiently to cause significant changes in the received signal. However, $\theta_n(t)$ will change by $2\pi$ whenever $\tau_n(t)$ changes by $1/f_c$, which is a relatively small number, and hence, $\theta_n(t)$ can change by $2\pi$ with relative small motions in the medium [4]. The changes in $\theta_n(t)$ results in channel fading as explained in Section II.

The delays $\tau_n(t)$ associated with the different propagation paths can be expected to change at different rates and in an unpredictable way. This implies that the received signal can be modeled as a random process. When there are a large number of independent random components in the received signal, the central limit theorem can be applied [5]:

Given $n$ independent random variables $x_i$, their sum is formed: $x = x_1 + x_2 + \ldots + x_n$. This is a random variable with mean $\eta = \eta_1 + \eta_2 + \ldots + \eta_n$ and variance $\sigma^2 = \sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2$. The central limit theorem states that under certain conditions, the

distribution $F(x)$ of $x$ approaches a normal distribution with the same mean and variance.

That is, the received signal may be modeled as a complex-valued Gaussian process. This means that the time-variant impulse response $h(\tau, t)$ is a complex-valued Gaussian random process in the $t$ variable. When the envelope of the impulse response is modeled as a zero-mean complex valued Gaussian process, it can be shown that the envelope $|h(\tau, t)|$ at any instant $t$ is Rayleigh-distributed. In this case the channel is said to be Rayleigh fading. In the case that there are fixed scatterers or signal reflections in addition to randomly moving scatterers in the channel, the impulse response can no longer be modeled as having zero-mean. In this case, the envelope has a Rice distribution and the channel is said to be Rice fading.

The time-variant transfer function is obtained by Fourier-transforming the impulse response with respect to the delay variable $\tau$ as shown in (4).

$$H(f, t) = F_r\{h(\tau, t)\} = \int_{-\infty}^{\infty} h(\tau, t) e^{-j2\pi f \tau} d\tau$$

(4)

The output Doppler-spread function is obtained by Fourier-transforming the time-variant transfer function with respect to the time variable $t$ as shown in (5). It describes the channel frequency response to the frequency $f + \nu$.

$$D(f, \nu) = F_r\{H(f, t)\} = \int_{-\infty}^{\infty} H(f, t) e^{-j2\pi \nu t} dt$$

(5)

The delay/Doppler-spread function is obtained by Fourier-transforming the time-variant impulse response with respect to the time variable $t$ as shown in (6), or by taking the inverse Fourier-transform of the output Doppler-spread function with respect to frequency. The delay/Doppler-spread function describes the complex gain of the channel in the delay interval $[\tau + d\tau]$ and the Doppler-shift interval $[\nu + d\nu]$.

$$S(\tau, \nu) = F_r\{h(\tau, t)\} = \int_{-\infty}^{\infty} h(\tau, t) e^{-j2\pi \nu t} dt$$

(6)

Figure 3 describes the relationships between the system functions. The relationships between system functions are assumed to be deterministic.
B. System functions of the WSSUS random LTV-channel

The characterization of the system functions can then be extended to practical channels that are randomly time-variant. Wide-sense stationary uncorrelated scattering (WSSUS) channels are an important class of practical channels. A randomly time-variant LTV-channel is described using 4-dimensional autocorrelation functions that correspond to the previously described system functions. The use of these 4-dimensional autocorrelation functions is rather unpractical; two assumptions are made in order to get 2-dimensional autocorrelation functions [4].

- The LTV-channel under consideration is wide-sense stationary.
- The LTV-channel under consideration is a multipath channel where the propagation paths are statistically independent or at least uncorrelated.

This LTV-channel is called a WSSUS-channel. The autocorrelation functions for a WSSUS-channel can be presented as \( P_h(\tau, \Delta t) \), \( P_H(\Delta f, \Delta t) \), \( P_D(\Delta f, v) \) and \( P_S(\tau, v) \) similarly as the previous system functions. The relationships between the autocorrelation functions are presented in Figure 4.

Through these functions the characteristic parameters of the WSSUS channel, i.e. delay spread \( T_m \), coherence bandwidth \( B_m \), coherence time \( T_D \), and Doppler-spread \( B_D \), can be obtained.

These multipath channel parameters are useful in order to compare different multipath channels. Delay spread quantifies the time dispersive properties of the channel, coherence bandwidth characterizes the channel in frequency domain and Doppler Spread and coherence time describe the time varying nature of the channel.

The RMS delay spread \( S \) is the square root of the second central moment of the power delay profile (PDP) \( P_h(\tau, 0) \) as shown in (7) [3].

\[
S = \sqrt{\int_0^\infty (\tau - D)^2 P_h(\tau) d\tau},
\]

where \( D \) is the mean excess delay of the channel. It is the first moment of the power delay profile (PDP) as shown in (8) [3].

\[
D = \sqrt{\int_0^\infty \tau P_h(\tau) d\tau} \int_0^\infty P_h(\tau) d\tau
\]

C. Multipath channel characteristics

The delay spread \( T_m \) quantifies the time dispersive properties of the channel, i.e. the delays of multipath components.

The Doppler spread \( B_D \) is a measure of the spectral broadening caused by the time rate of change of the mobile radio channel. It is defined as the range of frequencies over which the received Doppler spectrum is essentially non-zero. If the baseband signal bandwidth is much greater than \( B_D \), the effects of the Doppler spread are negligible at the receiver.

The coherence bandwidth is a statistical measure of the range of frequencies over which the channel can be considered flat. It is defined as the range of frequencies over which two different frequency components have a strong
possibility of amplitude correlation.

$$B_m \approx \frac{1}{T_m} \quad \quad \quad \quad (9)$$

The coherence time $T_D$ is the time domain dual of Doppler spread. It is used to characterize the time varying nature of the frequency dispersion of the channel in the time domain. The coherence time is actually a statistic measure of the time interval over which the channel impulse response is essentially invariant. The definition of coherence time implies that the channel affects differently two signals arriving with a time separation greater than $T_D$.

$$T_D \approx \frac{1}{B_D} \quad \quad \quad \quad (10)$$

Figure 5 sums up the multipath channel characteristics by presenting the relationships of spaced-frequency, spaced-time correlation function and scattering function, and derivation of Doppler spread and delay spread [4].

![Figure 5 Relationships of spaced-frequency, spaced-time correlation function and scattering function, and derivation of Doppler spread and delay spread.](image)

**IV. TAPPED DELAY LINE CHANNEL MODEL**

The Rayleigh and Rice distributions may not be suitable for modeling the behavior of a channel when the bandwidth of the channel increases. When a bandwidth $W >> B_m$ is available to the user, the wideband channel is generally characterized by the tapped delay line channel model. For a fixed number of taps, parameters characterizing the multipath behavior, such as excess delay, normalized amplitudes and amplitude distributions, are calculated. The tapped delay line channel is represented by a time-variant FIR-filter in complex equivalent low-pass signal domain. The tapped delay line model with $K$ tap coefficients $h_k(t), \; k = 0,1,\ldots,K-1$ is shown in Figure 6 [4].

![Figure 6 Tapped delay line presentation of a wideband multipath radio channel](image)

Since the total multipath spread is $T_m$, for all purposes, the tapped delay line model for the channel can be truncated at $K$ taps according to (11) [4].

$$K = (T_m W) + 1 \quad \quad \quad \quad (11)$$

The output function is given by (12).

$$y(t) = \sum_{k=1}^{K} x(t - \frac{k}{W}) h_k(t), \quad \quad \quad \quad (12)$$

The time-variant tap coefficients $h_k(t)$ are complex-valued stationary random processes. Since $h_k(t)$ represent the tap weights corresponding to the $K$ different delays $\tau = k/W, \; k = 0,1,2,\ldots,K-1$, the uncorrelated scattering assumption made in Section III implies that $h_k(t)$ are mutually uncorrelated [4].

The tapped delay line model with only one tap represents the narrowband channel case, and as the bandwidth of the transmitted signal increases the number of taps increases also. In the special case of Rayleigh fading, the magnitudes of the taps are Rayleigh distributed and the phases are uniformly distributed [4]. However, as the bandwidth increases even more, the Rayleigh or the Rice distributions are no longer valid. In such a case also the number of taps can increase to give very impractical models. One possibility is to use sparse channel models, in which a large part of the tap coefficients are set to zero. This leads to the situation where the tap delays do not necessarily have to be uniformly spaced.
V. UMTS VEHICULAR CHANNEL MODEL

The development of UMTS in recent years has created the need for channel models for the 2 GHz frequency band. UMTS operates at frequency of 2 GHz and the modulation bandwidth of UMTS is 5 MHz.

ITU has published a recommendation concerning WCDMA channel models. The ITU Recommendation ITU-R M.1225 covers indoor office, outdoor-to-indoor and vehicular environments. The vehicular environment model and its channel impulse response model is presented here. The number of taps, the time delay relative to the first tap, the average power relative to the strongest tap, and the Doppler spectrum of each tap characterize the model. For most of the time the RMS delay spread is relatively small, but there are occasions with ‘worst case’ multipath characteristics that lead to a much larger RMS delay spread. These ‘worst case’ situations occur relatively infrequently, but they can have a major impact on system performance. To model this behavior of the channel, two multipath channels are defined: channel A is the low delay spread case that occurs frequently, channel B is the medium delay spread case that also occurs frequently. Channel A is expected to be encountered 40% of the time, while for channel B the value is 55 % of the time. Table 1 shows the tapped-delay-line parameters of the vehicular test environment with high antennas [6].

VI. CONCLUSIONS

In this paper, radio channel modeling was presented from a theoretical point of view. The focus was on small-scale radio channel modeling, starting with multipath propagation. The linear time-variant channel model is a suitable approach and the extension to WSSUS channel characterization is particularly useful. Many radio channel characteristics can be derived from the LTV channel model. Finally, practical multipath channels are dealt with using the tapped delay line approach.

REFERENCES


HOMEWORK

Consider UMTS channel model. Can channel A be considered flat for UMTS signal? What about channel B?

Table 1 Tapped-delay-line parameters of the vehicular test environment.

<table>
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<th>Channel A</th>
<th>Channel B</th>
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