Modulation methods

S-72. 333 Physical layer methods in wireless communication systems

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16 November 2004

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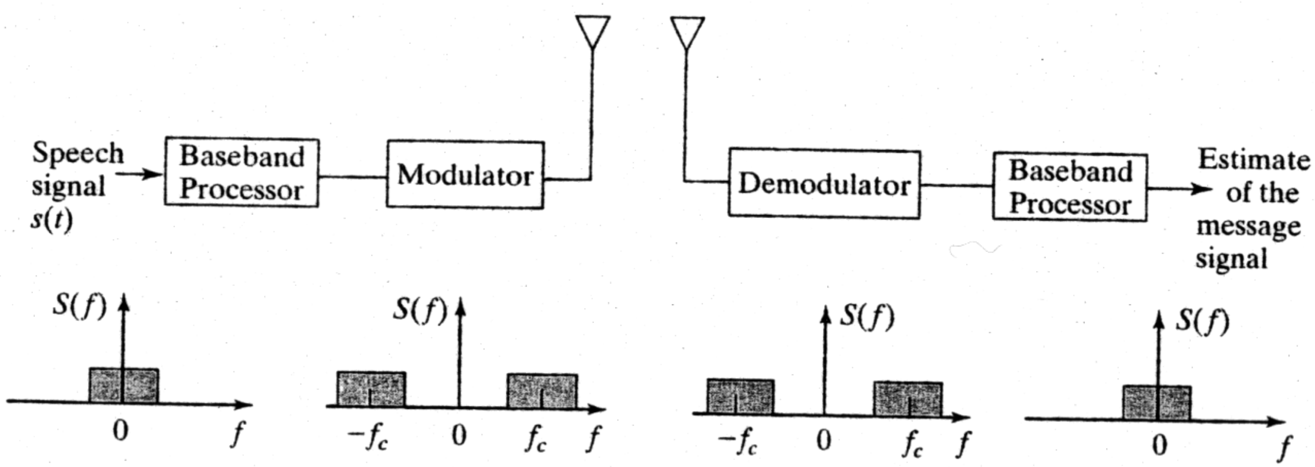
1 Introduction

Definition :

Process by which some characteristic of a carrier wave is varied in accordance with an information-bearing signal

Information-bearing signal ⟶ Modulating signal

Output of modulation process ⟶ Modulated signal



Three practical benefits from the use of modulation in wireless communication :

1) It is used to shift the spectral content of a message signal so that it lies inside the operating frequency band of the wireless communication channel

   Ex.: telephonic communication over cellular radio channel
   Voice ≈ 300-3100Hz → freq. assigned to cellular radio channel ≈ 900-1800MHz

2) It provides a mechanism for putting the information content of a message signal into a form that be less vulnerable to noise or interference

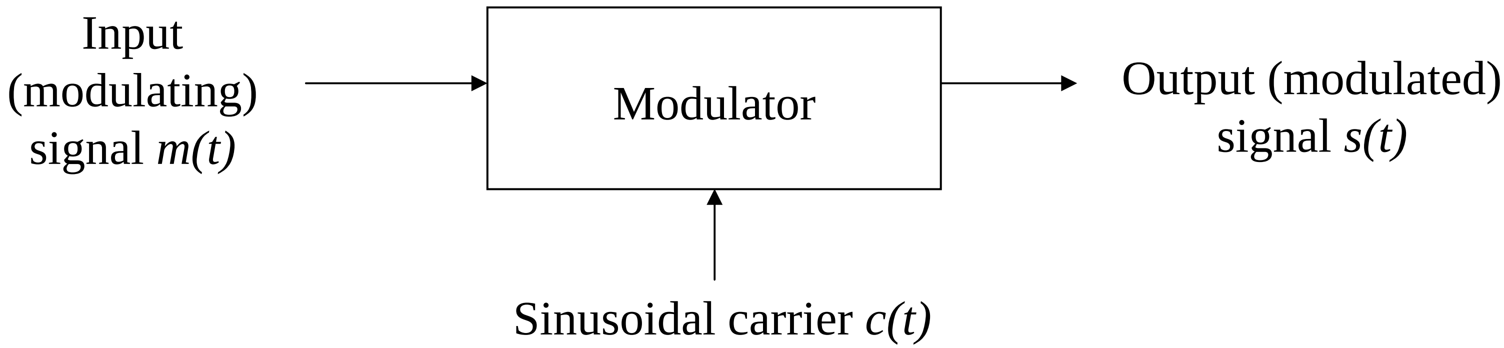
   Received signal ordinarily corrupted by noise → FM : improve system performance in presence of noise

3) It permits the use of multiple-access techniques

   → Simultaneous transmission of several different information-bearing signals over the same channel

2 Principal characteristics

2.1 Linear and Nonlinear Modulation Process



Linear Modulation :

→ Input-Output relation of modulator satisfies ***principle of superposition***

- Output of modulator produced by a number of inputs applied simultaneously is equal to the sum of the output that result when the inputs are applied one at a time → $M(i_1+i_2\ldots+i_n) = M(i_1)+M(i_2)\ldots+M(i_n)$
- If the input is scale by a certain factor, the output of the modulator is scaled by exactly the same factor

Nonlinear Modulation :

→ Input-Output relation of modulator **does not** (partially or fully) satisfies ***principle of superposition***

Linearity and nonlinearity has importance in both theoretical and practical aspects.

2.2 Analog and Digital Modulation Techniques

Analog modulation :

Modulation of analog signal → infinity of value of the modulated parameter of the modulated signal within a certain scale

Digital modulation :

Modulation of digital signal → finite number of value of the modulated parameter of the modulated signal

Ex.: QPSK → 4 values of phase

2.3 Amplitude and Angle Modulation Process

Carrier $C(t) = A_c \cos(2\pi f_c t + \theta)$

Three parameters :

$A_c \rightarrow$ Amplitude modulation : AM

$f_c \rightarrow$ Frequency modulation : FM

$\theta \rightarrow$ Phase modulation : PM

3 Linear Modulation Techniques

3.1 Binary Phase-Shift Keying

↳ Simplest form of digital phase modulation

Modulating signal = binary data stream = $m(t) = \sum_k b_k \; p(t - kT)$

Where $P(t)$ = ***basic pulse*** and T = ***bit duration***

$b_k = \begin{cases} +1 & \text{for binary symbol 1} \\ -1 & \text{for binary symbol 0} \end{cases}$

Binary symbol 0 → carrier phase $\theta(t) = 0$ radians

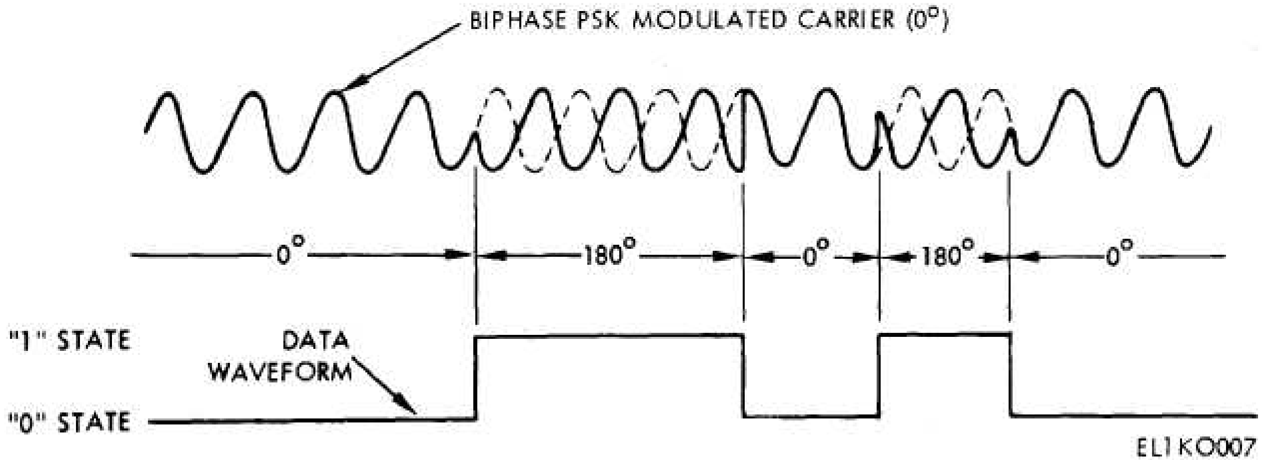
Binary symbol 1 → carrier phase $\theta(t) = \pi$ radians

$$S(t) = \begin{cases} A_c \cos(2\pi f_c t) & \text{for binary symbol 0} \\ A_c \cos(2\pi f_c t + \pi) & \text{for binary symbol 1} \end{cases}$$

As $\cos(\theta(t) + \pi) = -\cos(\theta(t))$

$$S(t) = \begin{cases} A_c \cos(2\pi f_c t) & \text{for binary symbol 0} \\ -A_c \cos(2\pi f_c t + \pi) & \text{for binary symbol 1} \end{cases}$$

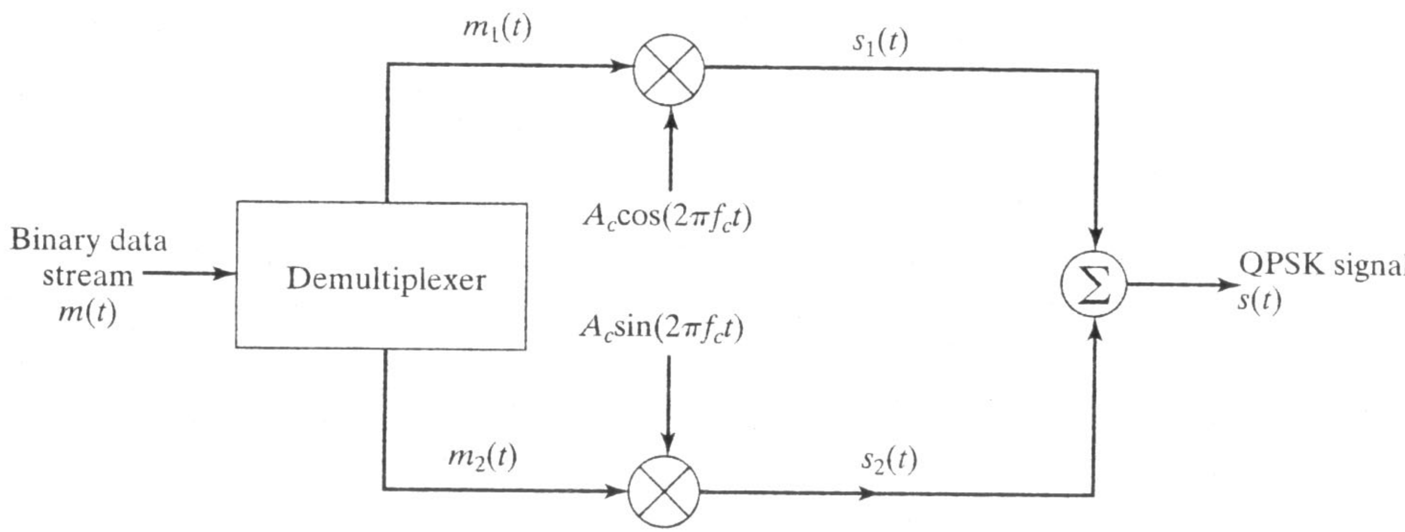
$\Rightarrow$ $S(t) = c(t)\, m(t)$



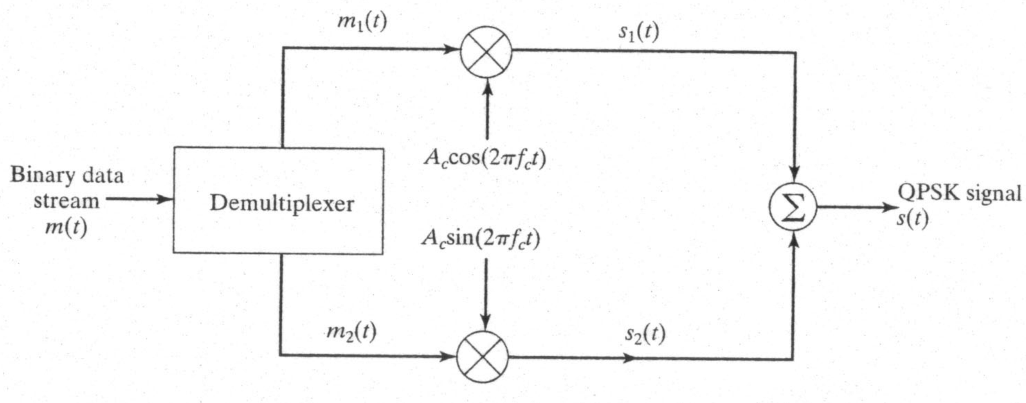
Inconvenient : transmission bandwidth = 2 x message bandwidth

3.2 Quadriphase-Shift Keying (QPSK)

Interest : transmission bandwidth = message bandwidth



- Phase of carrier can take 4 different values depending on each **dibit**
- QPSK : Parallel combination of 2 BPSK modulators that operate in phase quadrature with respect to each other



$$m_i(t) = \sum_k b_{k,i}\ p(t - kT) \quad \text{for } i = 1,2$$

$$B_{k,i} = \begin{cases} +1 & \text{for binary symbol 0} \\ -1 & \text{for binary symbol 1} \end{cases}$$

For rectangular pulse : $p(t) = \begin{cases} +1 & \text{for } 0 \leq t \leq 2T \\ 0 & \text{otherwise} \end{cases}$

$$S(t) = s_1(t) + s_2(t) = A_c\, m_1(t) \cos(2\pi f_c\, t) + A_c\, m_2(t) \sin(2\pi f_c\, t)$$

3.3 Offset Quadriphase-Shift Keying

Motivation :

QPSK : carrier phase may jump by ± 90° or ± 180° every 2 bit durations

Problem : filtering action can cause the carrier amplitude to fluctuate

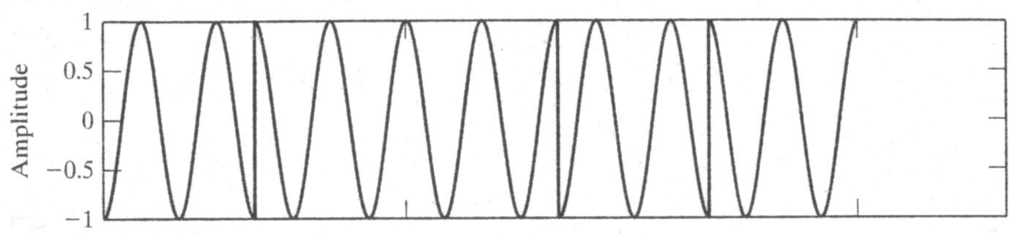
→ receiver produce additional symbol errors

To reduce fluctuation → *Offset Quadriphase-Shift Keying* (*staggered PSK*)

Principle :

Second substream $m_2(t)$ is delayed (offset) by a bit duration T

➡ phase transitions are confined in 0°, ± 90°



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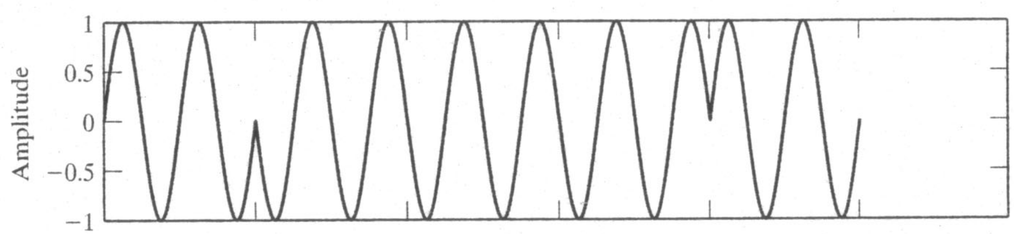


(b)

3.4 π/4-Shifted Quadriphase-Shift Keying

Motivation : similar to Offset Quadriphase-Shift Keying (OQPSK)

Carrier phase of QPSK can be (usually 1.) :

1. 0, $\pi/2$, π or $3\pi/2$
2. $\pi/4$, $3\pi/4$, $5\pi/4$ or $7\pi/4$

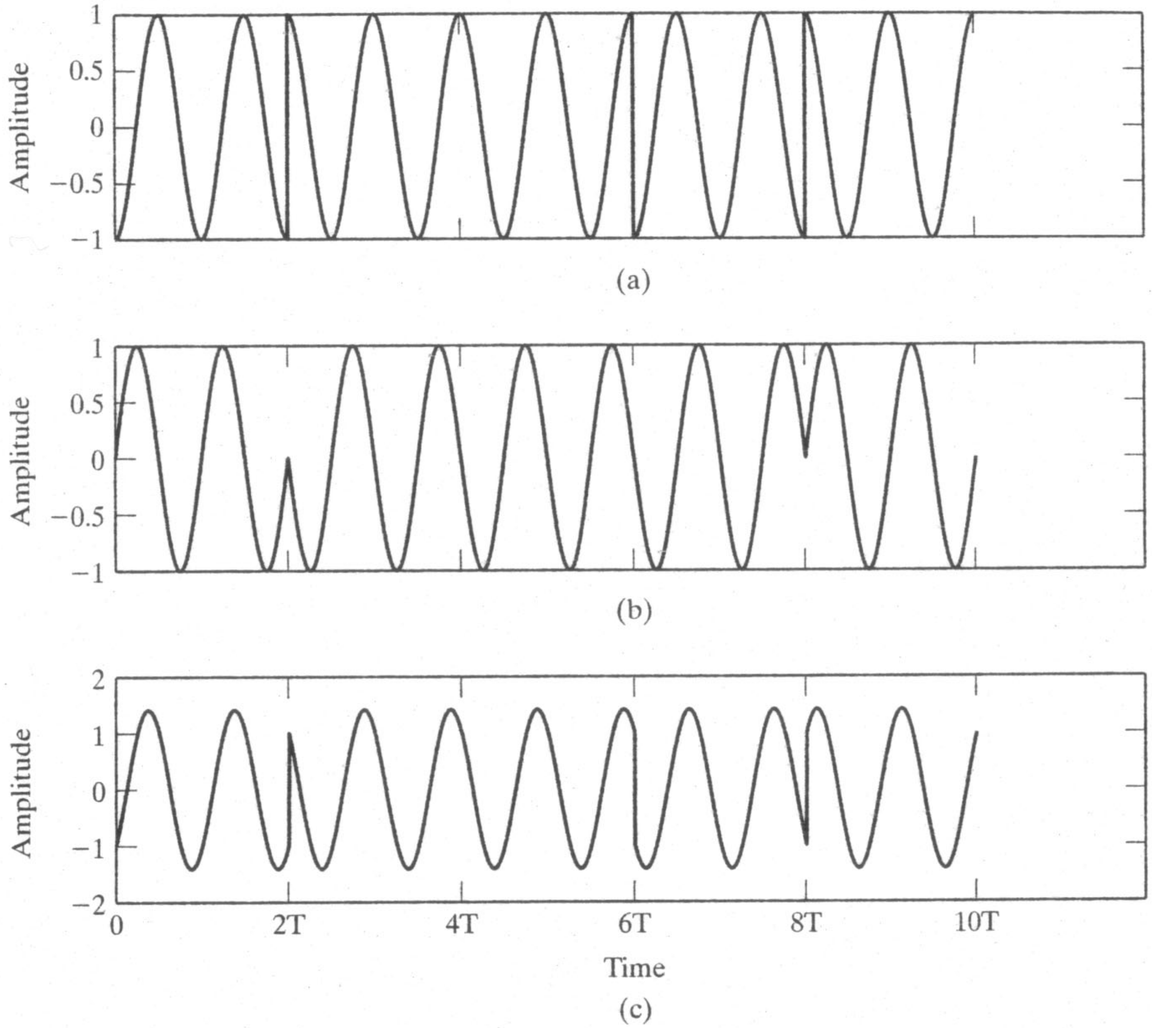
➡ π/4-Shifted QPSK uses alternatively 1. And 2.

↳ amplitude fluctuations during filtering are significantly reduced

- Reduced amplitude fluctuation becomes important when transmitter includes slightly nonlinear amplifier

↳ **π/4-Shifted QPSK** has been adopted in north American digital cellular time division multiple access (TDMA) standard IS-54 as well as the Japanese digital cellular standard

Difference between conventional, offset and π/4-Shifted QPSK



4 Pulse Shaping

Problems with rectangular pulse :

- Infinite spectrum → signal distortion when transmitted over band limited channel (wireless)

- *Memory* of wireless channel (multipath) → inter-symbol interference (ISI)

To overcome the 2 problems : pre-modulation filter ➡ Pulse shaping

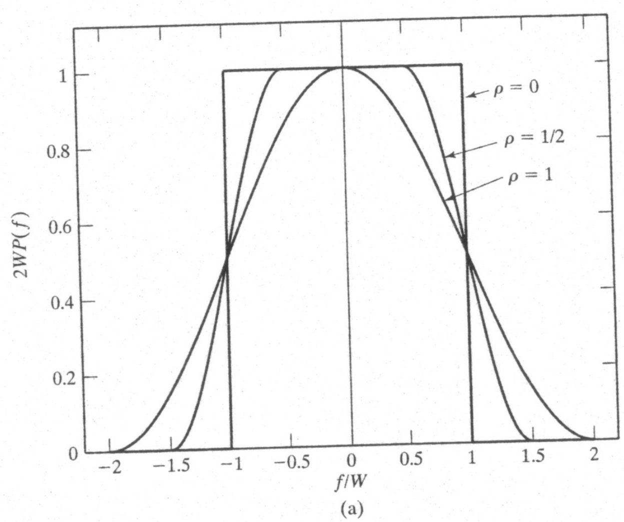
↳ use of fundamental theoretical work of Nyquist

Effects of ISI can be reduced to zero by shaping the overall frequency response $P(f)$ so as to consist of a flat portion and sinusoidal roll-off portions

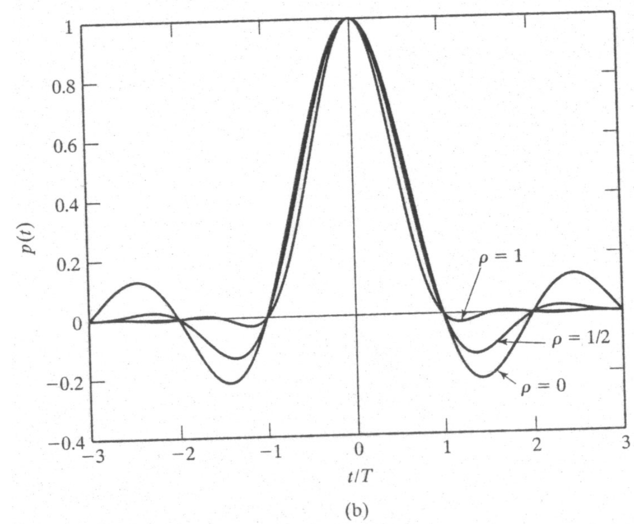
For a given data rate R bits/sec :
channel bandwidth may extend from minimum $W = R/2$ to adjustable value from W to $2W$

$$P(f)=\begin{cases} 1/2W & 0 \le |f| \le f_1 \\ \frac{1}{4W}\left[1+\cos\left(\frac{\pi}{2W\rho}\left(|f|-W(1-\rho)\right)\right)\right] & f_1 \le |f| \le 2W - f_1 \\ 0 & |f| \ge 2W - f_1 \end{cases}$$

roll-off factor : $\rho = 1 - (f_1 / W)$ = excess bandwidth over the *ideal* solution : $\rho = 0$



(a)



(b)

4.1 Root Raised-Cosine Pulse Shaping

- Spectrum of basic pulse defined by square root of $\frac{1}{4W}\left[1+\cos\left(\frac{\pi}{2W\rho}\left(|f|-W(1-\rho)\right)\right)\right]$

$$P(f) = \begin{cases} \frac{1}{\sqrt{2W}} & 0 \le |f| \le f_1 \\ \frac{1}{\sqrt{2W}}\cos\left(\frac{\pi}{4W\rho}\left(|f|-W(1-\rho)\right)\right) & f_1 \le |f| \le 2W - f_1 \\ 0 & |f| \ge 2W - f_1 \end{cases}$$

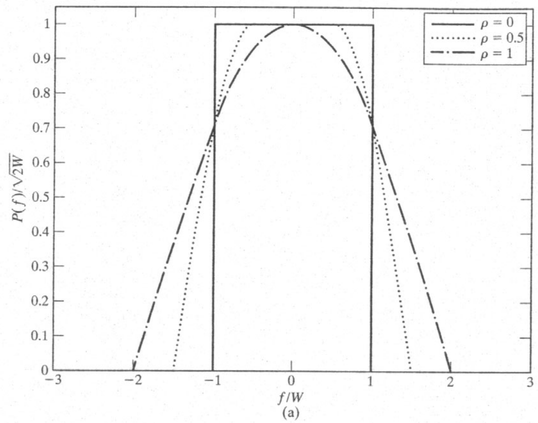
When transmitter includes pre-modulation filter with this transfer function and receiver include an identical filter :

overall pulse waveform will experience the spectrum $P^2(f)$ (regular raised cosine spectrum)

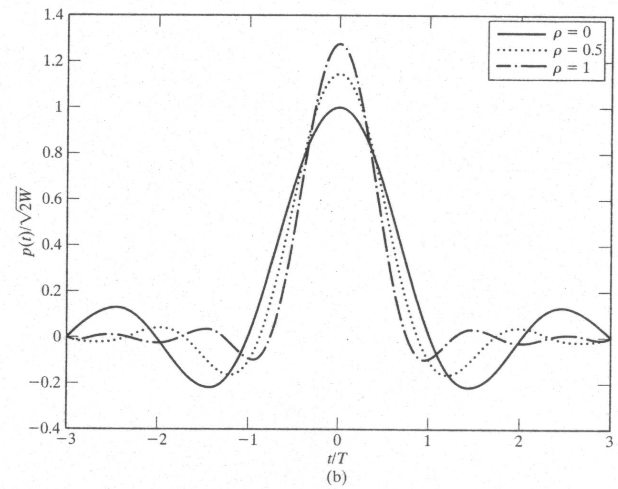
if channel affected by both flat fading and additive white noise :

→ Receiver maximize output signal-to-noise ratio

Inverse Fourier transform : $p(t) = \frac{\sqrt{2W}}{1-(8\rho Wt)^2}\left(\frac{\sin(2\pi W(1-\rho)t)}{2\pi Wt} + \frac{4\rho}{\pi}\cos(2\pi W(1+\rho)t)\right)$



P(f) Root raised–cosine spectrum



p(t)

<u>Regular RC waveform Vs Root RC waveform</u> :

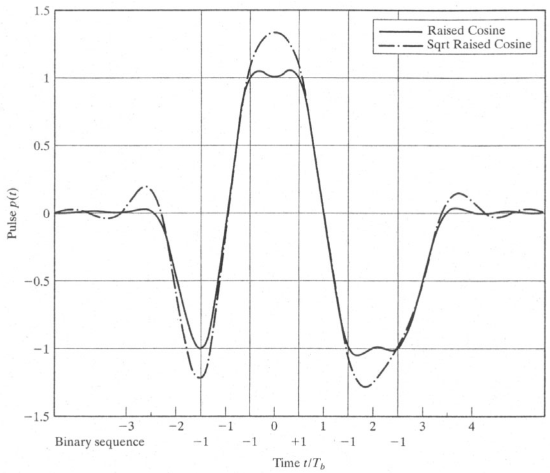


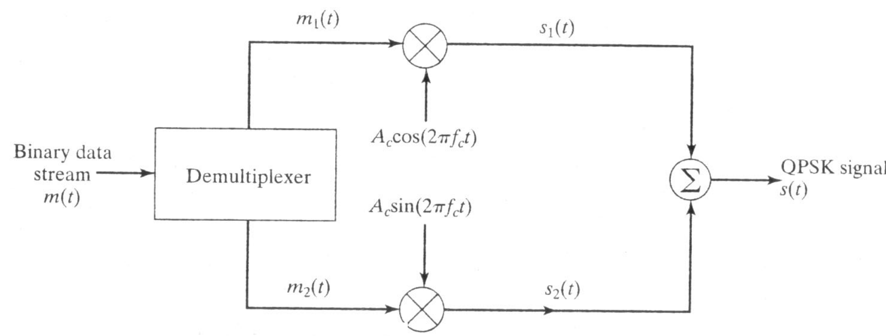
FIGURE 3.11 Two pulse trains for sequence 01100, one using regular RC pulse and the other using root RC pulse.

Root RC waveform occupies larger dynamic range than regular RC waveform

5 Complex Representation of Linear modulated signals

Linear modulation scheme may be viewed as special case of canonical representation of a band-pass signal :

$$s(t) = s_I(t)\cos(2\pi f_c t) - S_Q(t)\sin(2\pi f_c t)$$



$s_I(t)$: *in-phase component of* $s(t)$

$s_Q(t)$: *quadrature component of* $s(t)$

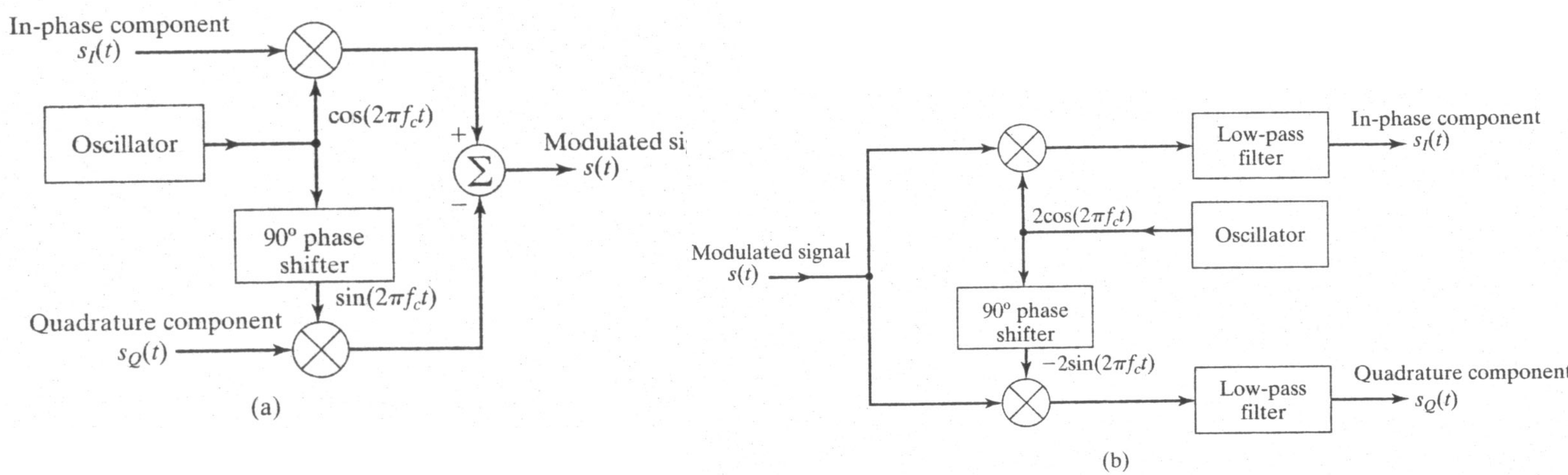
TABLE 3.1 Special Cases of the Canonical Equation (3.23).

	Type of modulation	In-phase component $s_I(t)$	Quadrature component $s_Q(t)$
Analog	Amplitude modulation	$A_c(1 + k_a m(t))$	0
	Double sideband–suppressed carrier modulation	$A_c m(t)$	0
Digital	Binary phase-shift keying	$A_c \sum_k b_k p(t - kT)$	0
	Quadriphase-shift keying	$A_c \sum_k b_{k,1} p(t - 2kT)$	$-A_c \sum_k b_{k,2} p(t - 2kT)$

complex envelope of *s(t)* : $\tilde{s}(t) = s_I(t) + js_Q(t)$ and $s(t) = \mathrm{Re}\{\tilde{s}(t)\exp(j2\pi f_c t)\}$

Synthesizer for constructing modulated signal from in-phase and quadrature components : *(a)*

Analyzer for deriving the in-phase and quadrature components : *(b)*



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6 Signal-Space Representation of Digitally Modulated Signals

Signal constellation (signal pattern) :

- Energy normalized version of in-phase component $s_I(t)$ = horizontal axis

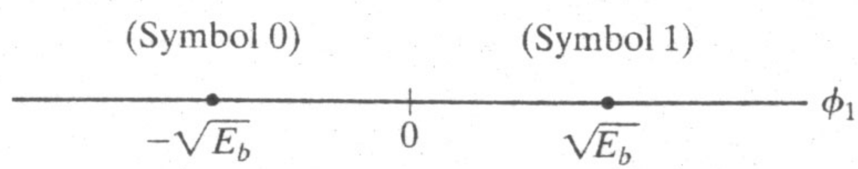
$$\phi_1 = \sqrt{\frac{2}{T}} \cos\left(2\pi\ f_c\ t\right) \qquad 0 \le t \le T$$

- Energy normalized version of quadrature component $s_Q(t)$ = vertical axis

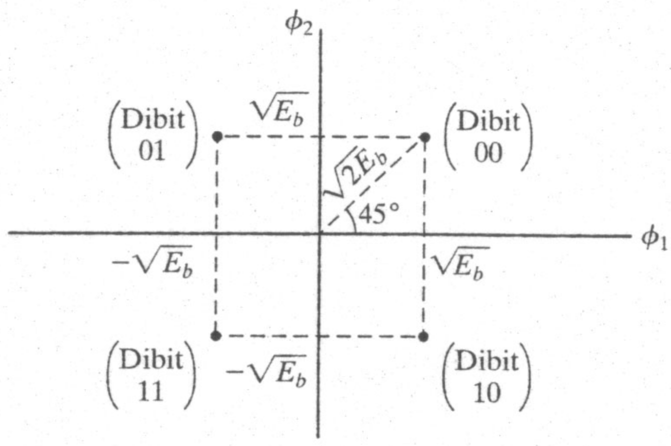
$$\phi_2 = \sqrt{\frac{2}{T}} \sin\left(2\pi\ f_c\ t\right) \qquad 0 \le t \le T$$

$$\int_0^T \phi_1(t)\, \phi_2(t)\, dt = 0$$ ➡ orthogonality of ϕ_1 and ϕ_2 over $0 \le t \le T$

BPSK :

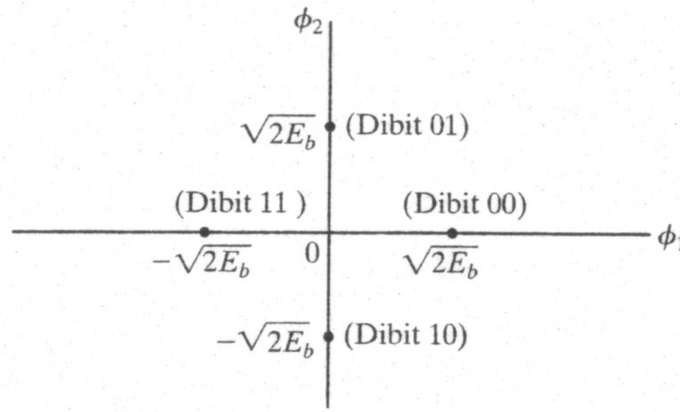


(a)



(b)

QPSK



(c)

Other version of QPSK

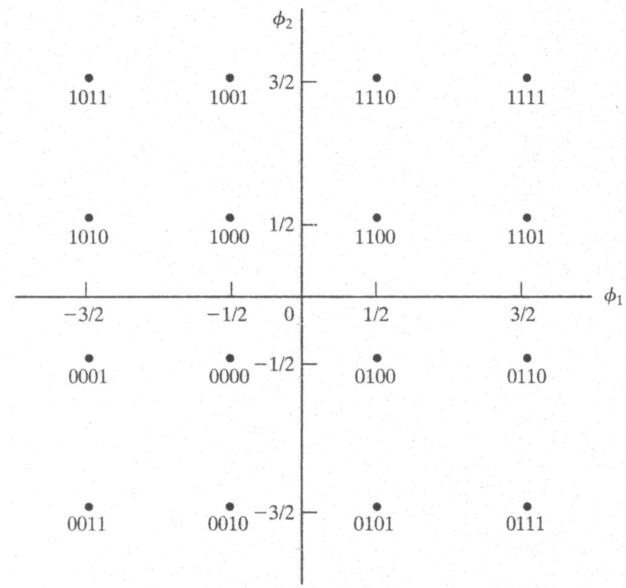
TABLE 3.2 Signal-space characterization of the QPSK signal constellation described in Fig. 3.14(b).

Input dibit	Gray-encoded phase of QPSK signal (radians)	Coordinates of message points	
		s_{i1}	s_{i2}
10	$7\pi/4$	$+\sqrt{E_b}$	$-\sqrt{E_b}$
11	$5\pi/4$	$-\sqrt{E_b}$	$-\sqrt{E_b}$
01	$3\pi/4$	$-\sqrt{E_b}$	$+\sqrt{E_b}$
00	$\pi/4$	$+\sqrt{E_b}$	$+\sqrt{E_b}$

Quadrature Amplitude Modulation

➡ Combination of amplitude modulation and phase modulation

Ex: 16-QAM



Remark:

Although QPSK can be transmitted over nonlinear channels,16-QAM need to be transmitted over linear channel

In QPSK energy transmitted remains fixed, although in 16-QAM energy transmitted is variable, depending on particular quad-bit

7 Nonlinear Modulation Techniques

➡ Preferably studied in ***polar*** form : $s(t) = a(t) \cos [2\pi f_c t + \theta(t)]$

Where envelope = $a(t) = \sqrt{s_I^2(t) + s_Q^2(t)}$

and phase = $\theta(t) = \tan^{-1}\left(\frac{s_Q(t)}{s_I(t)}\right)$

7.1 Frequency Modulation

$$f(t) = f_c + k_f m(t)$$

where k_f = sensitivity of the frequency modulator

f_c = frequency of un-modulated carrier

Transmission bandwidth :

➡ approximately given by Carson's rule : $B_T \approx 2\Delta f\left(1+\frac{1}{D}\right)$

Δf = *frequency deviation* : maximum deviation in the instantaneous frequency

D = *deviation ratio* : ratio of the frequency deviation to the highest frequency component contained in the modulating signal

Unlike AM :

Increasing of FM transmission ***bandwidth*** produces ***quadratic increase in signal-to-noise*** ratio at the output of the receiver

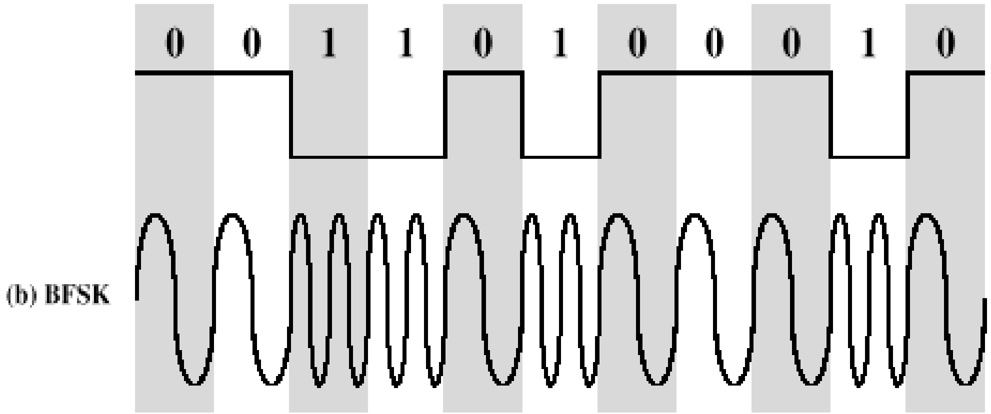
Thanks to this bandwidth-noise trade-off capability : FM was adopted in first generation of wireless communication systems (based on FDMA)

7.2 Binary Frequency-Shift Keying

Symbol 0 : sinusoid of frequency f_1

Symbol 1 : sinusoid of frequency f_2

$$s_i(t) = \sqrt{\frac{2E_b}{T}} \cos(2\pi f_i \, t) \qquad i = 1, 2$$



T = symbol (bit) duration

E_b = energy transmitted per bit

$$f_i = \frac{n_c + i}{T}$$

n_c = fixed integer and $i = 1, 2$ ➡ ***Sunde's FSK***

↳ ***continuous-phase signal*** : phase continuity is maintained everywhere, including the inter-bit switching time

➡ part of ***Continuous-Phase Frequency-Shifted Keying*** (CPFSK)

7.3 Continuous-Phase Frequency-Shifted Keying (CPFSK)

Goal : improve spectral efficiency and noise performance

Optimal parameters :

$$f_1 = f_c + \frac{h}{2T} \quad , \quad f_2 = f_c - \frac{h}{2T}$$

$$f_c = \frac{1}{2}\left(f_1 + f_2\right) \quad , \quad h = T\left(f_1 - f_2\right) = deviation \ \ ratio$$

$$\theta(T) - \theta(0) = \begin{cases} \pi h & \text{for symbol 1} \\ -\pi h & \text{for symbol 0} \end{cases}$$

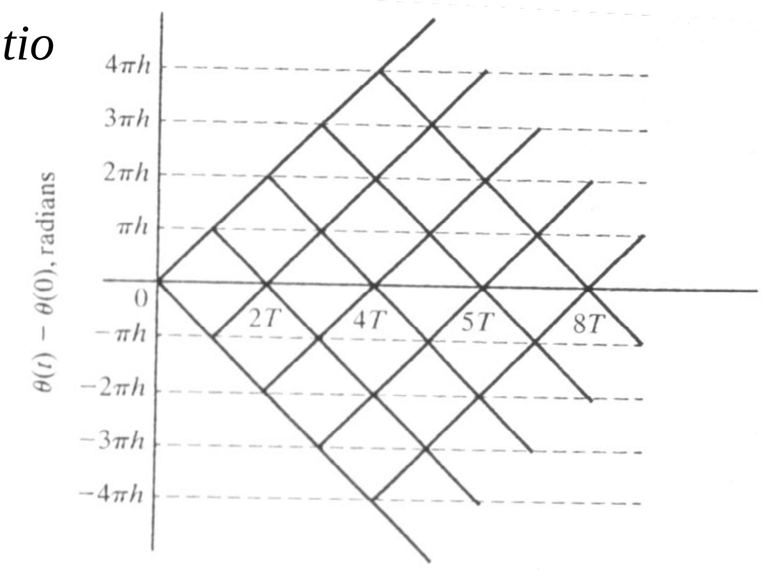


FIGURE 3.17 Phase tree of a CPFSK signal.

Memory :

a) In Sunde's FSK :

deviation ratio h = unity

↳ phase change over 1 bit interval = $\pm\,\pi$

As : change of π rad = change of –π rad modulo 2π

↳ no memory in Sunde's FSK !!

➡ Knowing change in *previous* interval provides no help in the *current* bit interval

b) If h = 1/2 :

phase can be only $\pm\pi/2$ at odd multiple of T, and 0 and π at even multiple of T

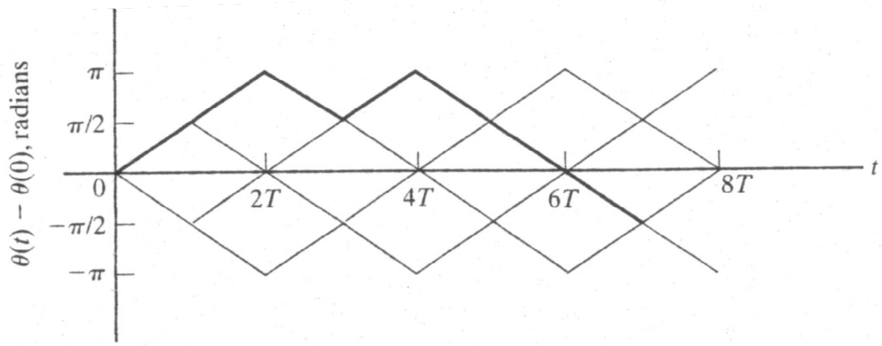


FIGURE 3.18 Phase trellis; boldfaced path represents the sequence 1101000.

If h = 1/2 :

Frequency deviation = ½ bit rate

↳ minimum frequency spacing that allows FSK signals to be coherently orthogonal

(no interference during process of detection)

➡ CPFSK signal with deviation ratio ½ is commonly referred as ***Minimum-Shift Keying*** (MSK)

TABLE 3.3 Transition characterization of MSK.

Transmitted Binary Symbol, $0 \le t \le T$	Phase States (radians)		Coordinates of Message Points	
	$\theta(0)$	$\theta(T)$	s_1	s_2
0	0	$-\pi/2$	$+\sqrt{E_b}$	$+\sqrt{E_b}$
1	π	$-\pi/2$	$-\sqrt{E_b}$	$+\sqrt{E_b}$
0	π	$+\pi/2$	$-\sqrt{E_b}$	$-\sqrt{E_b}$
1	0	$+\pi/2$	$+\sqrt{E_b}$	$-\sqrt{E_b}$

7.4 Power spectra of MSK signal

In phase component = ± $g_1(t)$ with $g_I(t)=\sqrt{\frac{2E_b}{T}}\cos\left(\frac{\pi t}{2T}\right)$

quadrature component = ± $g_2(t)$ with $g_Q(t)=\sqrt{\frac{2E_b}{T}}\sin\left(\frac{\pi t}{2T}\right)$

Base-band power spectral density = $\frac{32E_b}{\pi^2}\left[\frac{\cos(2\pi\, Tf)}{16\,T^2 f^2-1}\right]^2$

➡ base-band power spectral density decreases as the inverse fourth power of frequency (inverse square power of frequency for QPSK)

↳ MSK does not produce as much interference outside signal band of interest as does QPSK

7.5 Gaussian-Filtered MSK

Motivation : Adjacent channel interference of MSK not low enough for multi-user communication environment

Goal : modify the power spectrum of the signal into a ***compact form***

How : use of pre-modulation low-pass filter (base-band ***pulse shaping filter***)

➡ Polar ***non-return-to-zero*** (NRZ) binary data stream through base-band pulse-shaping filter with impulse response defined by a ***Gausssian*** function

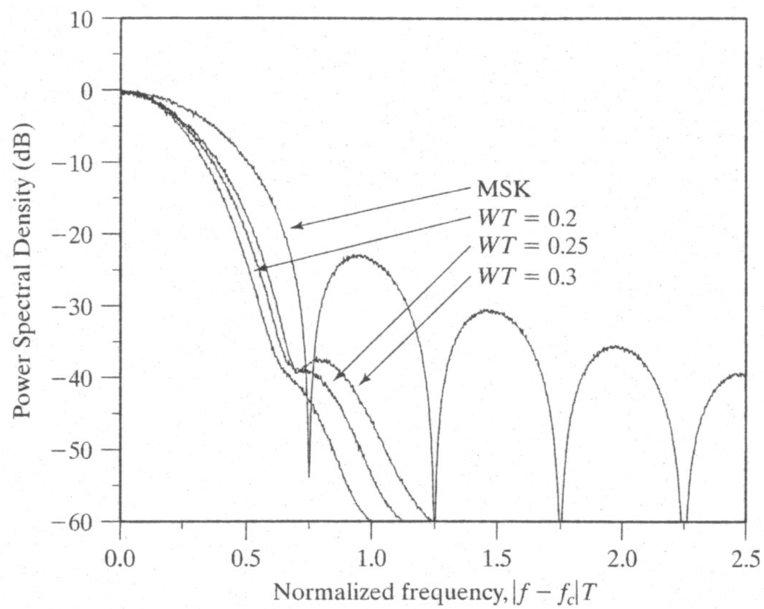
response of Gaussian filter to rectangular pulse of unit amplitude and duration T :

$$g(t) = \sqrt{\frac{2\pi}{\log 2}} W \int_{-T/2}^{T/2} \exp\left(-\frac{2\pi^2}{\log 2} W^2 (t-\tau)^2 \right) d\tau$$

W = 3dB base-band bandwidth

Parameter : time-bandwidth WT

Power spectra of MSK and GMSK signals:



Curve for limiting condition WT = ∞ ➡ ordinary MSK

Undesirable feature of GMSK :

modulation signal no longer confined into a single bit interval

↳ generation of ***controlled form of inter-symbol interference***

(which increases with decreasing WT)

➡ choice of WT offers trade-off between spectral compactness and reduction in receiver performance

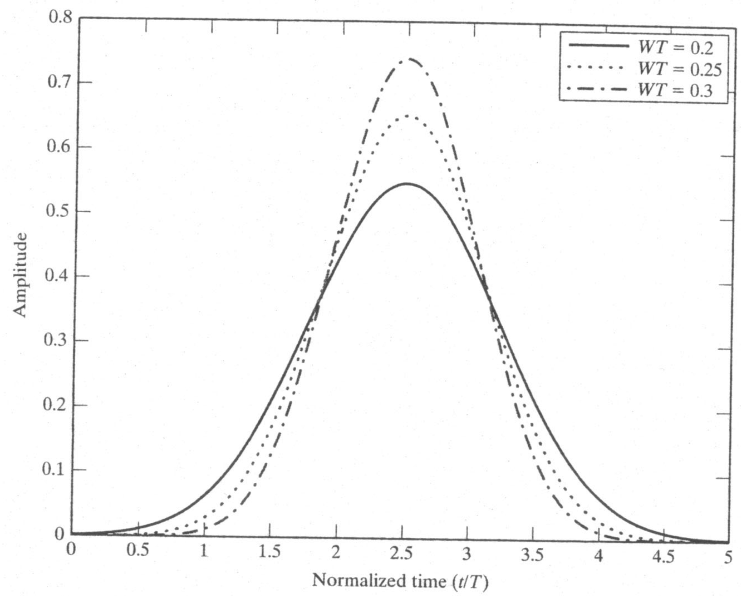
Compromise value : $WT = 0.3$ ➡ ensures that side-lobes drop by at least 40dB

↳ effects of side-lobes are negligible

Corresponding degradation in noise performance : 0.46dB

↳ small price to pay for the desirable compactness of GMSK signal

Frequency shaping pulse truncated at $t = \pm\, 2.5T$ and shifted in time by $2.5T$:



8 Comparison of Modulation Strategies for Wireless Communications

Linear or Nonlinear channel ?

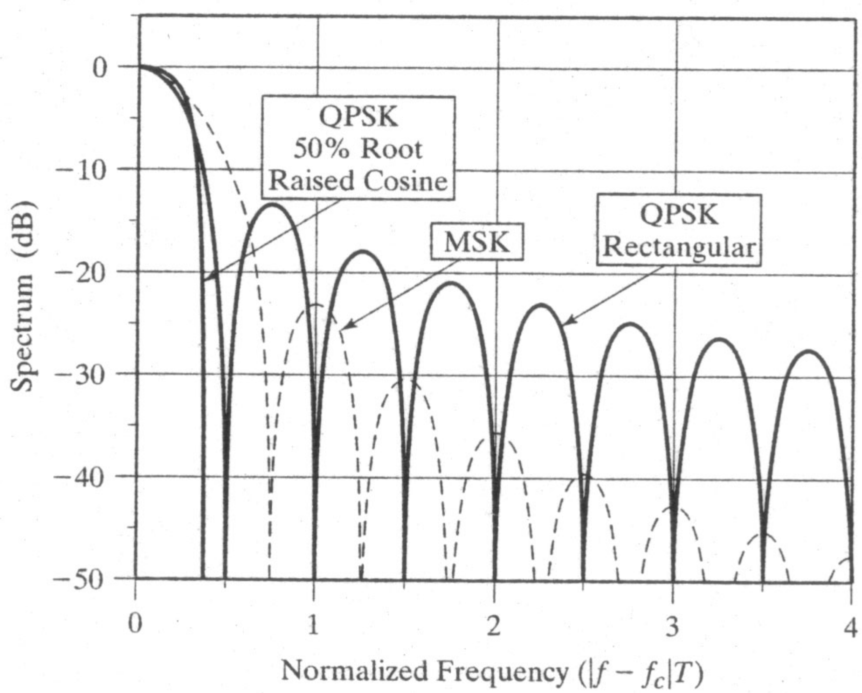
↳ Depends if the transmit power amplifier is operated in its linear or nonlinear region

8.1 Linear channel

Criterion : transmit spectrum

- MSK ↘ as the inverse fourth power
- QPSK ↘ as the inverse square

QPSK with root Raised Cosine pulse shaping has narrowest main lobe and has negligible side lobes



8.2 Nonlinear channel

Nonlinear effects depend upon ***envelope variation***

↳ No effect on rectangular QPSK, MSK and GMSK

➡ Effects on QPSK with root RC filtering

(rely on its envelope variation to produce compact spectrum)

Phase distortion :

Depend on type of modulation :

- Can be tolerate in BPSK

- Should be very small for 64-QAM

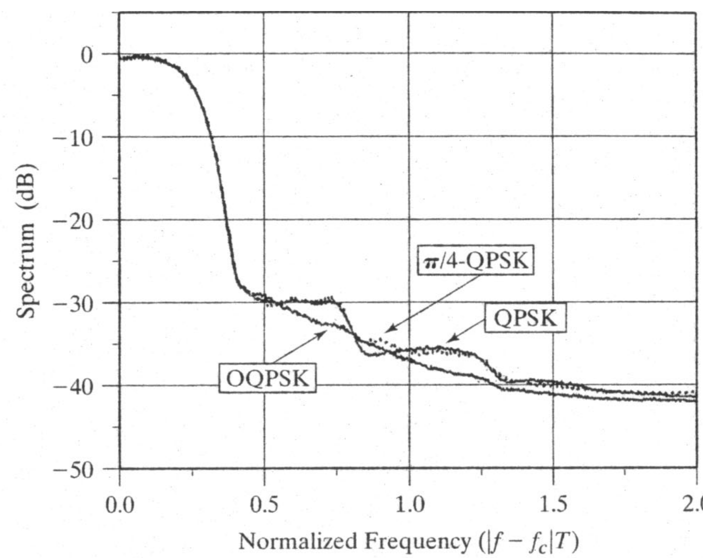


FIGURE 3.29 Comparison of different QPSK spectrum when passed through an ideal nonlinear amplifier with a 1 dB input backoff. All three modulation schemes use root-raised-cosine pulse shaping with 50% rolloff.

In practice :

Choice of modulation is a tradeoff between:

- transmit spectrum
- simplicity of detection
- error rate performance

QPSK with root raised-cosine filtering appears to be the method of choice

9 Performance : Bit Error Rate

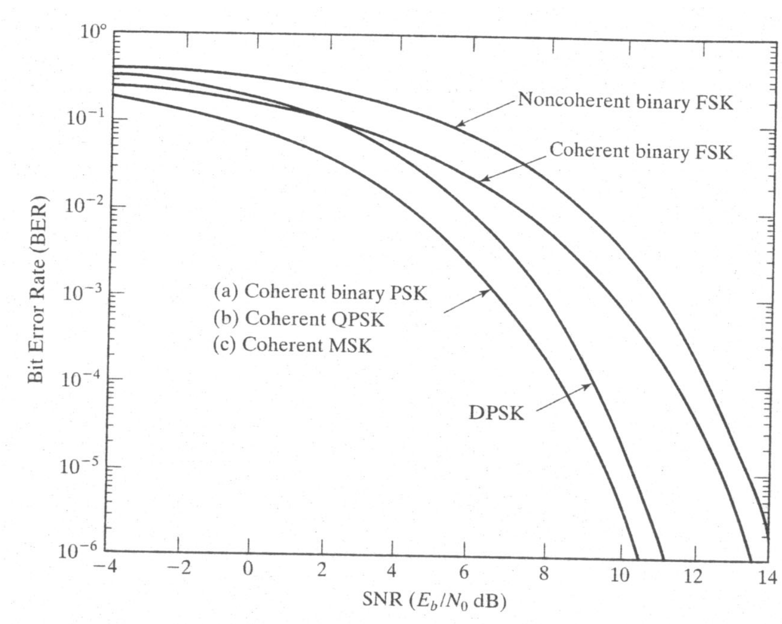
Performance of system measured in terms of *average probability of symbol error*

↳ Bit Error Rate (BER)

Signaling Scheme	BER (Additive white Gaussian noise channel)	BER (Slow Rayleigh fading channel)
(a) Coherent BPSK Coherent QPSK Coherent MSK	$\frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$	$\frac{1}{2}\left(1-\sqrt{\frac{\gamma_0}{1+\gamma_0}}\right)$
(b) Coherent BFSK	$\frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$	$\frac{1}{2}\left(1-\sqrt{\frac{\gamma_0}{2+\gamma_0}}\right)$
(c) Binary DPSK	$\frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$	$\frac{1}{2(1+\gamma_0)}$
(d) Noncoherent BFSK	$\frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)$	$\frac{1}{2+\gamma_0}$

Definitions:
E_b = transmitted energy per bit
N_0 = one-sided power spectral density of channel noise
γ_0 = mean value of the received energy per bit-to-noise spectral density ratio

For additive white Gaussian channel (AWG channel) :



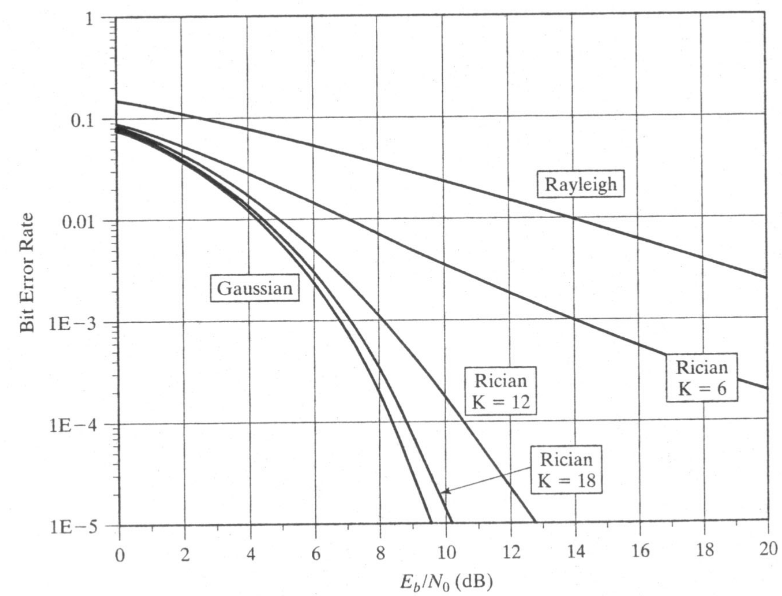


FIGURE 3.33 Comparison of performance of coherently-detected BPSK over different fading channels.

↳ Rayleigh fading process result in severe degradation

Homework

1) What is the problem of traditional QPSK ? How to overcome this problem ?

2) What is the interest of pulse shaping ?

3) How does Rayleigh fading channel affect the bit error rate when using BPSQ modulation, comparing with Gaussian noise channel ?