Space Time code for MIMO Systems

S72.333-Postgraduate Course

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Outline

- Introduction
- Space Time codes
- Alamouti codes
- Space-Time block codes
- Space-time Trellis Codes
- Differential Space-Time block codes
- Space-Time for OFDM systems
- Conclusions
- References

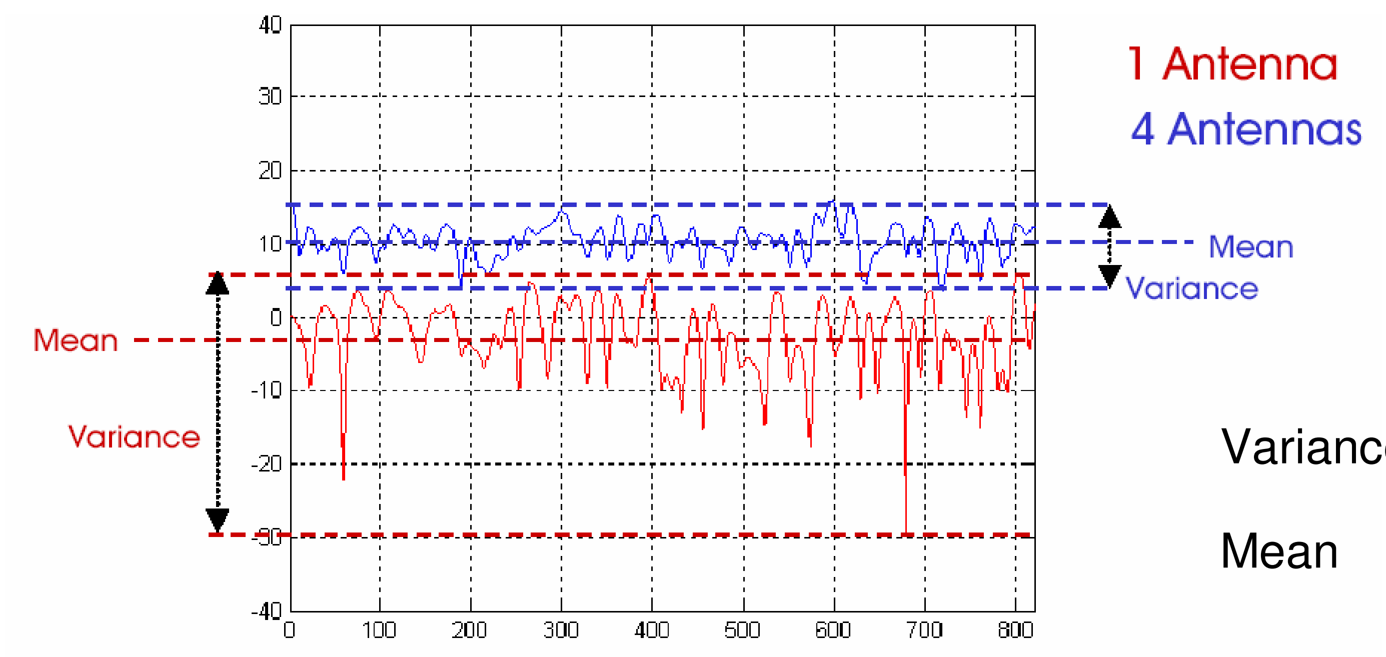
Introduction

- Depending on surrounding environment, a transmitted radio signal propagates through several different paths→ multipath propagation.
- The signal received by the receiver antenna consists of the superposition of various multipaths.
- The attenuation coefficients corresponding to different paths are assumed to be independent and identically distributed →
  - The path gain can be modeled as a complex Gaussian random variable → Rayleigh fading channel

Diversity Gain

- Signal power in a wireless system fluctuates. When this signal power drops significantly, the channel is said to be in fade.
- Diversity is used in wireless channels to combat the fading. **Receive diversity** and **transmit diversity** mitigate fading and **significantly improve link quality**.
- The receive antennas see independently faded versions of the same signals. The receiver combines these signals so that the resultant signal exhibits considerably reduced amplitude fading.
- Diversity order M_R x M_T
- MIMO turns multipath propagation into a benefit for the user

Diversity gain



Variance ⟷ Diversity

Mean ⟷ Array Gain

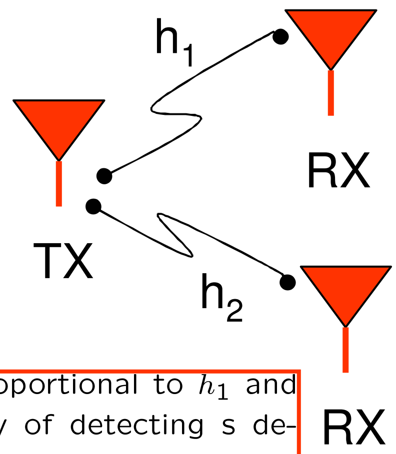
An introductory example

One transmit antenna and two receive antenna

The received signal:

$$y_1 = h_1 s + n_1$$
$$y_2 = h_2 s + n_2$$

S- Transmitted signal



To recover s

$$\begin{aligned} \hat{s} &= w_1 * y_1 + w_2 * y_2 \\ &= (w_1^* h_1 + w_2^* h_2) + w_1^* n_1 + w_2^* n_2 \end{aligned}$$

$$SNR = \frac{|w_1^* h_1 + w_2^* h_2|^2}{(|w_1|^2 + |w_2|^2)\sigma^2} E[|s|^2]$$

The resulting SNR is proportional to $(|h_1|^2 + |h_2|^2)$ If the fading is Rayleigh, then $(|h_1|^2 + |h_2|^2)$ is χ distributed, and we can show that the error probability of detecting s decay as SNR_a^{-2} in high SNR values.

w_1 and w_2 are chosen proportional to h_1 and h_2. The error probability of detecting s decay as SNR^{-2}. In single antenna case, the error probability of detecting s decay as SNR^{-1}. The diversity order of a system is the slope of the BER curved plotted versus the average SNR.

Diversity Gain=2

An introductory example

- Two transmit antenna and one receive antenna

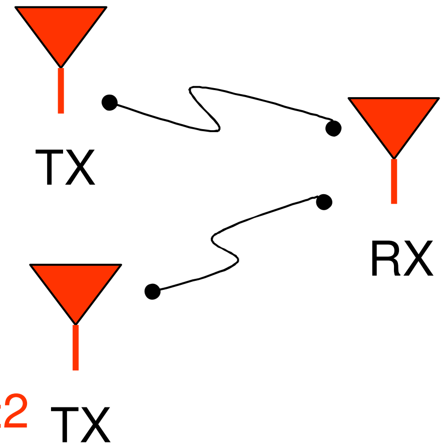
The symbol s is pre-weighted with w_1 and w_2
The received sample

$$y_1 = h_1 w_1 s + h_2 w_2 s + n$$

$$SNR = \frac{|h_1 w_1 + h_2 w_2|^2}{\sigma^2} E[|s|^2]$$

If w_1 and w_2 are fixed , the SNR has the same statistical distribution as $|h_1|^2$ (or $|h_2|^2$).

Diversity gain<2



- Channel Unknown: If the weights are not allowed to depend on h_1 and h_2 it is impossible to achieve diversity of order 2.
- Channel known : The error probability of detecting s decay as SNR^{-2}.

An introductory example

Using two time intervals

	A1	A2
t	S	
t+1		S

If two time intervals for the transmission is allowed.

Time=t - antenna 1 is used

Time=t+1 - antenna 2 is used

The RX signal

$$y_1 = h_1 s + n_1$$
$$y_2 = h_2 s + n_2$$

Diversity gain equal to 2 is achieved

Equal to 1x2 system
Diversity gain=2

Data rate is reduced !!

An introductory example

- Without channel knowledge at the transmitter, diversity can not be achieved.
- Using more than one time interval for the transmission, diversity gain is achieved
- Transmit diversity is easy to achieve if a sacrifice in information rate is acceptable.
- Space Time coding is concerned with
  - **Maximize the transmitted information rate**
  - **Minimize the error probability**

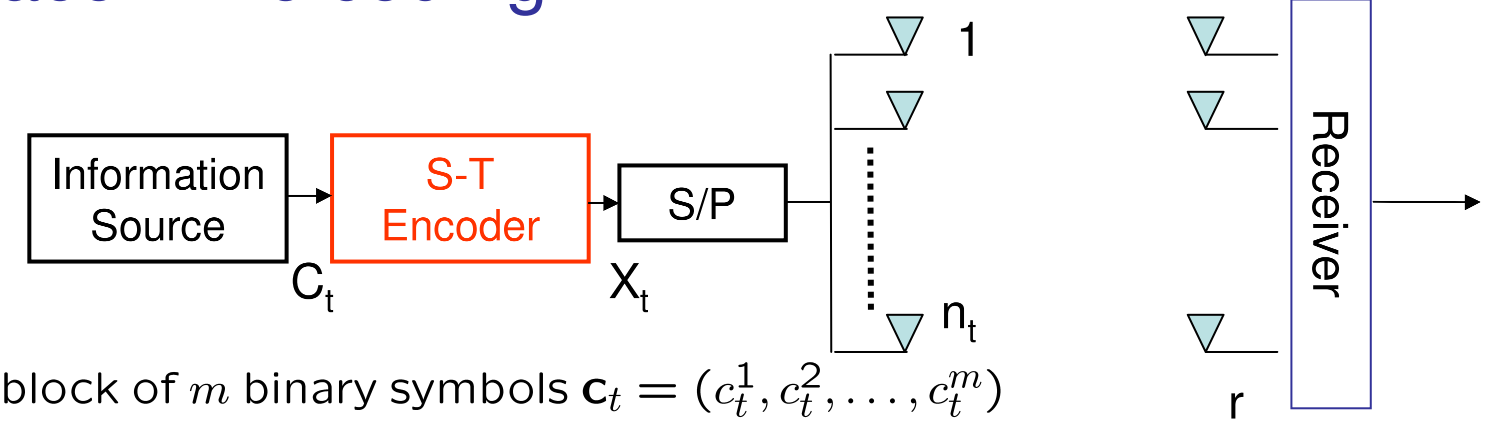
Transmit diversity

- Channel information at the transmitter
  - Beamforming methods
- Channel unknown at the transmitter
  - Space time coding:

Coding techniques designed for multiple antenna transmission.

Coding is performed by adding properly designed redundancy in both spatial and temporal domains which introduces correlation into the transmitted signal.

Space Time coding



A block of m binary symbols $\mathbf{c}_t = (c_t^1, c_t^2, \ldots, c_t^m)$ is fed into the ST encoder.

The ST encoder maps the block of m binary symbols into n_t modulation symbols from a signal set of $M = 2^m$ points $\mathbf{x}_t = (x_t^1, x_t^2, \ldots, x_t^{nt})$

The n_t parallel outputs are simultaneously transmitted by the different antennas.

Space Time coding

The channel matrix can be written as

$$H = \begin{bmatrix} h^t_{1,1} & h^t_{1,2} & \dots & h^t_{1,nt} \\ h_{2,1} & h_{2,2} & \dots & h^t_{2,nt} \\ \vdots & \vdots & \ddots & \vdots \\ h_{nr,1} & h_{nr,2} & \dots & h^t_{nr,nt} \end{bmatrix}$$

The received signal at antenna j

$$r^j_t = \sum_{i=1}^{nt} h^t_{j,i} x^i_t + n^j_t$$

The received signal vector

$\mathbf{r}_t = (r^1_t, r^2_t, \dots, r^{nr}_t)$

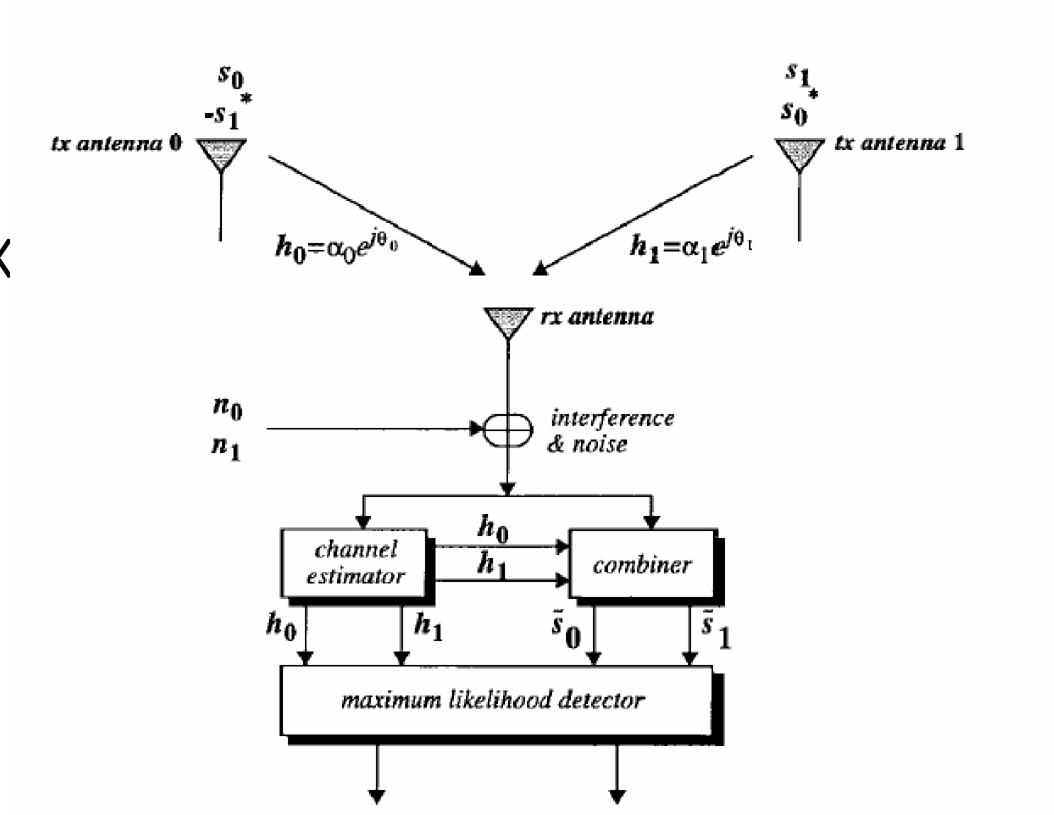
$\mathbf{r}_t = \mathbf{H}_t \mathbf{x}_t + \mathbf{n}_t$

The decision metric is computed based on the Squered Euclidean distance between the hypothesized received sequence and the actual received sequence

$$\sum_t \sum_{j=1}^{nr} \left| r^j_t - \sum_{i=1}^{nt} h^t_{j,i} x^i_t \right|^2$$

Alamouti code

- 2 by 1 orthogonal space time block code
- 2 TX antenna – 1 RX antenna



Alamouti code

Encoding and Transmission Sequence

- At a given symbol period, two signals are simultaneously transmitted from the two antennas. The signal transmitted from antenna zero is denoted by s_0

and from antenna one by s_1. During the next symbol period signal $-s_1^*$ is transmitted from antenna zero, and s_0^* signal is transmitted from antenna one where is the complex conjugate operation.

Space

Time

	A0	A1
t	S_0	S_1
t+T	$-S_1^*$	S_0^*

Alamouti code

Encoding and Transmission Sequence

- Assuming that fading is constant across two consecutive symbols

$$h_0(t) = h_0(t+T) = \alpha_0 e^{j\theta_0}$$
$$h_1(t) = h_1(t+T) = \alpha_1 e^{j\theta_1}$$

The received signal

$$\begin{aligned} r_0 &= r(t) = h_0 s_0 + h_1 s_1 + n_0 \\ r_1 &= r(t+T) = -h_0 s_1^* + h_1 s_0^* + n_1 \end{aligned}$$

Alamouti code

- *The Combining Scheme*

$$\hat{s}_0 = h_0^* r_0 + h_1 r_1^*$$
$$\hat{s}_1 = h_1^* r_0 - h_0 r_1^*$$

- *The Maximum Likelihood Decision Rule*

The decision statistics

$$\tilde{s}_0 = (\alpha_0^2 + \alpha_1^2) s_0 + h_0^* n_0 + h_1 n_1^*$$
$$\tilde{s}_1 = (\alpha_0^2 + \alpha_1^2) s_1 - h_0 n_1^* + h_1^* n_0$$

These combined signals are equivalent to that obtained from two branch MRRC.

The ML estimates of the transmitted symbols

$$\hat{s}_0 = arg \min_{\hat{s}_0 \epsilon S} d^2(\tilde{s}_0, \hat{s}_0)$$
$$\hat{s}_1 = arg \min_{\hat{s}_1 \epsilon S} d^2(\tilde{s}_1, \hat{s}_1)$$

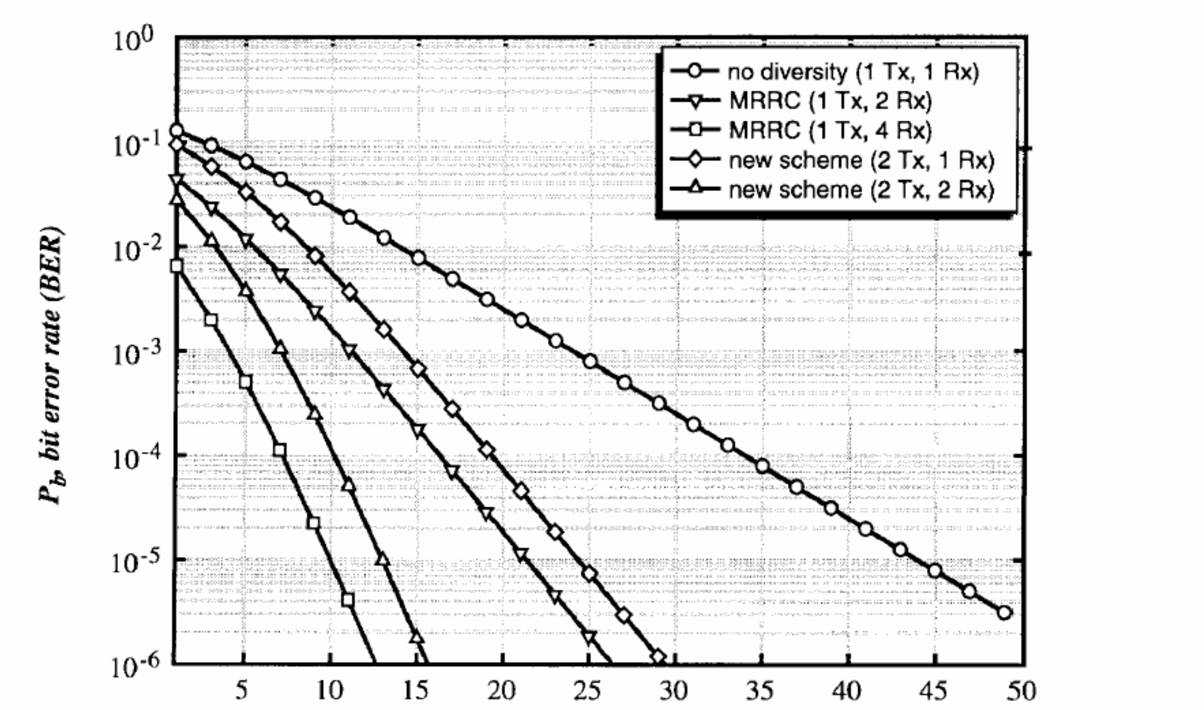
Diversity gain is equal to two branch MMRC !!

Properties of Alamouti code

- *Unitary*
  - *The product of its transmission matrix with its Hermitian transpose is equal to the 2 x 2 identity matrix.*
- *Full-rate complex code*
  - *Is the only complex S-T block code with a code rate of unity.*
- *Linearity*
  - *The Alamouti code is linear in the transmitted symbols.*
- *Optimality of capacity*
  - *For 2 transmit antennas and a single receive antenna, the Alamouti code is the only optimal S-T block code in terms of capacity*

Alamouti code- Performance

- Coherent BPSK with MRRC and two-branch transmit diversity in Rayleigh fading.



From: ALAMOUTI, Simple transmit diversity technique for wireless communications

Alamouti code- Performance

- The performance of Alamouti code with two transmitters and a single receiver is 3 dB worse than two-branch MRRC.
- The 3-dB penalty is incurred because is assumed that each transmit antenna radiates half the energy in order to ensure the same total radiated power as with one transmit antenna.
- If each transmit antenna was to radiate the same energy as the single transmit antenna for MRRC, , the performance would be identical.

Space Time Block Codes (STBC)

- Alamouti code can be generalized to an arbitrary number of antennas
- A S-T code is defined by an m x N_t transmission matrix
  - N_T – number of TX antennas
  - m– number of time periods for transmission of one block of coded symbols
- Fractional code rate
- Reduced Spectral efficiency
- Non-square transmission matrix
- Orthogonality of the transmission matrix only in the temporal sense
- Retain the property of having a very simple ML decoding algorithm based only in linear processing in the receiver

Space Time Block Codes (STBC)

3 transmit antennas
l=4 m=8

$$G_3 = \begin{bmatrix} s_1 & s_2 & s_3 \\ -s_2 & s_1 & s_4 \\ -s_3 & s_4 & s_1 \\ -s_4 & -s_3 & s_2 \\ s_1^* & s_2^* & s_3^* \\ -s_2^* & s_1^* & s_4^* \\ -s_3^* & s_4^* & s_1^* \\ -s_4^* & -s_3^* & s_2^* \end{bmatrix}$$

L-number of transmitted symbols

4 transmit antennas
l=4 m=8

$$G_4 = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2 & s_1 & s_4 & s_3 \\ -s_3 & s_4 & s_1 & -s_2 \\ -s_4 & -s_3 & s_2 & s_1 \\ s_1^* & s_2^* & s_3^* & s_4^* \\ -s_2^* & s_1^* & s_4^* & s_3^* \\ -s_3^* & s_4^* & s_1^* & -s_2^* \\ -s_4^* & -s_3^* & s_2^* & s_1^* \end{bmatrix}$$

- Fractional code rate
- The number of time slots across which the channel is required to have a constant fading envelope is increased by a factor of four !!

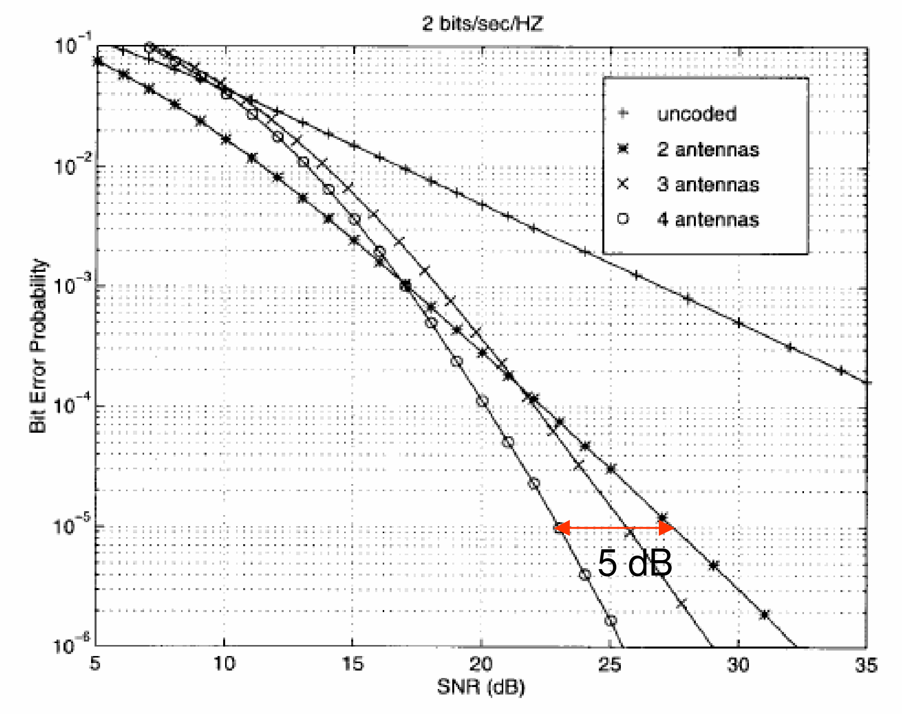
Space Time Block Codes (STBC)

- Parameters

Space Time code	Number of transmit antennas N_t	Number of transmitted symbol l	Number of time slots m	Rate $R=l/m$
Alamouti	2	2	2	1
G3	3	4	8	1/2
G4	4	4	8	1/2

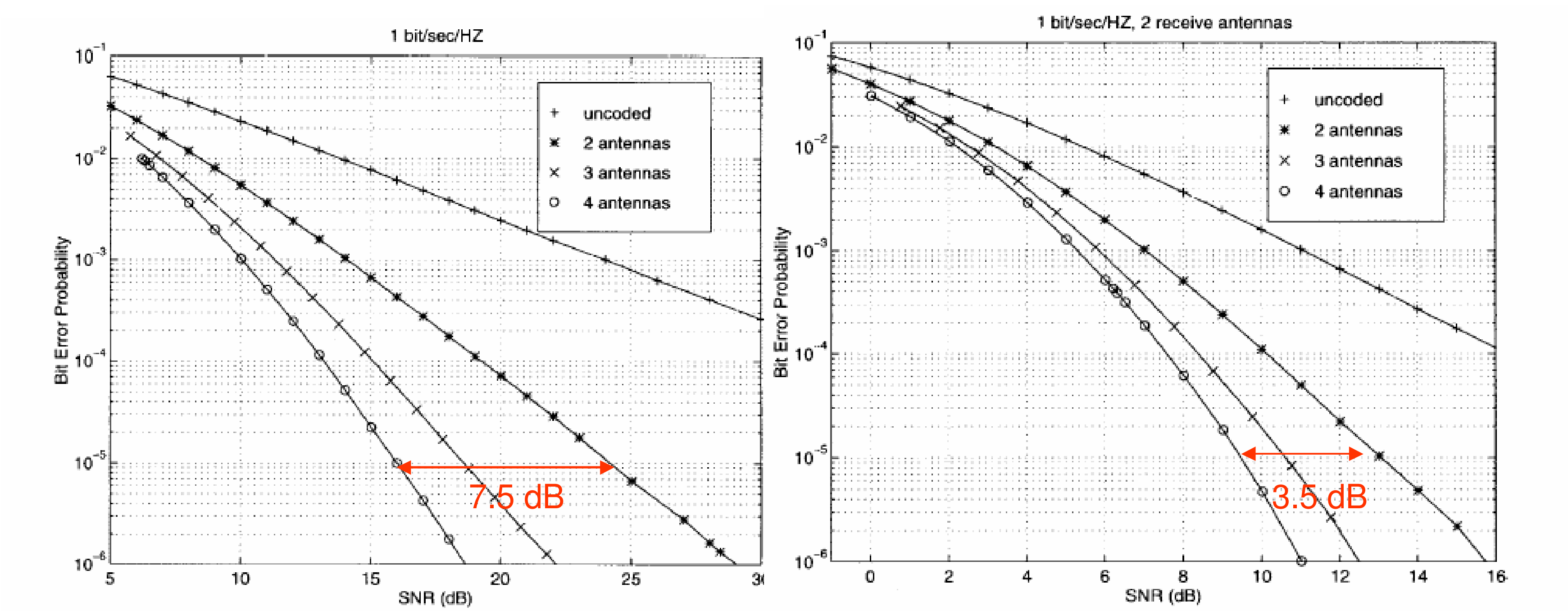
STBC - Performance

antennas	Modulation	Code	Code Rate
2	4 BPSK	G2	1
3	16 QAM	G3	1/2
4	16 QAM	G4	1/2



STBC – Performance [7]

antennas	Modulation
2	BPSK
3	4 DPSK
4	4 DPSK



Error probability in slow-fading channel

- The fading channel coefficients are constant within each frame.
- Codeword difference matrix B

$$X = \begin{bmatrix} x_1^1 - \hat{x}_1^1 & x_2^1 - \hat{x}_2^1 & \dots & x_L^1 - \hat{x}_L^1 \\ x_1^2 - \hat{x}_1^2 & x_2^2 - \hat{x}_2^2 & \dots & x_L^2 - \hat{x}_L^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{nT} - \hat{x}_1^{nT} & x_2^{nT} - \hat{x}_2^{nT} & \dots & x_L^{nT} - \hat{x}_L^{nT} \end{bmatrix}$$

- Codeword distance matrix A (Nt x Nt)

$$A(X,\hat{X}) = B((X,\hat{X}).B^H(X,\hat{X})$$

$$VA(X,\hat{X})V^H = \Delta$$

The rows of V are the eigenvectors of A matrix

The diagonal elements of Δ are the eigenvalues λ of A matrix

Error probability in slow-fading channel

- The Euclidean distance

$$d_h^2(X,\hat{X}) = \sum_{j=1}^{nR} h_j A(X,\hat{X}) h_j^H = \sum_{j=1}^{nR}\sum_{i=1}^{nT} \lambda_i |\beta_{j,i}|^2$$

$$\beta_{j,i} = h_j . v_i$$

Inner product

- The upper bound of the error probability is given by [2]

Rank(A)

Eigenvalues of matrix A

Diversity gain

$$P(x,\hat{x}) \le \left(\prod_{i=1}^{r} \lambda_i\right)^{-nR} \left(\frac{E_s}{4N_0}\right)^{-rnR}$$

$$P_{error} = G_c SNR^{-Gd}$$

Example 1 - A Time-Switched ST code

- Only one antenna is active in each time slot

$$X = \begin{bmatrix} x_t & 0 \\ 0 & x_t \end{bmatrix}$$

Given another codeword

$$\hat{X} = \begin{bmatrix} \hat{x}_t & 0 \\ 0 & \hat{x}_t \end{bmatrix}$$

The codeword difference matrix can be written

$$B(X, \hat{X}) = \begin{bmatrix} x_t - \hat{x}_t & 0 \\ 0 & x_t - \hat{x}_t \end{bmatrix}$$

Since $x_t - \hat{x}_t \neq 0$, the rank of $B(X, \hat{X})$ is $r = 2$. $A(X, \hat{X}) = B(X, \hat{X})B(X, \hat{X})^H$, the $rank(A) = rank(B)$.

- x_t is transmitted for antenna *1* at time *2t*
- x_t is transmitted for antenna *2* at time *2t+1.*
- *R=1/2*

Diversity gain = 2

Example 2 - Repetition code

- The same modulated symbols are transmitted from two antennas

$$X = \begin{bmatrix} x_t \\ x_t \end{bmatrix}$$

The codeword difference matrix can be written

$$B(X, \hat{X}) = \begin{bmatrix} x_t - \hat{x}_t \\ x_t - \hat{x}_t \end{bmatrix}$$

The rank of $B(X, \hat{X})$ is 1

- The repetition code has the same performance as a no diversity scheme (1x1 system) !!

S-T Code Design criteria

- The design criteria for slow Rayleigh fading channel depend on the value of rnR

$$P(x, \hat{x}) \leq \left(\prod_{i=1}^{r} \lambda_i \right)^{-nR} \left(\frac{E_s}{4N_0} \right)^{-rnR}$$

- The maximum possible value of rnR is $nT.nR$
- The error probability at high SNR is dominated by the minimum rank **r** of the matrix A over the all possible codewords pairs

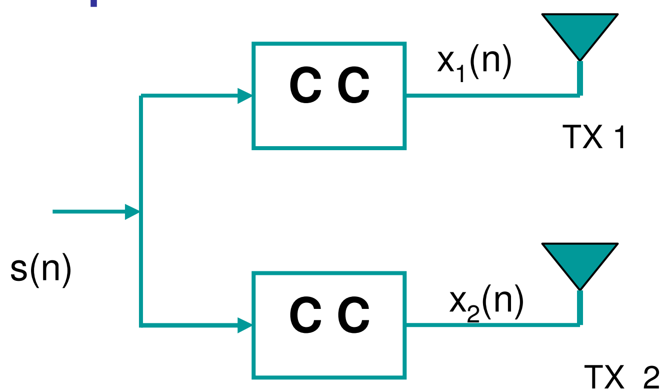
•Maximize the minimum rank r of matrix A over all pairs of distinct codewords

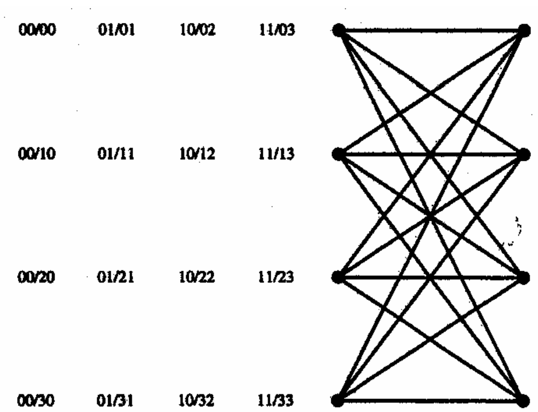
•Maximize the minimum product $\left(\prod_{i=1}^{r} \lambda_i \right)$ of matrix A

Space-Time Trellis Codes (STTC)

- A stream of data is encoded via Nt convolutional encoders to obtain N_t streams $x_1 \ldots x_{nt}$
- The design of STTC codes is a relatively hard problem.
- Advantages
  - Coding gain !!
  - Similar Diversity gain than STBC
- Disadvantages
  - Viterbi decoder.
  - The complexity of decoding algorithm grows exponentially with the memory length of the trellis code.

Space-Time Trellis Codes (STTC)





Example

- 4-state STTC
- Two transmit antennas
- CC g^1=[(0 2),(2 0)]

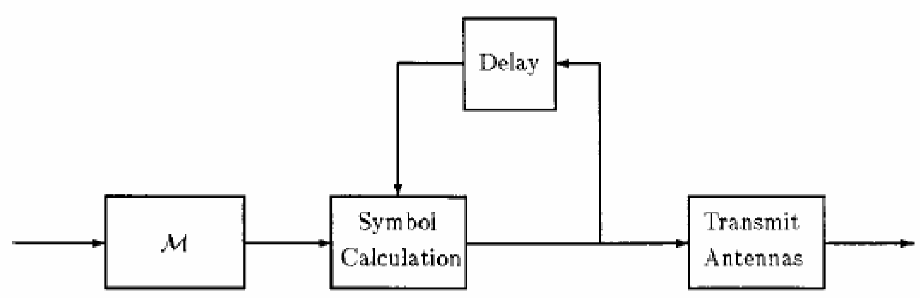
  g^2=[(0 1),(1 0)]

The encoder takes m=2 bits as its input at each time.

- Input Sequence **c**=(10, 01,11,00,01,…)
- Output sequence **x**=(02, 21,13,30,01,…)
- Antenna 1 $\mathbf{x}^1$=(0,2,1,3,0,…)
- Antenna 2 $\mathbf{x}^2$=(2,1,3,0,1,…)

Differential space time block codes

- Channel acknowledge in the receiver is necessary for STBC
  - Overhead
  - Channel estimation
  - Problems in high mobility channels
- DSTBC eliminates the need for channel estimation
- Very simple Maximum Likelihood decoding



Differential space time block codes

Time	Antenna 1	Antenna 2
	S_1	S_2
	$-S_2^*$	S_1^*
2t-1	S_{2t-1}	S_{2t}
2t	$-S_{2t}^*$	S_{2t-1}^*

No information

•At time *2t+1* a block of *2b* bits B (2t+1) arrives at the encoder. Using the mapping M → computes S_{2t+1} and S_{2t+2}

2t+1	S_{2t+1}	S_{2t+2}
2t+2	$-S_{2t+2}^*$	S_{2t+1}^*

The process is inductively repeated until the end of the frame

Differential space time block codes

- Example
  - BPSK symbols

  Mapping function M

M(0 0)	(1 0)
M(0 1)	(0 -1)
M(1 0)	(0 1)
M(1 1)	(-1 0)

2t-1	0.707	-0.707
2t	0.707	0.707

M

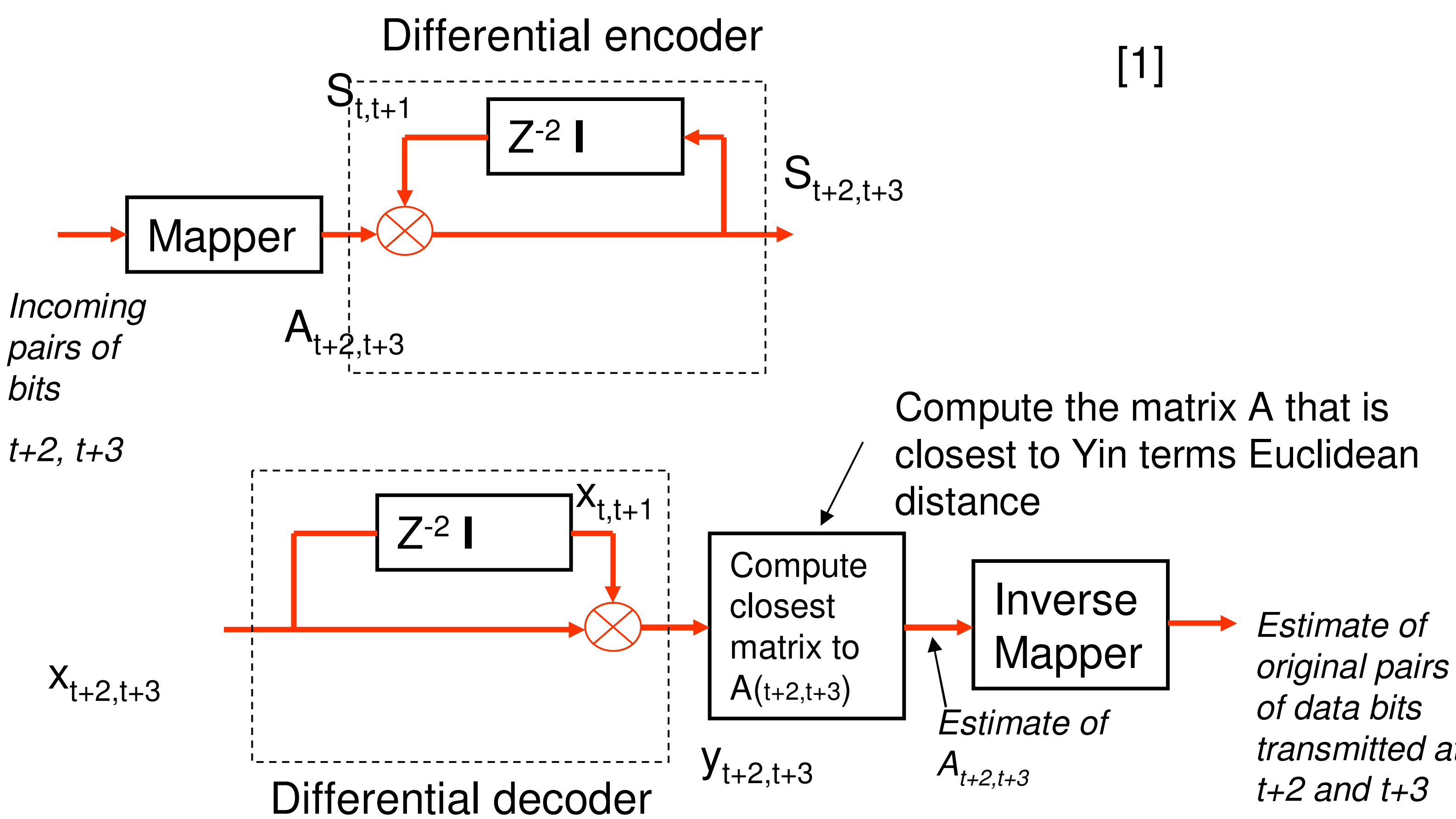
2t+1	0.707	-0.707
2t+2	0.707	0.707

M(1 0)=(0 1)

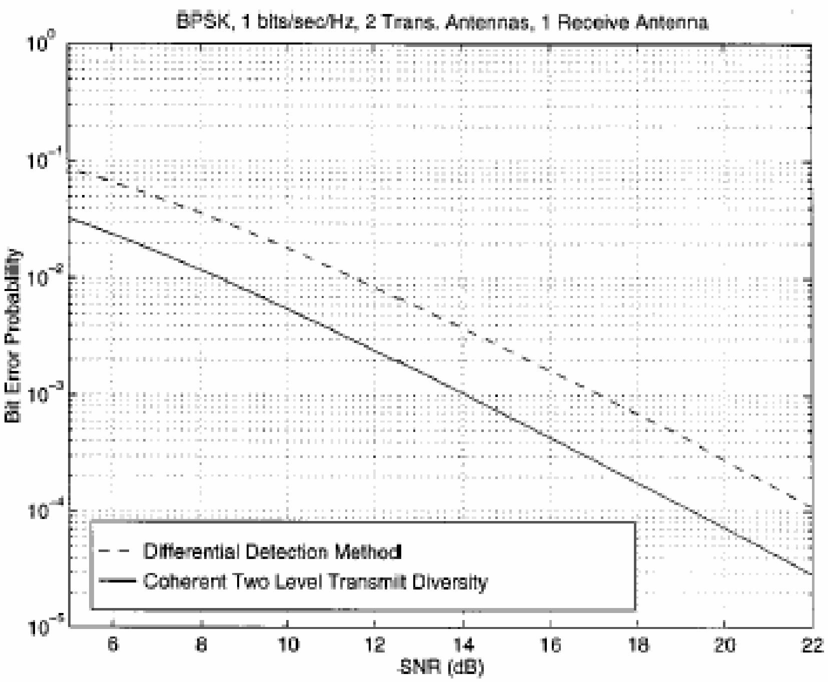
A=0

B=1

$$(S_{2t+1}, S_{2t+2})=A\,(S_{2t+1}, S_{2t+2})+B(S_{2t}^*,S_{2t-1}^*)$$


Differential encoder
[1]
S t,t+1
Z-2 I
S t+2,t+3
Mapper
Incoming pairs of bits
t+2, t+3
A t+2,t+3
Compute the matrix A that is closest to Yin terms Euclidean distance
Z-2 I
x t,t+1
x t+2,t+3
Compute closest matrix to A(t+2,t+3)
Inverse Mapper
Estimate of original pairs of data bits transmitted at t+2 and t+3
Estimate of A t+2,t+3
y t+2,t+3
Differential decoder

DSTBC-Performance [6]



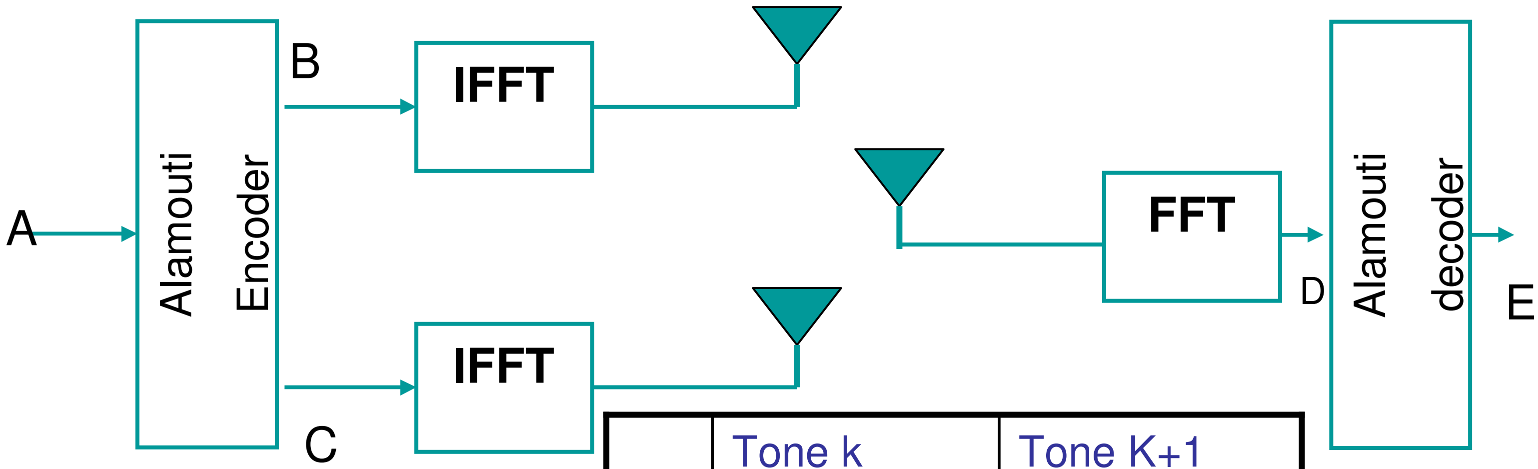
BPSK – 2 x 1 system

The DSTBC detection scheme is 3 dB worse than that of the transmit diversity scheme of employs coherent detection at high SNR.

Spatial Diversity coding for MIMO-OFDM [4]

- The time index is replaced by the tone index in OFDM
- Alamouti code requires that the channel remains constant over consecutive symbols periods.
- In OFDM context, the channel must remain constant over consecutive tones.
- Problems in frequency selective channels !!!

Spatial Diversity coding for MIMO-OFDM



	Tone k	Tone K+1
A	S_1	S_2
B	S_1	$-S_2^*$
C	s_2	S_1^*
D	$h_1s_1+h_2s_2$	$-h_1s_2^*+h_2s_1^*$
E	$H[k]^2\ s_1$	$H[k]^2\ s_2$

Spatial Diversity coding for MIMO-OFDM

- The receiver detected the transmitted symbols from the received signals on the two tones using the Alamouti detection technique.
- The use of consecutive tones is not strictly necessary, any pair of tones can be used as long as the associated channels are equal.
- The technique can be generalized over a large number of antennas to extract spatial diversity using STBC → The block size is $T \cdot N_T$.
- The channel must be identical over the T tones

Conclusions

- Alamouti code is the best option when 2 Transmission antennas is considered.
- Low complexity receiver is a good characteristic for STBC
- STTC provides coding gain. But Viterbi decoder must be implemented in the receiver.
- DSTBC can be considered in high mobility channels.

References

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- *[6] Tarokh, V.; Jafarkhani, H.,* **A differential detection scheme for transmit diversity,** Selected Areas in Communications, IEEE JSAC ,Vol. 18 ,July 2000 .
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Homework

- Alamouti code don’t provide coding gain .
  - Justify