Multiuser Detection for SDMA OFDM

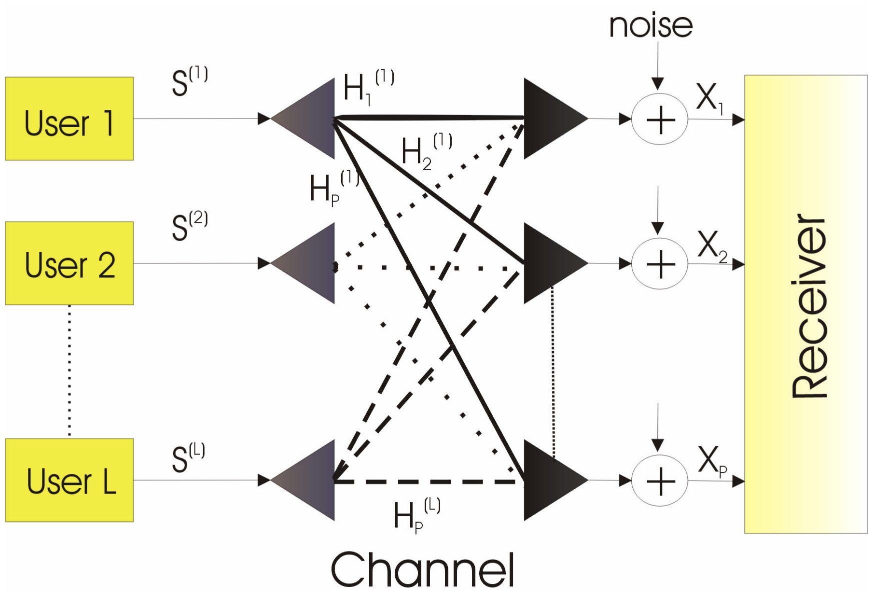
S7233-Postgraduate Course

Fernando Gregorio
gregorio@wooster.hut.fi

- Smart Antenna applications
  - Beamforming
  - Spatial Diversity Systems
  - Space Division Multiple Access (SDMA)

- Space Division Multiple Access (SDMA)
  - L different users
  - User-specific spatial signature
  - The signal signature generated by the channel over the transmitted signal acts like spreading code in a CDMA system.
  - Multiuser detection techniques known from CDMA can be applied in SDMA-OFDM

- Space Division Multiple Access (SDMA)



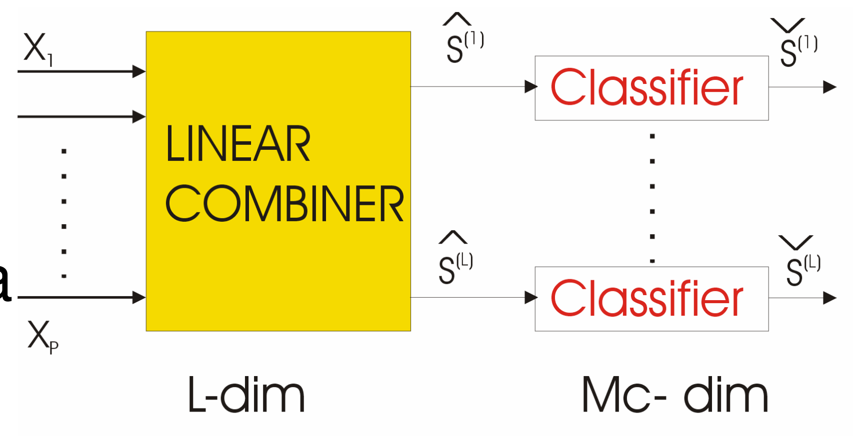
$$x = Hs + n$$

$$x = [x_1, x_2, \ldots, x_p]$$
$$s = [s^1, s^2, \ldots, s^L]$$
$$H = [H^1, H^2, \ldots, H^L]$$
$$H^L = [H_1^L, H_2^L, \ldots, H_p^L]$$

L users

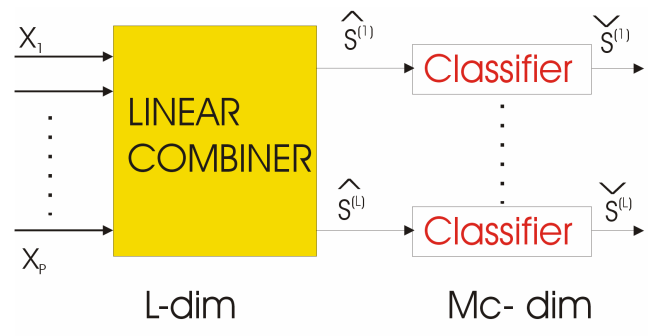
P – receive antennas

- Linear detection techniques
  - The different users transmitted signals are estimated with the aid of a linear combiner.
  - The residual interference caused by remaining users is neglected.



$$\hat{s} = \mathbf{W}^{\mathbf{H}}\mathbf{x}$$

- Linear detection techniques
  - L-th user



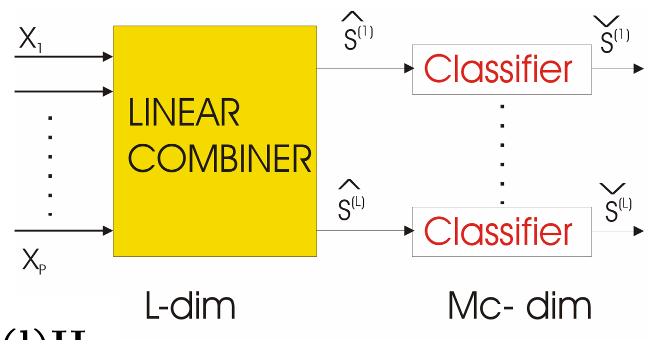
$$\begin{aligned}
\hat{s}^{l} &= \mathbf{w}^{(l)H}\mathbf{x} \\
&= \mathbf{w}^{(l)H}(\mathbf{H}\mathbf{s}+\mathbf{n}) \\
&= \underbrace{\mathbf{w}^{(l)H}\mathbf{H}^{l}\mathbf{s}^{l}}_{\text{Desired user}} + \underbrace{\mathbf{w}^{(l)H}\sum_{\substack{i=1 \\ i\neq l}}^{L}\mathbf{H}^{i}\mathbf{s}^{i}}_{\text{Interferening users'}} + \underbrace{\mathbf{w}^{(l)H}\mathbf{n}}_{\text{AWGN}}
\end{aligned}$$

Desired user

Interferening users'

AWGN

- Linear detection techniques
  - Statistical characterization



$$\hat{s}^l = \mathbf{w}^{(l)H}\mathbf{H}^l\mathbf{s}^l + \mathbf{w}^{(l)H}\sum_{\substack{i=1\\ i\neq l}}^{L}\mathbf{H}^i\mathbf{s}^i + \mathbf{w}^{(l)H}\mathbf{n}$$

Desired user

$$\sigma_S^{(l)2} = \mathbf{w}^{(l)H}\mathbf{R}^l_{a,S}\mathbf{w}^{(l)}$$
$$\mathbf{R}^l_{a,S} = \sigma_l^2\mathbf{H}^{(l)}\mathbf{H}^{(l)H}$$

Interferening users'

$$\sigma_I^{(l)2} = \mathbf{w}^{(l)H}\mathbf{R}^l_{a,I}\mathbf{w}^{(l)}$$
$$\mathbf{R}^l_{a,I} = \sum_{\substack{i=1\\ i\neq l}}^{L}\sigma_l^2\mathbf{H}^{(l)}\mathbf{H}^{(l)H}$$

AWGN

$$\sigma_N^{(l)2} = \mathbf{w}^{(l)H}\mathbf{R}^l_{a,N}\mathbf{w}^{(l)}$$
$$\mathbf{R}_{a,N} = \sigma_n^2\mathbf{I}$$

• Linear detection techniques

- Signal to Interference plus Noise Ratio

$$SINR^{(l)} = \frac{\sigma_S^{(l)2}}{\sigma_I^{(l)2} + \sigma_N^2}$$

- Signal to Interference Ratio

$$SIR^{(l)} = \frac{\sigma_S^{(l)2}}{\sigma_I^{(l)2}}$$

- Signal to Noise Ratio

$$SNR^{(l)} = \frac{\sigma_S^{(l)2}}{\sigma_N^2}$$

- Linear detector

•Least Squares Error

- •Zero Forcing
- •Maximize SNR at the receiver

•Minimum squares Error

- •Exploits the available statistical knowledge concerning the signals transmitted

- Least-squares Error detector
  - Estimation error $$\begin{aligned}\Delta\widehat{x} &= x - \widehat{x} \\ &= x - \mathbf{H}\hat{\mathbf{s}}\end{aligned}$$
  - Squared error

$$\begin{aligned}\|\Delta\widehat{x}\|^2 &= \Delta\widehat{x}^H \Delta\widehat{x} \\ &= x^H x - 2\Re(\widehat{s}^H \mathbf{p_{LS}}) + \widehat{\mathbf{s}}^{\mathbf{H}} \mathbf{Q_{LS}} \widehat{\mathbf{s}}\end{aligned}$$

  - Optimum value

$$\frac{\partial\|\Delta\widehat{x}\|^2}{\partial\widehat{s}}$$

$$\begin{aligned}\widehat{s}_{LS} &= \mathbf{Q_{LS}^{-1}} \mathbf{p_{LS}} \\ &= \mathbf{P_{LS}} \mathbf{x}\end{aligned}$$

$$\mathbf{P_{LS}} = (\mathbf{H^H H})^{-1} \mathbf{H^H}$$

Cross-correlation vector

$$\mathbf{p_{LS}} = \mathbf{H^H x}$$

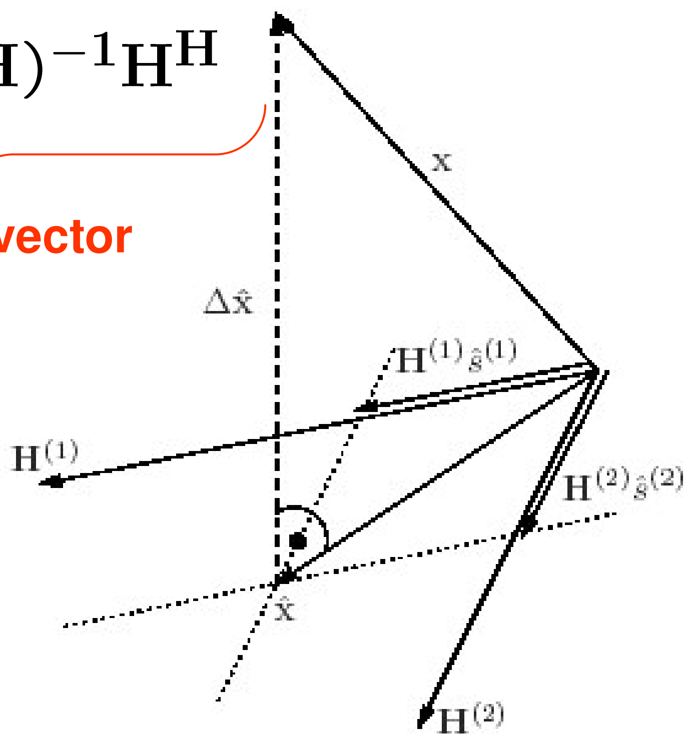
Auto-correlation matrix

$$\mathbf{Q_{LS}} = (\mathbf{H^H H})$$

- Least-squares Error detector

$$\widehat{s}_{LS} = \mathbf{Q}_{\mathbf{LS}}^{-1}\mathbf{p}_{\mathbf{LS}} = \mathbf{P}_{\mathbf{LS}}\mathbf{x}$$

$$\underbrace{\mathbf{P}_{\mathbf{LS}} = (\mathbf{H}^{\mathbf{H}}\mathbf{H})^{-1}\mathbf{H}^{\mathbf{H}}}_{\text{Projection vector}}$$



- The matrix $\mathbf{P}_{LS}$ projects the vector x of the P different antenna elements' received signals onto the column space of the channel matrix **H**

- Minimum Mean squares error detector
  - Cost Function

$$\begin{aligned}\Delta\hat{s} &= s - \hat{s} \\ &= s - (\mathbf{W^H x})\end{aligned}$$

  - The estimation error's autocorrelation matrix

$$\begin{aligned}\mathrm{R}_{\Delta\hat{\mathrm{s}}} &= E\{\Delta\hat{s}\Delta\hat{s}^H\} \\ &= \mathbf{P} - \mathbf{R_c^H W} - \mathbf{W^H R_c} + \mathbf{W^H R_a W}(1)\end{aligned}$$

$$\begin{aligned}\mathrm{R_c} &= E\{\mathrm{xs^H}\} \\ &= \mathrm{HP}\end{aligned}$$

$$\mathbf{P} = \mathbf{Diag}(\sigma_{\mathbf{1}}^{\mathbf{2}}, \sigma_{\mathbf{2}}^{\mathbf{2}}, \ldots, \sigma_{\mathbf{L}}^{\mathbf{2}})$$

$$\begin{aligned}\mathrm{R_a} &= E\{\mathrm{xx^H}\} \\ &= \mathbf{HPH^H} + \sigma_{\mathbf{n}}^{\mathbf{2}}\mathbf{I} \\ &= \sum_{l=1}^{L} \sigma_l^2 \mathbf{H^l H^{(l)H}} + \sigma_{\mathbf{n}}^{\mathbf{2}}\mathbf{I}\end{aligned}$$

- Minimum Mean squares error detector
  - The optimum weights are determined minimizing $E\{\|\Delta\widehat{s}\|^2\}$

$$\begin{aligned}\mathbf{W_{MMSE}} &= \mathbf{R_a^{-1}R_c}\\ &= (\mathbf{HPH^H} + \sigma_{\mathbf{n}}^{\mathbf{2}}\mathbf{I})^{-1}\mathbf{HP}\\ &= (\mathbf{HP_{SNR}H^H} + \sigma_{\mathbf{n}}^{\mathbf{2}}\mathbf{I})^{-1}\mathbf{HP_{SNR}}\end{aligned}$$

$$\mathbf{P_{SNR}} = \mathbf{Diag}(\mathbf{SNR^{(1)}}, \mathbf{SNR^{(2)}}, \ldots, \mathbf{SNR^{(L)}})$$

$$SNR^{(l)} = \frac{\sigma_l^2}{\sigma_n^2}$$

- Minimum Variance (MV) combining
  - LS
    - Information concerning with the AWGN process is not considered.
  - MMSE
    - Balance between the recovery signals transmitted and the suppression of the AWGN.
  - MV
    - Recover the original signals while ensuring a partial suppression of the AWGN.
    - The cost-function incorporates both a constraint on the desired user's effective transfer factor as well as the undesired signal's variance. $w_{MV}^{(l)} = \frac{g}{w_{MMSE}^{(l)H}\mathbf{H}^{\mathbf{l}}} w_{MMSE}^{(l)}$

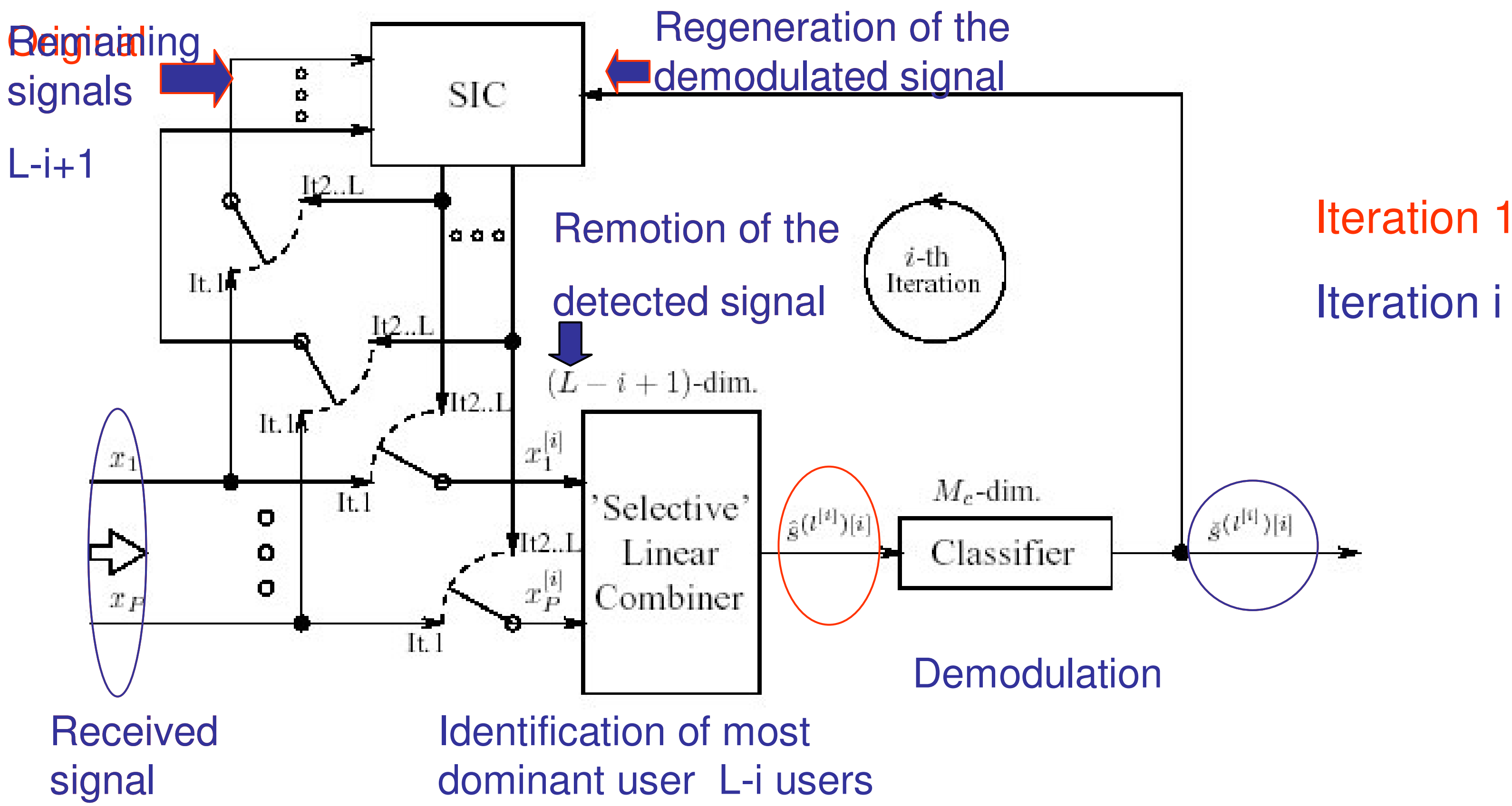
- Non-Linear Detection
  - Linear detector assumes that the different users' associated linear combiner output are corrupted only by AWGN
  - Linear combiner output signal contain residual interference which is not Gaussian distributed.
  - LS and MMSE sequential structure.

- Non-Linear Detection
  - The operation of classification can be embedd into the linear combination process.
  - The residual multi-user interference observed at the classifier's input is reduced
  - Successive Interference Cancellation (SIC)
  - Parallel Interference Cancellation (PIC)

- Successive Interference Cancellation (SIC)
  - Only the specific user having the highest SINR, SIR or SNR in each iteration at the output of the LS or MMSE combiner is detected.
  - Having detected this user's signal, the corresponding demodulated signal is subtracted from the composite signal received by the different antenna elements.
  - New iteration.

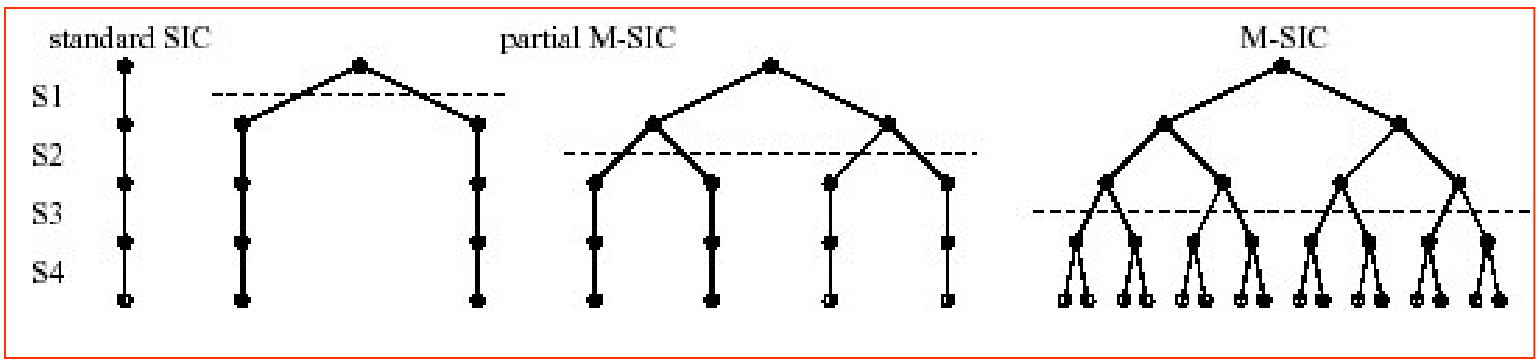
- Successive Interference Cancellation (SIC)
  1. Initialization
  2. Detection stage
  3. Calculation of remaining user's weight matrix
  4. Selection of the most dominant user
  5. Detection of the most dominant user
  6. Demodulation of the most dominant user.
  7. Removing of the most important user contribution.
  8. New iteration

- Successive Interference Cancellation (SIC)



- M-Successive Interference Cancellation (SIC)
  - Error propagation problems in standard SIC.
  - MSIC-> Track from each detection stage not only the single most likely symbol decision, but an increased number of $M \cdot M_c$ most likely tentative symbol decisions, where M_c denotes the number of constellation points associated with a specific modulation scheme.

- M-SIC (M = 2)
  - First detection step
    - We have a total of M = 2 possible symbol decisions.
  - Second detection step
    - M^2 = 4 tentative symbol decisions. and correspondingly,
  - i-th detection stage
    - M^i possible tentative symbol decisions.



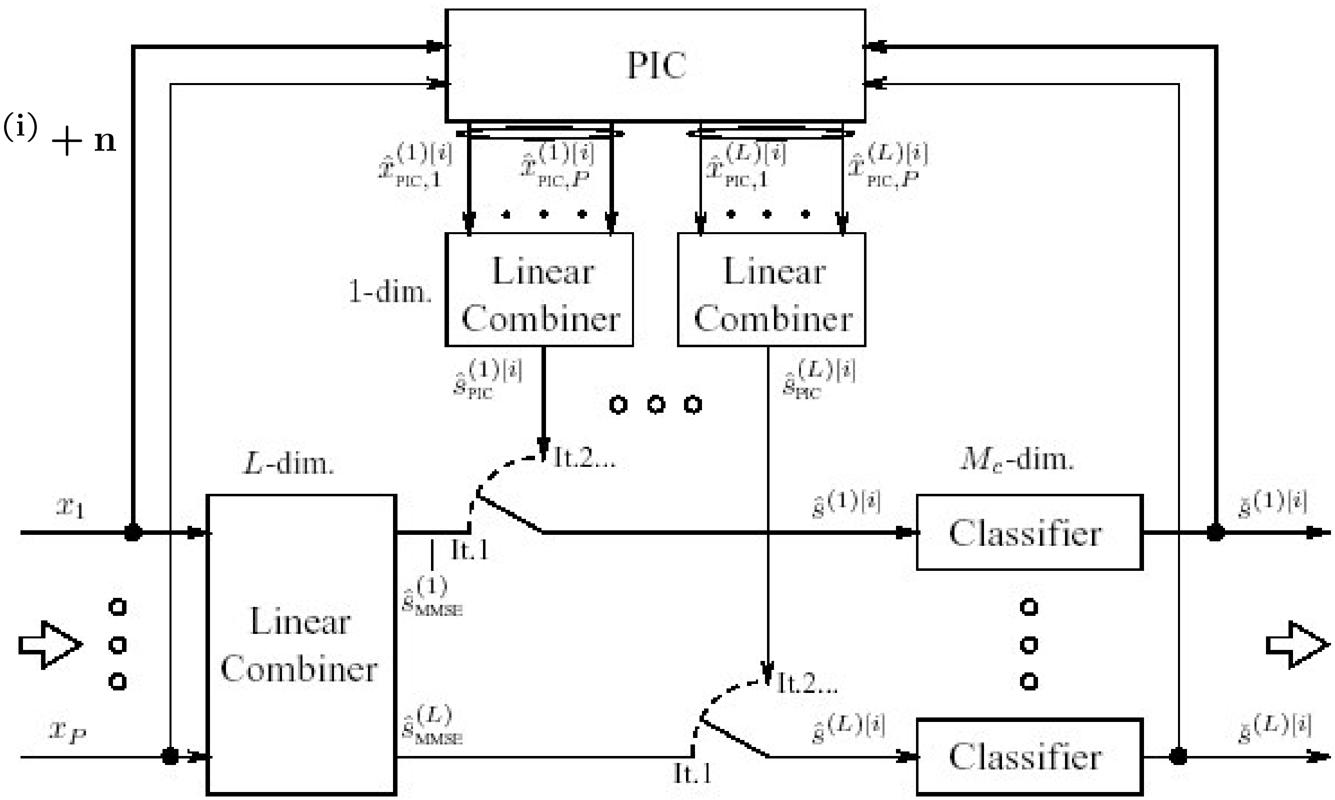
- Partial M-SIC
  - The performance improvement potentially observed for the M-SIC scheme compared to

    the standard SIC arrangement is achieved at the cost of a significantly increased computational complexity.
  - For sufficiently high SNRs the standard SIC detector's performance is predetermined by the bit- or symbol-error probabilities incurred during the first detection stage.
  - If the most dominant user's associated symbol decision is erroneous, its effects potentially propagate to all other users' decisions conducted in the following detection stages.

- Partial M-SIC
  - The symbol error probability specifically of the first detection stage should be as low as possible.
  - The tentative symbol decisions carried out at later detection stages become automatically more reliable as a result of the system's increased diversity order due to removing the previously detected users.
  - M > 1 number of tentative symbol decisions at each detection node are retained, characterized by its associated updated P-dimensional vector of received signals only up to the specific $L_{pM\text{-}SIC}$-th stage in the detection process.
  - At later detection stages only one symbol decision is retained, as in standard SIC scheme.

- PIC

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n}$$
$$= \mathbf{H}^{(l)}\mathbf{s}^{(l)} + \sum_{\substack{i=1 \\ i \neq l}}^{L} \mathbf{H}^{(i)}\mathbf{s}^{(i)} + \mathbf{n}$$



- PIC
- First-stage MMSE detection
  - Combining
    - During the first PIC iteration each user is detected by means of the MMSE combiner.
  - Classification demodulation
    - the linear combiner's output vector $\widehat{s}^{[1]} = \widehat{s}_{MMSE}$ is demodulated resulting in the vector $\breve{s}^{[1]}$ of symbols that are most likely to have been transmitted by the L different users.

- PIC
- i-th stage PIC detection (i>1)
  - PIC
    - During the *i-th* PIC iteration a potentially improved estimate $\hat{s}_{PIC}^{(l)[i]}$ of the complex symbol $s^{(l)}$ transmitted by the *l-th* user is obtained upon subtracting in a first step the *L-1* interfering users' estimated signal contributions, from the original vector *x* of signals received by the different antenna elements
  - Combining
    - Extract an estimate $\hat{s}_{PIC}^{(l)[i]}$ of the signal $s^{(l)}$ transmitted by the $l-th$ user from the l-th user's PIC-related array output vector $\hat{x}_{PIC}^{(l)[i]}$.
  - Classification / demodulation
    - Delivers the symbol $\breve{s}^{(l)[i]}$ that is most likely to have been transmitted by the *l-th* user.

- Maximum Likelihood detection

- Comparison
  - BER
  - Complexity