MULTIUSER DETECTION FOR SDMA OFDM

Fernando H. Gregorio

Helsinki University of Technology
Signal Processing Laboratory,
POB 3000, FIN-02015 HUT, Finland
E-mail:Fernando.Gregorio@hut.fi

1. INTRODUCTION

Smart antenna designs have emerged in recent years. They are applied with the main objective of combating the effects of multipath fading on the desired signal and suppressing interfering signals, thereby increasing both the performance and capacity of wireless systems. The more known applications of smart antenna are beamforming arrangements and spatial diversity systems. A third application of smart antennas is Space Division Multiple Access (SDMA)[1].

Space-Division-Multiple-Access (SDMA) communication systems have recently drawn wide interests. In these systems the L different users transmitted signals are separated at the base-station (BS) with the aid of their unique, user-specific spatial signature, which is constituted by the P-element vector of channel transfer factors between the users single transmit antenna and the P different receiver antenna elements at the BS, upon assuming flat-fading channel conditions such as in each of the OFDM subcarriers. In simple conceptual terms, it is possible to argue that the spatial signature generated by the channel over the transmitted signal acts like CDMA spreading code in a conventional CDMA system.

Multiuser detection techniques known from Code-Division-Multiple-Access (CDMA) can be applied in SDMA-OFDM transceivers. Some of these techniques are the Least-Squares (LS), Minimum Mean-Square Error (MMSE) , Successive Interference Cancellation (SIC), Parallel Interference Cancellation (PIC) and Maximum Likelihood (ML) detection [2].

Multiuser detection methods can be classified in two classes of linear and non-linear detection techniques. In the group of linear detection methods appear LS and MMSE detection, in which no a priori knowledge of the remaining users transmitted symbols is required for the detection of a specific user. In the case of SIC, PIC and ML detection, non-linear methods, a priori knowledge is involved, which must be provided by the non-linear classification operation involved in the demodulation process. In this paper a description of SDMA-OFDM is introduced in section 2, linear and non-linear techniques will be developed in section 2 and 3. Implementation complexity and performance will be evaluated in section 4. Conclusions are included in the last section.

2. SPACE-DIVISION-MULTIPLE-ACCESS (SDMA)

A Space-Division-Multiple-Access (SDMA) uplink transmission scenario, where each of the L simultaneous users is equipped with a single transmission antenna, while the receiver capitalizes on a P-element antenna front-end is illustrated in Figure 1.

The vector complex signals, $x[n,k]$, received by the P-element antenna array in the $k-th$ subcarrier of the n-th OFDM symbol is constituted by the superposition of the independently faded signals associated with the L users sharing the same space-frequency resource. The received signal was corrupted by the Gaussian noise at the array elements (the indices $[n,k]$ have been omitted for notational convenience). This signal can be written as

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{x} = [\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_p}]$ is the received signals vector, $\mathbf{s} = [\mathbf{s^1}, \mathbf{s^2}, \ldots, \mathbf{s^L}]$ is the transmitted signals vector and $\mathbf{n}$ is the array noise vector. The frequency domain channel transfer factor matrix $\mathbf{H}$ is constituted by the set of channel transfer factor vectors $\mathbf{H^l}$ with $l = 1, \ldots, L$ of the L users:

$$\mathbf{H} = [\mathbf{H^1}, \mathbf{H^2}, \ldots, \mathbf{H^l}] \tag{2}$$

each of which hosts the frequency domain channel transfer factors between the single transmitter antenna associated with a particular user l and the reception antenna elements $p = 1, \ldots, P$:

$$\mathbf{H^l} = [\mathbf{H^l_1}, \mathbf{H^l_2}, \ldots, \mathbf{H^l_P}] \tag{3}$$

with $l\epsilon\{1, \ldots, L\}$.

For detection techniques analysis will be assumed that the complex data signal s^l transmitted by the $l-th$ user has zero-mean and a variance of σ_l^2. The AWGN noise process n_p at any antenna array element p exhibits also zero-mean

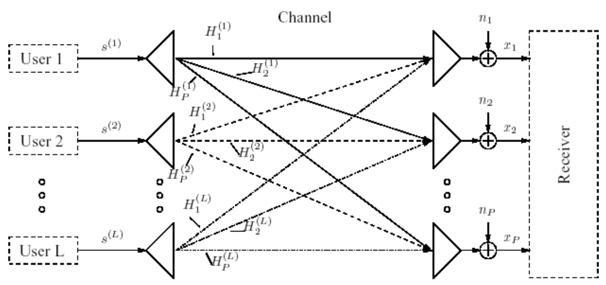

Fig. 1. SDMA MIMO channel scenario

and a variance of σ_n^2. The frequency domain channel transfer factors $\mathbf{H_P^l}$ of the different array elements p or users l are independent, stationary, complex Gaussian distributed processes with zero-mean and unit variance.

3. LINEAR DETECTION TECHNIQUES

The employment of linear detector is motivated by the observation that in the context of the optimum Maximum Likelihood detector to be discussed in next section a potentially excessive complexity of ML detector.

In a linear detector the different users transmitted signals are estimated with the aid of a linear combiner. These signal estimates would then be demodulated separately for each of the L users upon neglecting the residual interference caused by the remaining users in a specific users combiner output signal.

An estimate $\widehat{s}$ of the vector of transmitted signals s of the L simultaneous users is generated by linearly combining the signals received by the P different receiver antenna elements with the aid of the weight matrix $\mathbf{W}$, resulting in:

$$\widehat{s} = \mathbf{W^H x} \tag{4}$$

using previous equation, and considering the l user's associated vector component

$$\begin{aligned} \widehat{s}^l &= \mathbf{w}^(\mathbf{l})\mathbf{Hx} \\ &= \mathbf{w^{(l)H}}(\mathbf{Hs}+\mathbf{n}) \\ &= \mathbf{w^{(l)H}H^l s^l} + \mathbf{w^{(l)H}}\sum_{\substack{\mathbf{i=1}\\ \mathbf{i\neq l}}}^{\mathbf{L}} \mathbf{H^i s^i} + \mathbf{w^{(l)H} n} \end{aligned} \tag{5}$$

where the weight vector $\mathbf{w^{(l)}}$ is the $l-th$ column of the weight matrix $\mathbf{W}$. We observe from Equation 5 that the combiners output signal is constituted by three additive components. The first term

$$\widehat{s}_S^l = \mathbf{w^{(l)H}H^l s^l} \tag{6}$$

denotes the desired users associated contribution which variance is given by $\sigma_S^{(l)2} = \mathbf{w^{(l)H}R^l_{a,S}w^{(l)}}$ where $\mathbf{R^l_{a,S}} = \sigma_\mathbf{l}^\mathbf{2}\mathbf{H^{(l)}H^{(l)H}}$ is the autocorrelation matrix of the desired user.

The second term

$$\widehat{s}_I^l = \mathbf{w^{(l)H}}\sum_{\substack{\mathbf{i=1}\\ \mathbf{i\neq l}}}^{\mathbf{L}} \mathbf{H^i s^i} \tag{7}$$

denotes the interfering users residual contribution. The variance is given by $\sigma_I^{(l)2} = \mathbf{w^{(l)H}R^l_{a,I}w^{(l)}}$ where $\mathbf{R^l_{a,I}} = \sum_{\substack{\mathbf{i=1}\\ \mathbf{i\neq l}}}^{\mathbf{L}} \sigma_\mathbf{l}^\mathbf{2}\mathbf{H^{(l)}H^{(l)H}}$ is the autocorrelation matrix of the interfering user signals.

Finally, the last term

$$\widehat{s}_N^l = \mathbf{w^{(l)H}n} \tag{8}$$

is related to the AWGN which variance is $\sigma_N^{(l)2} = \mathbf{w^{(l)H}R^l_{a,N}w^{(l)}}$ where $\mathbf{R_{a,N}} = \sigma_\mathbf{n}^\mathbf{2}\mathbf{I}$ is the diagonal noise autocorrelation matrix .

The undesired signals auto-correlation matrix is related to the sum of the residual interference plus the AWGN expressed as $\mathbf{R^l_{a,I+N}} = \mathbf{R^l_{a,I}} + \mathbf{R_{a,N}}$.

Three different performance measures can be defined on the basis of the desired signals variance $\sigma_S^{(l)2}$, the interfering signals variance $\sigma_I^{(l)2}$ and the noise variance σ_N^2. These measures can be employed for characterizing the quality of the linear combiners output signal. These are the Signal-to-Interference plus- Noise Ratio (SINR) at the combiners output, defined as

$$SINR^{(l)} = \frac{\sigma_S^{(l)2}}{\sigma_I^{(l)2} + \sigma_N^2} \tag{9}$$

the Signal-to-Interference Ratio (SIR), defined as

$$SIR^{(l)} = \frac{\sigma_S^{(l)2}}{\sigma_I^{(l)2}} \tag{10}$$

and the Signal-to-Noise Ratio (SNR) given by

$$SNR^{(l)} = \frac{\sigma_S^{(l)2}}{\sigma_N^2} \tag{11}$$

3.1. Least-Squares Error detector

The Least-Squares (LS) error or Zero-Forcing (ZF) combiner attempts to recover the vector $s[n,k]$ of signals transmitted by the L different users in the $k-th$ subcarrier of the $n-th$ OFDM symbol period, regardless of the signal quality quantified in terms of the SNR at the reception antennas.

Assuming perfect knowledge of the channel transfer factor matrix H an estimate $\widehat{x}$ of the vector of signals received by the P different antenna elements in a specific subcarrier is given by $\widehat{x} = \mathbf{H}\widehat{\mathbf{s}}$. The estimation error in the received signal's domain can be expressed as

$$\begin{aligned}\Delta\widehat{x} &= x - \widehat{x} \\ &= x - \mathbf{H}\widehat{\mathbf{s}}\end{aligned}$$

The squared error is given as

$$\begin{aligned}\|\Delta\widehat{x}\|^2 &= \Delta\widehat{x}^H\Delta\widehat{x} \\ &= x^H x - 2\Re(\widehat{s}^H \mathbf{p_{LS}}) + \widehat{\mathbf{s}}^{\mathbf{H}}\mathbf{Q_{LS}}\widehat{\mathbf{s}} \quad (12)\end{aligned}$$

where $\mathbf{p_{LS}} = \mathbf{H^H x}$ is the cross-correlation vector and $\mathbf{Q_{LS}} = \mathbf{H^H H}$ is the auto-correlation matrix.

In order to determine the desired vector representing the estimated transmitted signals of the L users is obtained minimizing the squared error. In the optimum point of operation, associated with the weight matrix having the optimum weights, the conjugate gradient $\frac{\partial\|\delta\widehat{x}\|^2}{\partial\widehat{s}}$ is equal to zero. After some mathematical manipulations we obtain

$$\widehat{s}_{LS} = \mathbf{Q_{LS}^{-1} p_{LS}}$$

Substituting the values of the auto-correlation and cross-correlation matrix, the vector LS of estimated transmitted signals of the L simultaneous users can be written as

$$\widehat{s}_{LS} = \mathbf{P_{LS} x}$$

where the projection matrix PLS is $\mathbf{P_{LS}} = (\mathbf{H^H H})^{\mathbf{-1}}\mathbf{H^H}$. More specifically, the matrix $\mathbf{P_{LS}}$ projects the vector x of the P different antenna elements' received signals onto the column space of the channel matrix $\mathbf{H}$.

The average estimation Mean-Squared error(MSE) evaluated in the transmitted signal domain is given by

$$\begin{aligned}M\bar{S}E_{LS} &= \frac{1}{L}Trace(\mathbf{R_{\Delta\widehat{s}_{LS}}}) \\ &= \frac{1}{L}\sigma_n^2 Trace((\mathbf{H^H H})^{\mathbf{-1}})\end{aligned}$$

The $l-th$ user's associated minimum MSE is given as the $l-th$ diagonal element of the matrix $\mathbf{R_{\Delta\widehat{s}_{LS}}}$

$$\begin{aligned}MSE_{LS}^{(l)} &= \sigma_n^2 \mathbf{w_{LS}^{(l)H} w_{LS}^{(l)}} \\ &= \sigma_n^2((\mathbf{H^H H})^{\mathbf{-1}})_{[1,1]} \quad (13)\end{aligned}$$

3.2. Minimum Mean-Squares Error detector

In contrast to the LS combiner, the Minimum Mean-Square Error (MMSE) detectors associated MMSE combiner exploits the available statistical knowledge concerning the signals transmitted by the different users, as well as that related to the AWGN at the receiver antenna elements.

The cost-function employed directly reflects the quality of the combiner weights in the transmitted signals' domain. The vector $\Delta\widehat{s}$ of the L simultaneous users' estimation errors evaluated in the transmitted signals' domain can be defined as

$$\begin{aligned}\Delta\widehat{s} &= s - \widehat{s} \\ &= s - (\mathbf{W^H x})\end{aligned}$$

The estimation error's auto-correlation matrix $\mathbf{R_{\Delta\widehat{s}}}$ is given by

$$\begin{aligned}\mathbf{R_{\Delta\widehat{s}}} &= E\{\Delta\widehat{s}\Delta\widehat{s}^H\} \\ &= \mathbf{P} - \mathbf{R_c^H W} - \mathbf{W^H R_c} + \mathbf{W^H R_a W} \quad (14)\end{aligned}$$

where $\mathbf{R_c}$ is the cross-correlation matrix of the received and transmitted signals

$$\begin{aligned}\mathbf{R_c} &= E\{\mathbf{xs^H}\} \\ &= \mathbf{HP} \quad (15)\end{aligned}$$

The matrix $\mathbf{P}$ is the diagonal matrix of the different users' associated transmit powers or signal variances, given by $\mathbf{P} = \mathbf{Diag}(\sigma_\mathbf{1}^\mathbf{2}, \sigma_\mathbf{2}^\mathbf{2}, \ldots, \sigma_\mathbf{L}^\mathbf{2})$.

$\mathbf{R_a}$ is the auto-correlation matrix of the received signals

$$\begin{aligned}\mathbf{R_a} &= E\{\mathbf{xx^H}\} \\ &= \mathbf{HPH^H} + \sigma_\mathbf{n}^\mathbf{2}\mathbf{I} \\ &= \sum_{l=1}^{L}\sigma_l^2\mathbf{H^l H^{(l)H}} + \sigma_\mathbf{n}^\mathbf{2}\mathbf{I}\end{aligned}$$

The sum of the auto-correlation matrices $R_{a,S}$, $R_{a,I}$ and $R_{a,N}$ constitutes the auto-correlation matrix R_a.

$$\mathbf{R_a} = \mathbf{R_{a,S}^l} + \mathbf{R_{a,I+N}^l} \quad (16)$$

Determining the weight matrix on the basis of evaluating the gradient of the total-mean square estimation error $E\{\|\Delta\widehat{s}\|^2\}$ with respect to the different users total mean-square estimation error results in the standard form of the MMSE combiner, which is related to the right-inverse of the channel matrix H.

The total-mean square estimation error is given by

$$\begin{aligned}E\{\|\Delta\widehat{s}\|^2\} &= Trace(\mathbf{R_{\Delta\widehat{s}}}) \\ &= Trace(\mathbf{P}) - \mathbf{Trace}(\mathbf{R_c^H W}) - \\ &- Trace(\mathbf{W^H R_c}) + \mathbf{Trace}(\mathbf{W^H R_a W}) \quad (17)\end{aligned}$$

The matrix $\mathbf{W}$ of the optimum weigths can be determined minimizing $E\{\|\Delta\widehat{s}\|^2\}$. Applying derivates and mathematical manipulations, the optimum weights are

$$\begin{aligned}\mathbf{W_{MMSE}} &= \mathbf{R_a^{-1} R_c} \\ &= (\mathbf{HPH^H} + \sigma_\mathbf{n}^\mathbf{2}\mathbf{I})^{\mathbf{-1}}\mathbf{HP} \\ &= (\mathbf{HP_{SNR}H^H} + \sigma_\mathbf{n}^\mathbf{2}\mathbf{I})^{\mathbf{-1}}\mathbf{HP_{SNR}} \quad (18)\end{aligned}$$

where $\mathbf{P_{SNR}}$ is the diagonal matrix of the different users' associated SNRs at the receiver antennas $\mathbf{P_{SNR}} = \mathbf{Diag}(\mathbf{SNR^{(1)}}, \mathbf{SNR^{(2)}}, \ldots, \mathbf{SNR^{(L)}})$ in which the $l-th$ user SNR is given by $SNR^{(l)} = \frac{\sigma_l^2}{\sigma_n^2}$.

The autocorrelation matrix of the estimation error associated with the different users's transmitted signal is expressed as $\mathbf{R_{\Delta\hat{s}_{MMSE}}} = \mathbf{P} - \mathbf{R_c^H W_{MMSE}}$.

The average estimation Mean-Squared error(MSE) evaluated in the transmitted signal domain is given by

$$\bar{MSE}_{MMSE} = \frac{1}{L} Trace(\mathbf{R_{\Delta\hat{s}_{MMSE}}}) \tag{19}$$

The $l-th$ user's associated minimum MSE is given as the $l-th$ diagonal element of the matrix $\mathbf{R_{\Delta\hat{s}_{LS}}}$

$$\begin{aligned} MSE_{MMSE}^{(l)} &= \sigma_l^2(1 - \mathbf{H^{(l)H} R_a^{-1} H^{(l)}} \sigma_\mathbf{l}^\mathbf{2}) \\ &= \sigma_l^2(1 - \mathbf{H^{(l)H} w_{MMSE}^{(l)}}) \end{aligned} \tag{20}$$

3.3. Minimum Variance (MV) Combining

In LS combiners philosophy was to fully recover the original signal transmitted without relying on any information concerning the AWGN process, which corrupts the signal received by the different antenna elements. By contrast, the philosophy of the MMSE combiner was to strike a balance between the recovery of the signals transmitted and the suppression of the AWGN.

An attractive compromise is constituted by the MV approach, which aims for recovering the original signals transmitted while ensuring a partial suppression of the AWGN based on the knowledge of its statistics. In other words, the $l-th$ users associated weight vector $w^{(l)}$ has to be adjusted such, that its transfer factor assumes a specific predefined value of $g = w^{(l)H} H^{(l)}$.

Usually the MV combiner is derived by minimizing a Lagrangian cost-function, which incorporates both a constraint on the desired users effective transfer factor, as well as the undesired signals variance. However the different combiners associated weight vectors, namely those of the MMSE, MV and Maximum SINR combiners, differ only by a scalar multiplier. Hence, the MV-related weight vector $w_{MV}^{(l)}$ of the $l-th$ user can be directly inferred from the MMSE-related weight vector $w_{MMSE}^{(l)}$ by simple normalization according to

$$w_{MV}^{(l)} = \frac{g}{w_{MMSE}^{(l)H} \mathbf{H^l}} w_{MMSE}^{(l)} \tag{21}$$

4. NON-LINEAR DETECTION

In linear detection, the strategy is first to provide linear estimates of the different users transmitted signals and then to perform the non-linear classification- or demodulation separately for each user. This philosophy was based on the assumption that the different users associated linear combiner output signals are corrupted only by the residual AWGN. In fact the linear combiners output signals also contain residual interference, which is not Gaussian distributed and hence represents an important source of further information.

Instead of sequentially performing the operations of linear combining and classification or demodulation as in the linear detectors , a more effective strategy is to embed the demodulation into the process of linear combining, which is known from the family of classic channel equalizers as decision-feedback. As a result, the residual multi-user interference observed at the classifiers inputs is reduced. Hence, the classifiers accuracy due to neglecting the residual interference is less impaired.

Two of the most prominent multi-user detection techniques known from CDMA communications, which incorporate these ideas are the SIC and PIC detection techniques. These techniques are also applicable in the context of communicating over flat-fading channels as observed for example on an OFDM subcarrier basis.

4.1. SIC Detection

The philosophy of the Successive Interference Cancellation (SIC) assisted detector is motivated by two observations. First of all, we note that for a specific sub-carrier the MSE and SINR at the output of the LS or MMSE combiner might substantially differ for the different users, depending on their spatial signatures. Secondly, upon increasing the MIMO systems diversity order the MSE performance of the LS or MMSE combiner and correspondingly the systems BER performance is improved as a consequence of assigning a higher grade of diversity to mitigate the effects of fading.

Hence, an attractive strategy, which has recently drawn wide interests is to detect only the specific user having the highest SINR, SIR or SNR in each iteration at the output of the LS or MMSE combiner. Having detected this users signal, the corresponding remodulated signal is subtracted from the composite signal received by the different antenna elements. Furthermore, the channel transfer factor matrix and the SNR matrix formulated in the context of the MMSE combiner and its left-inverse related form are updated accordingly.

In Figure 4 the SIC detector block diagram is shown. During the first iteration the signals x_p, with $p = 1, \ldots, P$ received by the different antenna elements are directly fed into the selective linear combiner, where we have $x^{[1]} = x$ at the detection stage or iteration of $i = 1$. The task of the selective linear combiner is to identify the most dominant remaining user in terms of its $SINR$ at the combiner output from the set of $(L - i + 1)$ remaining users during the $i - th$ detection stage and to provide its signal estimate $\hat{s}^{(l[i])[i]}$ at the combiner's output. The selection of the most dominant user can be expressed as

$$l^{[i]} = arg\,max(SNR^{(l)[i]}) \tag{22}$$

Under the assumption that the $l^{[i]}-th$ user has been found to be the most dominant one among the $L^{[i]}$ remaining users at the $i-th$ detection stage, the detect user's transmitted signal is

$$\hat{s}^{(l[i])[i]} = \mathbf{w^{(l[i])[i]H}x^{[i]}} \tag{23}$$

The selected $l^{[i]}-th$ user's linear signal estimate $\hat{s}^{(l[i])[i]}$ is then classified- or demodulated according to

$$\check{s}^{(l[i])[i]} =_{\check{s}/\sigma_i \epsilon M_c}^{arg\,min} \left| \frac{1}{H_{eff}^{(l[i])[i]}} \hat{s}^{(l[i])[i]} - \check{s} \right|^2 \tag{24}$$

yielding the amplified constellation point $\check{s}^{(l[i])[i]}$ that is most likely to have been transmitted by the $l^{[i]}-th$user.

Now the corresponding modulated signal can be regenerated. The influence of the $l^{[i]}-th$ user's modulated signal is then removed from the vector $x^{[1]}$ of signals received by the different antenna elements with the aid of the SIC module. This cancellation operation is described by

$$x^{[i+1]} = x^{[i]} - \mathbf{H^{(l^{[}i])}\check{s}^{(l[i])[i]}} \tag{25}$$

The influence of the $l^{[i]}-th$ users associated channel transfer factor vector $H^{(l[i])}$ is eliminated from the auto-correlation matrix $R_a^{[i]H}$, yielding the reduced-dimensional matrix

$$R_a^{[i]H} \rightarrow R_a^{[i+1]H} \tag{26}$$

which new size is $(L^{[i]}-1) \times (L^{[i]}-1)$.

The first iteration $(i=1)$ is deemed to have been completed, when the decontaminated signal appears at the output of the SIC stage. Hence, beginning with the second SIC iteration the selective linear combiner's input, namely the decontaminated vector $x^{[i]}$ of signals received by the different antenna elements, which contains only the influence of the $(L-i+1)$ remaining users, is constituted by the output of the SIC module, provided that correct symbol decisions were conducted in the previous detection stages.

The role of the switches is to indicate that at the first detection stage the SIC is directly fed with the signals received by the different array elements, while during the remaining iterations of $i = 2, ..., L$ with the partially decontaminated composite signal of the remaining $(L-i+1)$ users.

M-SIC The standard SIC detectors performance is impaired as a result of the error-propagation occurring between the different consecutive detection stages. A viable strategy of reducing the error propagation effects is to track from each detection stage not only the single most likely symbol decision, but an increased number of $M \leq M_c$ most likely tentative symbol decisions, where M_c denotes the number of constellation points associated with a specific modulation scheme.

To provide an example, for $M = 2$ in the first detection stage we have a total of $M = 2$ possible symbol decisions, while in the second detection stage $M^2 = 4$ tentative symbol decisions and correspondingly, in the $i-th$ detection stage we encounter M^i possible tentative symbol decisions. Associated with each tentative symbol decision there is a specific updated vector of signals, generated by canceling the effects of the most dominant $L-i+1$ number of users from the P-dimensional vector of signals received by the P number of different antenna elements. Hence, in the following detection stage the MMSE combining has to be performed separately for the different updated P-dimensional vectors of received signals. Correspondingly, the number of parallel tentative symbol decisions to be tracked is increased by the factor of M compared to that of the current detection stage. This process can conveniently be portrayed with the aid of a tree-structure, as shown at Figure 2, where M = 2 was used. Specifically, each detection node represents an updated P-dimensional vector of signals received by the P different antenna elements, while the branches are associated with the various tentative symbol decisions at the $i = 1, \ldots, L$ detection stages. Note that the first detection node at the top of the figure is associated with the original P-dimensional vector of signals received by the different antenna elements. In the final detection stage, after the subtraction of the least dominant user's estimated P-dimensional signal contribution, a decision must be made concerning which specific combination of L number of symbols - represented by the branches connecting the different detection nodes - has most likely been transmitted by the L different users in the specific subcarrier considered. A suitable criterion for performing this decision is given by the Euclidean distance between the original P-dimensional vector of signals received by the P different antenna elements and the estimated P-dimensional vector of received signals based on the tentative symbol decisions and upon taking into account the effects of the channel.

The performance improvement potentially observed for the M-SIC scheme compared to the standard SIC arrangement is achieved at the cost of a significantly increased computational complexity. This is since the number of parallel tentative symbol decisions associated with a specific detection stage is a factor of M higher than that of the previous detection stage, and hence in the last detection stage we potentially have to consider M^L number of different tentative symbol decisions.

Partial M-SIC

A viable approach of further reducing the associated computational complexity is motivated by the observation that for sufficiently high SNRs the standard SIC detectors performance is predetermined by the bit- or symbol-error probabilities incurred during the first detection stage. This is, because if the most dominant users associated symbol decision is erroneous, its effects potentially propagate to all other users decisions conducted in the following detection

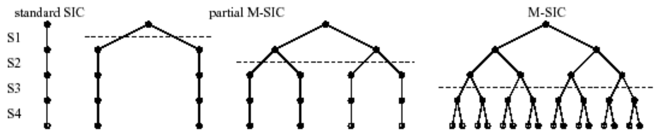

Fig. 2. a) Standard SIC b) Partial M-SIC c)M-SIC

stages.

The symbol error probability specifically of the first detection stage should be as low as possible, while the tentative symbol decisions carried out at later detection stages become automatically more reliable as a result of the systems increased diversity order due to removing the previously detected users. Hence, our suggestion is to retain $M > 1$ number of tentative symbol decisions at each detection node, characterized by its associated updated P-dimensional vector of received signals only up to the specific $L_{pM-SIC}-th$ stage in the detection process.

4.2. PIC Detection

PIC detector's structure is shown in Figure 6. From the second section, the specific structure of the vector $\mathbf{x}$ of signals received by the different antenna elements can be written

$$\begin{aligned} \mathbf{x} &= \mathbf{Hs} + \mathbf{n} \\ &= \mathbf{H^{(l)}s^{(l)}} + \sum_{\substack{\mathbf{i=1} \\ \mathbf{i \neq l}}}^{\mathbf{L}} \mathbf{H^{(i)}s^{(i)}} + \mathbf{n} \end{aligned} \quad (27)$$

Specifically, from the component representation given by the last equation we observe that the array output vector x is composed of the $l-th$ user's signal contribution vector and the $L-1$ interfering users' signal contribution vectors plus the AWGN vector. Hence, if initial estimates $\check{s}^{(i)}$ with $i\epsilon\{1,\ldots,L\}$ of the interfering users' transmitted signals would be available, a noisy estimate $\widehat{x}^{(l)}$ of the $l-th$ user's signal contribution could be obtained upon removing the $L-1$ interfering users' estimated signal contributions given by $\mathbf{H^{(i)}\check{s}^{(i)}}$ with $i\epsilon\{1,\ldots,L\}$ from the vector x of signals received by the different antenna elements. An estimate $\widehat{s}^{(l)}$ of the $l-th$ user's transmitted signal could then be inferred by linear antenna diversity combining. The PIC detector operation can be resumed in the following explanation

First-Stage - MMSE Detection

- Combining

  During the first PIC iteration each user is detected by means of the MMSE combiner.

- Classification/Demodulation

  Then the linear combiner's output vector $\widehat{s}^{[1]} = \widehat{s}_{MMSE}$ is demodulated resulting in the vector $\check{s}^{[1]}$ of symbols that are most likely to have been transmitted by the L different users.

$i-th$ **Stage: PIC Detection**

- Parallel Interference Cancellation

  During the $i-th$ PIC iteration where $i \geq 2$ a potentially improved estimate $\widehat{s}_{PIC}^{(l)[i]}$ of the complex symbol $s^{(l)}$ transmitted by the $l-th$ user is obtained upon subtracting in a first step the $L-1$ interfering users' estimated signal contributions, from the original vector x of signals received by the different antenna elements, which can be expressed as

$$\widehat{x}_{PIC}^{(l)[i]} = \mathbf{x} - \sum_{\substack{\mathbf{j=1} \\ \mathbf{j \neq l}}}^{\mathbf{L}} \mathbf{H^{(j)}\check{s}^{(j)[i-1]}} \quad (28)$$

- Combining

  The final task is hence to extract an estimate $\widehat{s}_{PIC}^{(l)[i]}$ of the signal $s^{(l)}$ transmitted by the $l-th$ user from the $l-th$ user's PIC-related array output vector $\widehat{x}_{PIC}^{(l)[i]}$. This results in the weight vector $w_{MMSE}^{(l)[i]}$ given by

$$w_{MMSE}^{(l)[i]} = \frac{\mathbf{H^{l}}}{\|\mathbf{H^{l}}\|^2 + (1/SNR^{(l)}} \quad (29)$$

  With the aid of this weight vector an estimate $\widehat{s}^{(l)[i]} = \widehat{s}_{PIC}^{(l)[i]}$ of the $l-th$ user's transmitted signal $s^{(l)}$ can then be extracted from the vector $\widehat{x}_{PIC}^{(l)[i]}$ seen at the output of the linear MMSE combiner $\widehat{s}^{(l)[i]} = w_{MMSE}^{(l)[i]H}\widehat{x}_{PIC}^{(l)[i]}$.

- Classification/Demodulation

  The above PIC and MMSE-combining steps are again followed by the classification, demodulation stage which obeys

$$\check{s}^{(l[i])[i]} =_{\check{s}/\sigma_i\epsilon M_c}^{arg\ min} \left| \frac{1}{H_{eff}^{(l[i])[i]}} \widehat{s}^{(l[i])[i]} - \check{s} \right|^2 \quad (30)$$

  where the l-th user's effective channel transfer factor $H_{eff}^{(l)[i]}$ is given by $H_{eff}^{(l)[i]} = w_{MMSE}^{(l)[i]}\mathbf{H^{(l)}}$

  In other words, the classification/demodulation operation delivers the symbol $\check{s}^{(l)[i]}$ that is most likely to have been transmitted by the $l-thuser$.

  The $i-th$ PIC iteration described above potentially has to be performed for all the different SDMA users namely, for $l = 1,\ldots,L$

5. MAXIMUM LIKELIHOOD (ML) DETECTION

Maximum Likelihood (ML) detector is optimum from a statistical point of view. An associated disadvantage is its potentially excessive computational complexity, which results from the strategy of jointly detecting the L different users. This implies assessing the M_c^L possible combinations of symbols transmitted by the L different users by evaluating their Euclidean distance from the received signal, upon taking into account the effects of the channel.

The definition of the vector x of signals received by the P different antenna elements is $\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n}$.

We observe that $x\ CN(\mathbf{H}\mathbf{s}, \mathbf{R_n})$, namely x is a sample of an $L-dimensional$ multi-variate complex Gaussian distribution, having a vector of mean values given by $\mathbf{H}\mathbf{s}$ and a covariance matrix $\mathbf{R_n} = \sigma_\mathbf{n}^\mathbf{2}\mathbf{I}$ implying that the different noise contributions are assumed to be uncorrelated.

In simple verbal terms the ML detector finds the specific $L - dimensional$ vector of $M_c - ary$ symbols, which is most likely to have been transmitted. In more formal terms ML detection is based on the idea of maximizing the a posteriori probability $P(\check{s}|\mathbf{x}, \mathbf{H})$.

This maximization procedure can be expressed as:

$$\check{s}_{ML} = ^{arg\ max}_{\check{s} \epsilon M^L} P(\check{s}|\mathbf{x}, \mathbf{H}) \quad (31)$$

As observed in this equation , determining the ML symbol estimate requires comparing the Euclidean distance between the vector x of signals actually received by the different antenna elements and the vector H of signals, which would be received in the absence of AWGN, for all the different vectors of symbol combinations contained in the set M^L. The complexity associated with this evaluation might potentially be excessive, depending on the M_c^L number of vectors contained in the trial-set M^L.

6. COMPARISON OF DIFFERENT DETECTION TECHNIQUES

6.1. BER performance

In Figure 3 the different detectors SDMA-OFDM related BER performance are compared for an uncoded scenario. As expected, the best performance is exhibited by the most complex ML detector, closely followed by the M-SIC scheme, where M = 2. By contrast, a significant BER degradation is observed for the standard SIC scheme potentially as a result of the effects of error propagation through the different detection stages. The second worst performance is exhibited by the PIC arrangement, while a further degradation by about 1.25dB is incurred upon employing the rudimentary MMSE detection. Specifically, the PIC detectors performance was impaired by the lower-power users, potentially propagating errors to those users, which benefited from a relatively high SNR at the first-stage combiner output.

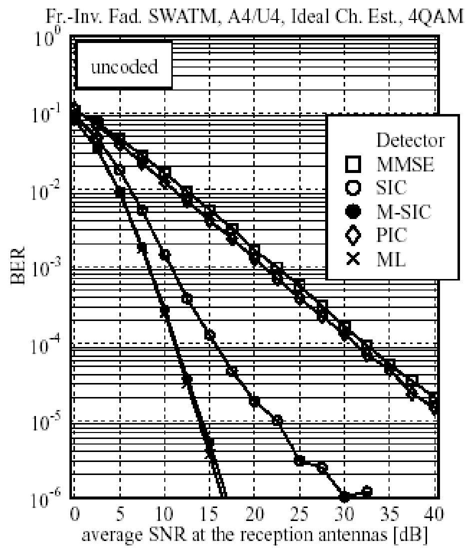

Fig. 3. BER performance

	MMSE	std. SIC	M-SIC	PIC	ML
C^{C*C}	133.33	344.58	454.58	197.33	4608
C^{C+C}	135.33	335.33	467.33	233.33	5632
$C^{R\leq.R}$	16	25	97	32	256

Table 1. Computational Complexity of the different detection schemes

6.2. Complexity

Having compared the various detection techniques, namely MMSE, SIC, M-SIC, PIC and ML in terms of the associated system's BER performance, in this section we will compare them with respect to their computational complexity.

Table 1 shows the computational complexity of the different detection schemes, namely MMSE, standard SIC, M-SIC, PIC and ML detection quantified in terms of the number of complex multiplications and additions C^{C*C}, C^{C+C} as well as the number of real-valued comparisons $C^{R\leq.R}$ for a scenario of $L = P = 4$ simultaneous users and reception antennas; specifically for M-SIC the number of tentative symbol decisions per detection node was equal to $M = 2$, while in all scenarios $M_c = 4$ constellation points were assumed, which is for example the case in conjunction with $4 - QAM$ modulation.

As expected, the lowest computational complexity expressed in terms of the number of multiplications is exhibited by the MMSE detector, followed by PIC, standard SIC and M-SIC, while the highest complexity is exhibited by the optimum ML detector.

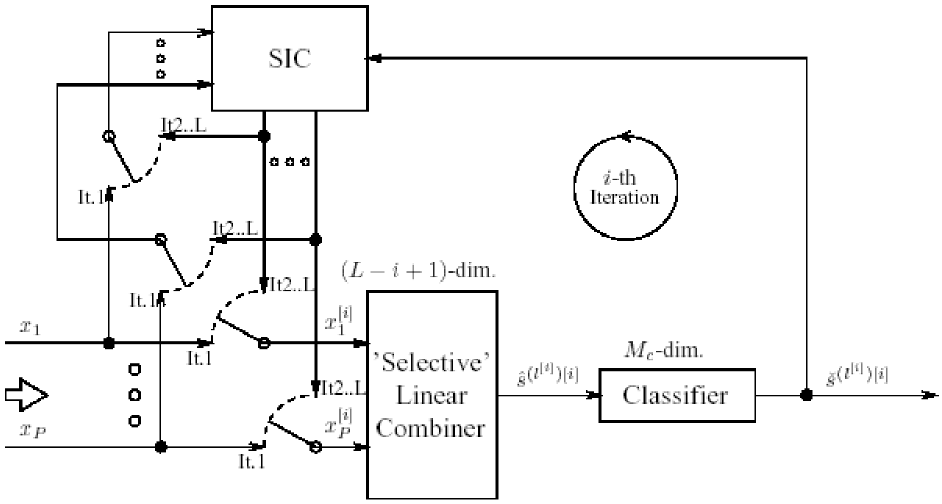

Fig. 4. SIC detector

Description	Instruction
Initialization	$\mathbf{x}^{[1]} = \mathbf{x}$; $\mathbf{R}_{\overline{a}}^{[1]H} = \mathbf{P}\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}$; $\mathbf{R}_c^{[1]H} = \mathbf{P}\mathbf{H}^H$; $L^{[1]} = L$
i-th iteration:	
Weight calc.	$\mathbf{R}_{\overline{a}}^{[i]H}\mathbf{W}_{\text{MMSE}}^{[i]H} = \mathbf{R}_c^{[i]H} \quad \Longleftrightarrow \quad \mathbf{W}_{\text{MMSE}}^{[i]H} = \mathbf{R}_{\overline{a}}^{[i]H-1}\mathbf{R}_c^{[i]H}$
Selection	$\text{SNR}^{(l)[i]} = \frac{\mathbf{w}^{(l)[i]H}\mathbf{R}_{a,S}^{(l)}\mathbf{w}^{(l)[i]}}{\mathbf{w}^{(l)[i]H}\mathbf{R}_{a,N}\mathbf{w}^{(l)[i]}}$, $l \in \mathcal{L}^{[i]}$, $l^{[i]} = \underset{l \in \mathcal{L}^{[i]}}{argmax}(\text{SNR}^{(l)[i]})$
Combining	$\hat{\mathbf{s}}^{(l^{[i]})[i]} = \mathbf{w}^{(l^{[i]})[i]H}\mathbf{x}^{[i]}$
Demodulation	$\check{\mathbf{s}}^{(l^{[i]})[i]} = arg \underset{\mathbf{s}/\sigma_{l^{[i]}} \in \mathcal{M}_c}{min} \left\| \frac{1}{H_{\text{eff}}^{(l^{[i]})[i]}}\hat{\mathbf{s}}^{(l^{[i]})[i]} - \check{\mathbf{s}} \right\|^2$, $H_{\text{eff}}^{(l^{[i]})[i]} = \mathbf{w}^{(l^{[i]})[i]H}\mathbf{H}^{(l^{[i]})}$
Updating	$\mathbf{x}^{[i+1]} = \mathbf{x}^{[i]} - \mathbf{H}^{(l^{[i]})}\check{\mathbf{s}}^{(l^{[i]})[i]}$
	$\mathbf{R}_{\overline{a}}^{[i]H} \longrightarrow \mathbf{R}_{\overline{a}}^{[i+1]H} \in \mathbb{C}^{(L^{[i]}-1)\times(L^{[i]}-1)}$
	$\mathbf{R}_c^{[i]H} \longrightarrow \mathbf{R}_c^{[i+1]H} \in \mathbb{C}^{(L^{[i]}-1)\times P}$
	$L^{[i+1]} = L^{[i]} - 1$
Return	Start $(i+1)$-th iteration

Fig. 5. Summary of SIC detector

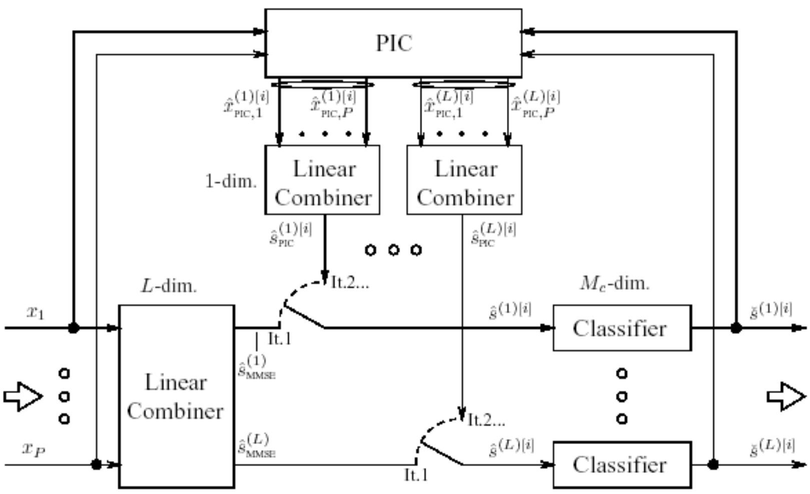

Fig. 6. PIC detector

description	instruction
First-Stage - MMSE Det.	
Calc. MMSE weight matrix	$\mathbf{W}_{\text{MMSE}}^{[1]} = \mathbf{H}\mathbf{P}_{\text{SNR}}(\mathbf{H}^H\mathbf{H}\mathbf{P}_{\text{SNR}} + \mathbf{I})^{-1} \in \mathbb{C}^{P\times L}$
Detection	$\hat{\mathrm{s}}_{\text{MMSE}} = \mathbf{W}_{\text{MMSE}}^{[1]H}\mathbf{x} \in \mathbb{C}^{L\times 1}, \quad \hat{s}^{(l)[1]} = \hat{s}_{\text{MMSE}}, \; l = 1, \ldots, L$
Demodulation, $l = 1, \ldots, L$	$\check{s}^{(l)[1]} = arg \min_{\check{s}/\sigma_l \in \mathcal{M}_c} \left\| \frac{1}{H_{\text{eff}}^{(l)[1]}} \hat{s}^{(l)[1]} - \check{s} \right\|^2, H_{\text{eff}}^{(l)[1]} = \mathrm{w}_{\text{MMSE}}^{(l)[1]}\mathbf{H}^{(l)}$
i-th Stage - PIC ($l = 1, .., L$)	
Subtraction	$\hat{\mathrm{x}}_{\text{PIC}}^{(l)[i]} = \mathbf{x} - \sum_{j=1, j\neq l}^{L} \mathbf{H}^{(j)}\check{s}^{(j)[i-1]} \in \mathbb{C}^{P\times 1}$
Calc. MMSE weight vectors	$\mathrm{w}_{\text{MMSE}}^{(l)[i]} = \frac{\mathbf{H}^{(l)}}{\lVert\mathbf{H}^{(l)}\rVert_2^2 + \frac{1}{\text{SNR}^{(l)}}} \in \mathbb{C}^{P\times 1}$
Detection	$\hat{s}_{\text{PIC}}^{(l)[i]} = \mathrm{w}_{\text{MMSE}}^{(l)[i]H}\hat{\mathrm{x}}_{\text{PIC}}^{(l)[i]}, \quad \hat{s}^{(l)[i]} = \hat{s}_{\text{PIC}}^{(l)[i]}$
Demodulation	$\check{s}^{(l)[i]} = arg \min_{\check{s}/\sigma_l \in \mathcal{M}_c} \left\| \frac{1}{H_{\text{eff}}^{(l)[i]}} \hat{s}^{(l)[i]} - \check{s} \right\|^2, H_{\text{eff}}^{(l)[i]} = \mathrm{w}_{\text{MMSE}}^{(l)[i]H}\mathbf{H}^{(l)}$

Fig. 7. Summary of PIC detector

7. REFERENCES

[1] L. Hanzo, M. Munster, B. J. Choi, and T. Keller, *OFDM and MC-CDMA for broadband Multi-User Communication , WLANs and Broadcasting*, John Wiley Sons, LTD, 2003.

[2] S. Verdu, *Multiuser Detection*, Cambridge University Press, 1998.

8. HOMEWORK

In few words explains the advantages of $M-SIC$ detection over standard SIC detection.