Orthogonal Frequency Division Multiplexing

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Abstract—**Orthogonal Frequency Division Multiplexing is one of the radio interface techniques that has gained attraction recently. In this paper, the basics behind the OFDM is discussed. The key issue in OFDM is the division of the frequency-selective transmission channel into several subchannels, which can be characterized as flat fading channels. Another significant property of the OFDM is the IDFT / DFT pair. All signal processing is made in the frequency domain and before transmission the signal is transformed to the time domain. OFDM is very tolerant to ISI and it is spectrally efficient. On the other hand, OFDM is very susceptible to phase and frequency offsets.**

Index Terms—**Orthogonal Frequency Division Multiplexing, Fourier transformation, subchannel.**

I. INTRODUCTION

ORTHOGONAL Frequency Division Multiplexing (OFDM) has become an attractive technique and gained more popularity recently. Many new communication systems have selected OFDM because its good properties, e.g. tolerance to inter-symbol interference (ISI) and good spectral efficiency. Although the idea of OFDM was developed in the 60's, the major boost for OFDM was the lowered prices for integrated circuits and the possibility to use fast Fourier-transform.

At the moment OFDM is used in wireline and wireless communications. Systems such as ADSL, Power Line Communications, WiMAX, wireless lans, digital radio and digital television are using OFDM. In this paper some insight is given to the basic operation and to the theory of OFDM.

II. ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

Orthogonal Frequency Division Multiplexing (OFDM) is technique based on multi carrier modulation (MCM) and frequency division multiplexing (FDM). OFDM can be considered as a modulation or multiplexing method. The basic idea behind multi carrier modulation is to divide the signal bandwidth into parallel subcarriers or narrow strips of bandwidth. Unlike traditional MCM system, where subcarriers are non-overlapping, OFDM uses subcarriers that are mathematically orthogonal; information can be sent on parallel overlapping subcarriers, from which information can be extracted individually. These properties help to reduce interference caused by neighboring carriers and makes OFDM based systems more spectrally efficient as shown in Figure 1.

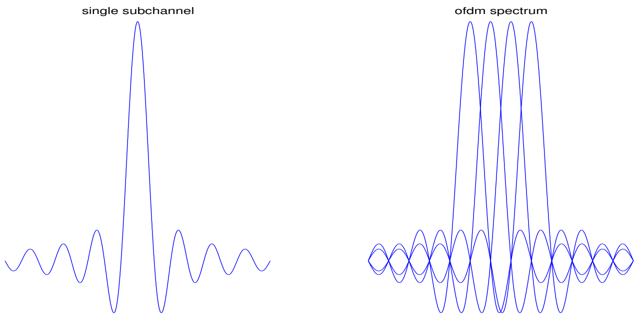

Figure 1. MCM and OFDM spectra.

Dividing channel into smaller subchannels helps OFDM to combat against frequency selective fading. Narrow subchannel bandwidths leads to each subchannel to experience flat fading channel in the transmission medium. Other advantages of OFDM based systems are the simplicity of implementation, robustness to channel impairments and narrowband interference. It allows the use of advanced antenna techniques.

A. *OFDM Symbol Creation*

The OFDM symbol is created in the digital domain before transmission. Serial data is first mapped using common methods e.g. BPSK or 16-QAM. This data stream is converted into N parallel streams, which are to be converted into an OFDM symbol. An OFDM symbol generated by an N-subcarrier OFDM system, symbol consists of N samples and then the OFDM symbol is

$$x_k = \sum_{n=0}^{N-1} a_n e^{\frac{j2\pi kn}{N}} \tag{1}$$

Where a_n is the data symbol on the n-th subcarrier. Equation(1) is equivalent to the N-point inverse discrete Fourier transform (IDFT). Spacing of subcarriers and frequencies are carefully selected to achieve subcarrier orthogonality. Orthogonality by definition means, that the average value over time T of multiplication of two signals is zero.

$$\frac{1}{T}\int_{T} x(t)\,y(t)\,dt = 0 \tag{2}$$

Equation(2) means that the signals are uncorrelated i.e. they are two different and independent signals. In OFDM, Sinc -shaped pulses are used as subcarrier spectra. According the properties of sinc-pulses, zero crossings are located at the multiples of 1/T. The use of sinc-pulses and subcarrier center frequency f_i selection with equation(3), subcarrier orthogonality is maintained.

$$f_i = f_c + \frac{i}{T} \quad i = \frac{-N}{2} \ldots \frac{N}{2} \tag{3}$$

Where f_c is the channel center frequency and N is the number of subchannels. This way each subcarrier has the maximum at its own center frequency and zero at the center frequency of the other subcarrier as shown in.

After serial-to-parallel conversion, inverse discrete Fourier Transform (IDFT) is applied to each stream. In practice, this transform can be implemented very efficiently by the inverse fast Fourier Transform (IFFT). This equals transition from frequency-domain to time-domain. After IFFT, all parallel data is summed and transmitted.

One of the most important features in OFDM system is the division of the frequency selective channel into smaller subchannels. These subchannels can be considered to be equal to coherence bandwidth, in which the channel is behaving like flat fading channel, if the system has been correctly designed. Whole OFDM symbol experiences frequency selective fading channel and the subcarrier signals flat fading channel as shown in Figure 2.

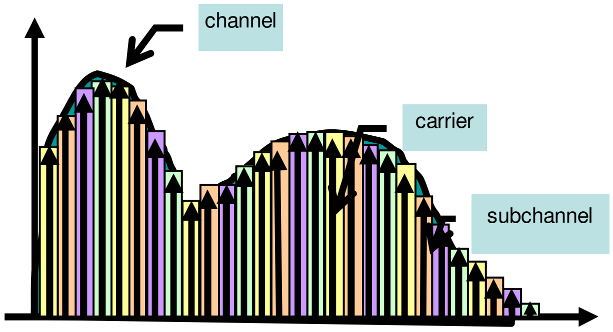

Figure 2. Flat fading subchannels.

The coherence bandwidth of the channel is proportional to the inverse of the delay spread, Bm = 1/Tm, and is a measure of frequency selectivity of the channel. When the coherence bandwidth is larger than the symbol bandwidth, channel is flat fading. Frequency selective fading occurs when the symbol bandwidth is larger than coherence bandwidth.

Delayed copies of the symbol cause intersymbol interference (ISI). ISI causes errors to the received symbol sequence and requires some form of error detection or correction. With out possibility to correct the received symbol, re-transmission is required to achieve reliable transmission. Moving scatterers, transmitter or receiver cause Doppler-shift. In OFDM, Doppler-shift causes subcarriers to shift on adjacent subcarrier. This phenomenon is called intercarrier interference (ICI). ICI is ‘crosstalk’ between different subcarriers, which means that they are no longer orthogonal.

E.g. scatterers moving 120 km/h causes 250 Hz Doppler-shift. As subchannel bandwidths are 312,5 kHz, Doppler-shift has no significant meaning.

Conversion from fast serial data stream into N slower parallel data streams enables possibility to use longer symbol periods. Longer transmission times allow more delay spread than shorter symbol durations. This property makes OFDM suitable for difficult multipath environments, because longer symbol times make OFDM robust against ISI. Even though OFDM is very resilient to ISI, it is very susceptible to frequency offsets and phase noise. Minor variations in frequencies yield directly to loss of orthogonality.

B. Guard time

To combat intersymbol interference a guard time is inserted between consecutive OFDM symbols. The guard time allows multipath components to fade away before the information is extracted from the next symbol.

Guard time is set to be larger than the delay spread. This way ISI caused by multipath propagation is almost completely removed. As long as the delay spread is smaller than the guard time, there is no limitation in multipath component signal levels. This still leaves interference introduced by copies of the same signal.

The guard time is usually implemented with a cyclic extension of the symbol. Part of the signal end is placed in the front of the signal. This effectively extends signal period and still maintains orthogonality of the waveform. In practice guard time is only added to OFDM symbol, not to all subcarriers.

Because signal waveform in guard time is a cyclic extension of the signal, every multipath component has an integer number of cycles in fast Fourier Transform (FFT) integration time. FFT integration time is the same as symbol time. This yields to same phase sine waves to sum up to a sine wave. If delay spread exceeds guard time, orthogonality is lost and phase transitions cause interference.

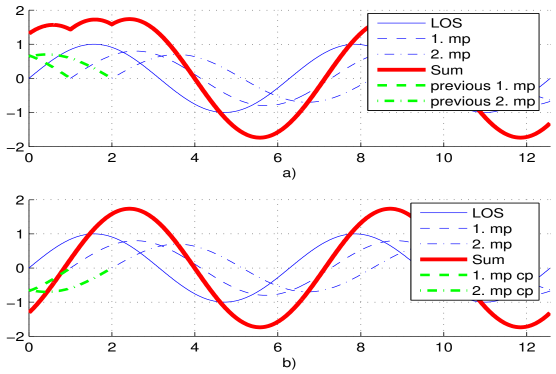

Figure 3. Sine summation.

In Figure 3 a) there is no ISI and all multipath components are within guard time. In Figure 3 b) delay spread exceeds guard time and ISI is introduced. Multipath components are shifted in phase, which causes phase shift and amplitude decrease in received signal.

C. Channel coding and interleaving

1) Channel coding

The goal of channel coding, or error control coding, is to improve bit error ratio (BER) performance by adding structured redundancy to the transmitted data. Channel coding means that additional redundant bits are added to the signal to enable error detection and error correction. Channel impairments can cause errors to the signal; these impairments can be e.g. noise, fading, interference or jamming. Basic channel coding methods are block coding and convolutional coding. In OFDM channel coding is done with convolutional coding, because convolutional coding offer good performance with low implementation cost. Coding is performed on serial data before symbol mapping.

Convolutional coding operates with bit streams and has memory that utilizes previous bits to encode or decode following bits. Convolutional encoder is defined with three variables: number of output bits n, number of input bits k and memory depth L. Encoder maps k input bits into n output bits. From memory length can be derived constraint length C using equation(4). Constraint length tells how many output bits are influenced with single input bit. The error correction capacity is related with this value.

$$C = n\left(L+1\right) \tag{4}$$

Code efficiency is measured by code rate, given by equation(5). Varying variables k and n, coding rate can be modified. Coding rate can also be modified by puncturing the coded data stream. Code puncturing involves removing certain channel coded bits.

$$R_c = \frac{k}{n} \tag{5}$$

The basic measure of channel coding performance is coding gain, which is the increase in signal-to-noise ratio (SNR) in AWGN –channel (Additive White Gaussian Noise). Coding gain achieved with convolutional coding over an uncoded BPSK or QPSK system is given with equation(6) [1].

$$C_{gain} = 10\log_{10}\left(R_c d_{free}\right) \tag{6}$$

Where d_{free} is the free distance, defined as the minimum Hamming distance between two different code words.

The simplicity of implementing convolutional codes comes from possibility to represent codes with generator polynomials. Generator polynomials are impulse responses from each decoder output.

2) Interleaving

Because of frequency selective fading, in OFDM certain subchannels can be located in a deep fades in channel and information carried by these subcarriers are lost. This effect causes errors to occur in bursts rather than being randomly scattered. Even the most forward error correction (FEC) codes are not designed to deal with error bursts. To make errors appear more randomly, interleaving is performed on the coded bit stream. Interleaving is a way to permute bits in a certain way and at the receiver reverse permutation is performed. A commonly used interleaving method is block interleaving. In block interleaving data is written in to a matrix row-by-row and read out column-by-column.

D. Windowing

Sharp phase transitions in bit sequences, caused by modulation, can result to out-of-band spectrum to decrease slowly according to a sinc-function. Windowing is applied to make spectrum go down more rapidly. Windowing is applied to individual OFDM symbols. A commonly used window type is the raised cosine window, where the channel bandwidth may extend from the minimum value W = R/2 to an adjustable value from W to 2W. A periodic waveform is truncated to a single OFDM symbol length by applying windowing in time domain. Figure 4 depicts windowing and shows the effect of the roll-off factor.

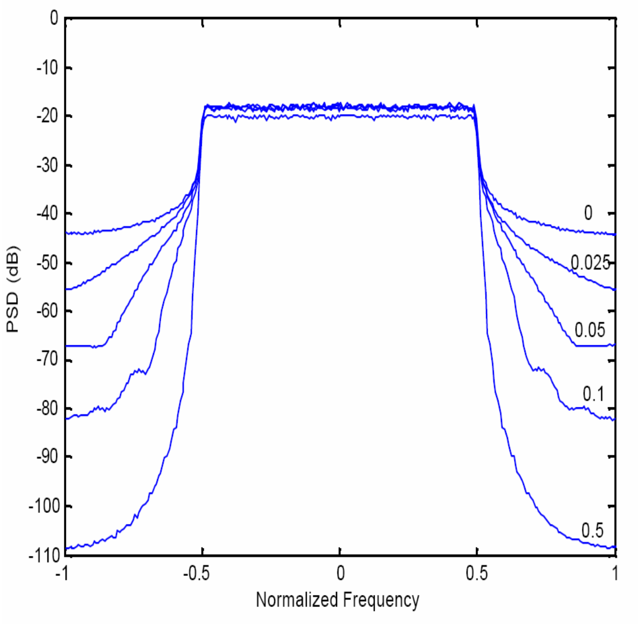

Figure 4. Effect of roll-off factor with 128 subcarriers, β=0…0,5.

E. Equalization and channel estimation

Although ISI has been almost completely eliminated with the use of guard time, new problems arise from varying channel. An OFDM system is developed to endure signal copies produced by multipath propagation, but multipath environment introduces also fading effects. Reliable detection of an OFDM symbol in time-varying multipath propagation environment requires channel estimates and equalization. The channel transfer function can have deep fades or zeros at some of the frequencies and to recover the information located around these locations, received signal form has to be equalized.

Radio channels are fading both in time and in frequency. Hence, a channel estimator has to estimate time-varying amplitudes and phases of all subcarriers [6]. Channel estimates can be formed from known subcarrier values known as pilots. Pilot symbols are inserted in the time and frequency dimensions. Pilot spacing, in time and in frequency domain, has to follow sampling theorem. With this condition, pilot spacing in time D_t and in frequency D_f are given with equations(7) and (8) [3].

$$D_t \le \frac{1}{2 f_d T} \tag{7}$$

$$D_f \le \frac{1}{2 f_s \tau_{\max}} \tag{8}$$

Where f_d, f_s, T and τ_{max} are maximum Doppler-frequency, subcarrier spacing, OFDM symbol duration and excess delay, respectively.

In principle, the channel estimation for OFDM system is performed by the two following steps. First, the channel transfer function at the positions of the pilot symbols is obtained by dividing the received pilot symbols by the known transmitted pilot symbols. Secondly, the channel transfer function at the position of the data symbols is obtained by interpolating the channel transfer function at the positions of pilot symbols [4].

The quality of channel estimate depends of the number of pilots and scale of the channel variations. Increasing the number of pilots improves the estimate, but on the contrary it decreases spectral efficiency or ergodic capacity and vice versa. Channel estimates at pilot locations can be obtained several methods e.g. least-squares (LS) or linear minimum mean square error estimator (LMMSE) [5].LMMSE is more elaborate method compared to LS and it is capable of reducing the effect of the error term.

The performance of the interpolation step depends on the used interpolation technique. Various interpolation methods are available e.g. Linear, Wiener, Si, Cubic and Gaussian interpolation. Each interpolation method makes trade-offs between complexity and suitability for different channel models. E.g. linear interpolation is very simple to implement, but it is only recommended to be used in the time direction for a slowly time-selective channel [4].

In [3] a low pass filter (LPF) combined with an interpolator for channel estimation is proposed. LPF removes the noise components while preserving the channel impulse response. Channel impulse response is obtained via IFFT of the estimated channel transfer function. It can be assumed that the most powerful multipath is contained within the guard time and the remainder outside the guard time is considered as noise components.

After achieving the channel transfer function for all subchannels, received signal form can be equalized. At the receiver, each subcarrier sample is multiplied with the corresponding coefficient of the channel equalizer. Also for equalization, there are several existing methods. Equalization can be made with e.g. zero-forcing (ZF) criterion, minimum mean square error (MMSE) criterion. ZF equalizers are relatively simple, but they are optimal for lineal channels. It enhances noise at low power levels i.e. deep fades or zeros at the channel transfer function. MMSE equalizer makes trade-off between ISI minimization and noise. MMSE requires good knowledge of the channel and it is difficult to implement in practice, because the radio channel is unknown and time-varying. This yields to use of adaptive or decision feedback equalizers.

F. Peak to average power ratio

An OFDM signal consists of a number of independent subcarriers, which can give a large peak-to-average power ratio (PAPR) when added coherently. When N signals are added with the same phase, they produce a peak power that is N times the average power. As a result, linear behavior of the system over a large dynamic range is needed and the efficiency of the output amplifier is reduced. The average power must be kept low in order to prevent the transmitter amplifier saturation. Minimizing the PAPR allows higher average powers to be transmitted and improves the SNR at the receiver. Several methods are proposed for reducing PAPR, which can be divided in to three categories: signal distortion, coding techniques and scrambling techniques [6].

Signal distortion techniques include clipping, peak windowing and peak cancellation to reduce PAPR. This method is very simple, but it produces in-band distortion and out-of-band spectral leakage. Spectral leakage is reduced with peak windowing. Coding techniques tries to reduce PAPR by excluding the symbols that have high PAPR and uses FEC to recover the excluded symbols. The basic idea behind scrambling techniques is that for each symbol, the input sequence is scrambled by a certain number of scrambling sequences and the output signal with the lowest PAPR is transmitted together with information about the actual scrambling code.

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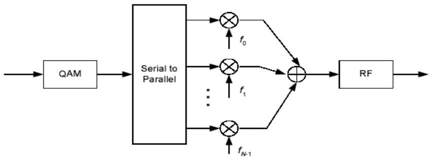
HOMEWORK

- Derive expression for OFDM-signal
- Use 4 subchannels and 4QAM
- Input data sequence:
  11 01 00 10
- Subcarrier frequencies are:
  $-2f_c$ $-1f_c$ $1f_c$ $2f_c$



OFDM transmitter

4QAM