SPACE TIME CODING FOR MIMO SYSTEMS

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ABSTRACT

With space-time codes (STC) the same information is transmitted in appropriate manner simultaneously from different transmit antennas to obtain transmit diversity. The main idea of transmit diversity is that if a message is lost in a channel with probability p and we can transmit replicas of the message over n independent such channels, the loss probability becomes p^n. Using diversity, more reliability is given to the symbols which allows employing higher order constellation resulting in higher throughput.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) communication technology has received significant recent attention due to the rapid development of high-speed broadband wireless communication systems employing multiple transmit and receive antennas. Information theoretic results show that MIMO systems can offer significant capacity gains over traditional single-input single-output channels.

This increase in capacity is enabled by the fact that in rich scattering wireless environments, the signals from each individual transmitter appear highly uncorrelated at each of the receive antennas. When conveyed through uncorrelated channels between the transmitter and the receiver, the signals corresponding to each of the individual transmit antennas have attained different spatial signatures. The receiver can exploit these differences in spatial signatures to separate the signals originated from different transmit antennas.

In Space-Time Coding (STC) systems , the same information symbol stream is transmitted from different transmit antennas in appropriate manner to obtain transmit diversity.

This paper is organized as follows. In Section 2 diversity concepts are introduced. In Section 3,4 and 5 space-time coding techniques are discussed. Error probability is introduced in section 6. Differential STBC (DSTBC) is presented in section 7. STC for OFDM are included in the section 8. Finally conclusions and references.

2. DIVERSITY

Depending on surrounding environment, a transmitted radio signal propagates through several different paths. This phenomenon is often referred as multipath propagation [3]. The signal received by the receiver antenna consists of the superposition of various multipaths. If there is Non-Line of Sight components between the transmitter and receiver , the attenuation coefficients corresponding to different paths are assumed to be independent and identically distributed. In which case the central limit theorem applies and the resulting path can be modeled as a complex Gaussian random variable. In this case, the channel is said to be Rayleigh.

Signal power in a wireless system fluctuates. When this signal power drops significantly, the channel is said to be in fade. Diversity is used in wireless channels to combat the fading. Receive diversity and transmit diversity mitigate fading and significantly improve link quality. The receive antennas see independently faded versions of the same signals. The receiver combines these signals so that the resultant signal exhibits considerably reduced amplitude fading.

In most scattering environments, antenna diversity is a practical, effective and, hence, a widely applied technique for reducing the effect of multipath fading . The classical approach is to use multiple antennas at the receiver and perform combining or selection and switching in order to improve the quality of the received signal. The major problem with using the receive diversity approach is the cost, size, and power of the remote units. The use of multiple antennas and radio frequency (RF) chains makes the remote units larger and more expensive. As a result, diversity techniques have almost exclusively been applied to base stations to improve their reception quality. A base station often serves hundreds to thousands of remote units. It is therefore more economical to add equipment to base stations rather than the remote units. For this reason, transmit diversity schemes are very attractive.

In recent years it has been realized that many of the benefits as well as substantial amount of the performance gain of receive diversity can be reproduced by using multiple antennas at the transmitter to achieve transmit diversity.

The use of transmit diversity at the base stations in a cellular o wireless local area network has atracted an special interest; this is so primarily because a performance increase is possible without adding extra antennas, power consumption or significant complexity to the mobile. Also, the cost of the extra transmit antenna at the base station can be shared among all users.

3. SPACE TIME CODING

An effective and practical way to approaching the capacity of MIMO wireless channels is to employ Space-Time Coding (STC). STC is a coding technique designed to be used with multiple transmit antennas. Coding is performed in both spatial and temporal domains to introduce correlation between signals transmitted for various antennas at various time periods [2]. Space-Time coding can achieve transmit diversity and power gain over spatially uncoded systems without sacrificing the bandwidth. There several approaches in coding structures, Space-Time Block Coding (STBC), Space Time Trellis Coding (STTC), Differential Space-Time Block Coding (DSTBC) that will be presented in the next sections.

3.1. STC - An introductory example

- One transmit antenna and two receive antennas

  Considering a system with two receive antennas and one transmit antenna. If the fading is frequency flat, the twor received signals can be written as

$$\begin{aligned} y_1 &= h_1 s + n_1 \\ y_2 &= h_2 s + n_2 \end{aligned} \tag{1}$$

  where h_1 and h_2 are the channel gains, s is the transmitted signal and n_1 ,n_2 are mutually uncorrelated noise terms. By the following linear combination, it is possible to recover s

$$\begin{aligned} \hat{s} &= w_1 * y_1 + w_2 * y_2 \\ &= (w_1^* h_1 + w_2^* h_2) + w_1^* n_1 + w_2^* n_2 \end{aligned} \tag{2}$$

  where w_1 and w_2 are the weights. The SNR in $\hat{s}$ is given by

$$SNR = \frac{|w_1^* h_1 + w_2^* h_2|^2}{(|w_1|^2 + |w_2|^2)\sigma^2} E[|s|^2]$$

  where σ^2 is the power noise. We can choose w_1 and w_2 proportional to h_1 and h_2 that maximizes the SNR. The resulting SNR is proportional to $(|h_1|^2 + |h_2|^2)$ If the fading is Rayleigh, then $(|h_1|^2 + |h_2|^2)$ is χ distributed, and we can show that the error probability of detecting s decay as SNR_a^{-2} in high SNR values. In single antenna case, the error probability of detecting s decay as SNR^{-1}. The diversity order of the system is the slope of the BER curved plotted versus the average SNR. In our example diversity gain equal to 2 was reached.

- Two transmit antennas and one receive antenna At a given time instant, the symbol s pre-weighted with w_1 and w_2 is transmitted. The received sample can be written

$$y = h_1 w_1 s + h_2 w_2 s + n \tag{3}$$

  where n is the noise sample. The SNR in y is given by

$$SNR = \frac{|h_1 w_1 + h_2 w_2|^2}{\sigma^2} E[|s|^2] \tag{4}$$

  If w_1 and $w_2$are fixed , the SNR has the same statistical distribution as $|h_1|^2$ (or $|h_2|^2$). Therefore, if the weights are not allowed to depend on h_1 and h_2 it is impossible to achieve diversity of order two. However, if we assume that the transmitter knows the channel, and w_1 and w_2 are chosen to be functions of h_1 and h_2, it is possible to achieve an error probability that behaves SNR^{-2}.

  We have seen that without channel knowledge at the transmitter, diversity can not be achieved. However, if two time intervals for the transmission is allowed, we can achieve diversity of order two easily. At Time=t, antenna 1 is used and at Time=t+1 antenna 2 is used.

  In this case, the received samples signal at different time instants are

$$\begin{aligned} y_1 &= h_1 s + n_1 \\ y_2 &= h_2 s + n_2 \end{aligned} \tag{5}$$

  Equation 5 is of the same form than equation 1, so that the error associated with this method is equal to that for the previous case (1×2 system). In this case diversity gain equal to 2 is achieved but a sacrifice in information rate is necessary.

Space Time coding is concerned with the harder and interesting topic, How maximize the transmitted information rate at the same time that minimize the error probability.

4. ALAMOUTI CODE

Figure 1 shows the baseband representation of the Alamouti two branch transmit diversity scheme [1]. The scheme uses two transmit antennas and one receive antenna and may be defined by the following three functions

- The encoding and transmission sequence of information

  At a given symbol period, two signals are simultaneously transmitted from the two antennas. The signal transmitted from antenna zero is denoted by S_0 and from antenna one by S_1 . During the next symbol period signal S_1^* is transmitted from antenna zero, and signal $S_0^*$is transmitted from antenna one. The encoding, however, may also be done in space and frequency. Instead of two adjacent symbol periods, two adjacent carriers may be used (spacefrequency coding). The 2×2 space time code is written in matrix form as

$$S = \begin{bmatrix} s_0 & s_1 \\ -s_1^* & s_0^* \end{bmatrix} \tag{6}$$

The channel at time t may be modeled by a complex multiplicative distortion $h_0(t)$ for transmit antenna zero and $h_0(t)$ for transmit antenna one. Assuming that fading is constant across two consecutive symbols, we can write

$$\begin{aligned} h_0(t) &= h_0(t+T) = \alpha_0 e^{j\theta_0} \\ h_1(t) &= h_1(t+T) = \alpha_1 e^{j\theta_1} \end{aligned} \tag{7}$$

The received signal can be written

$$\begin{aligned} r_0 &= r(t) = h_0 s_0 + h_1 s_1 + n_0 \\ r_1 &= r(t+T) = -h_0 s_1^* + h_1 s_0^* + n_1 \end{aligned} \tag{8}$$

where r_0 and r_1 are the received signals at time t and $t+T$.

- The combining scheme at the receiver

  The combiner builds the following two combined signals that are sent to the maximum likelihood detector

$$\begin{aligned} \hat{s}_0 &= h_0^* r_0 + h_1 r_1^* \\ \hat{s}_1 &= h_1^* r_0 - h_0 r_1^* \end{aligned} \tag{9}$$

The decision statistics can be expressed as

$$\widetilde{s}_0 = (\alpha_0^2 + \alpha_1^2) s_0 + h_0^* n_0 + h_1 n_1^* \tag{10}$$
$$\widetilde{s}_1 = (\alpha_0^2 + \alpha_1^2) s_1 - h_0 n_1^* + h_1^* n_0 \tag{11}$$

- The decision rule for maximum likelihood detection

  These combined signals are then sent to the maximum likelihood detector.

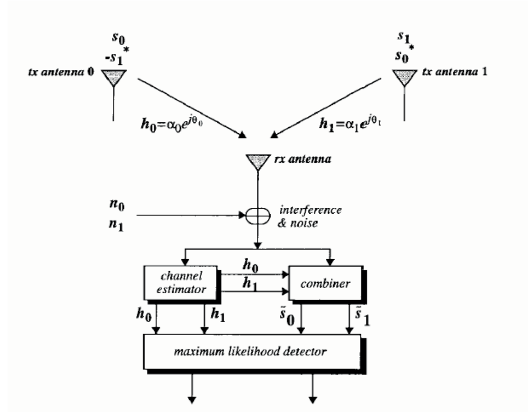

Fig. 1. Alamouti transceiver structure

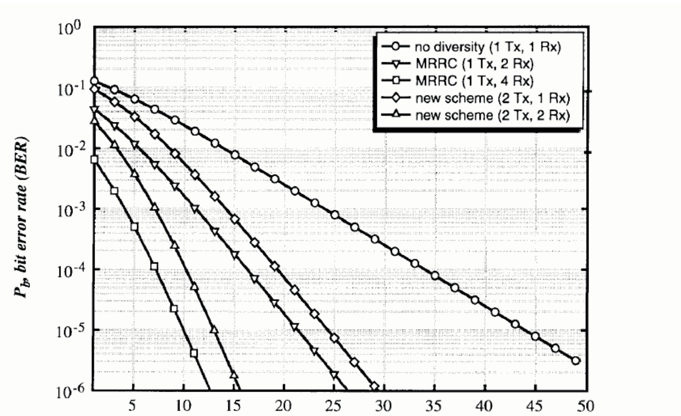

Fig. 2. BER Alamouti code compared with MRCC system

The resulting combined signals in 11 are equivalent to that obtained from two-branch MRRC. The only difference is phase rotations on the noise components which do not degrade the effective SNR.

Therefore, the resulting diversity order from the Alamouti transmit diversity scheme with one receiver is equal to that of two-branch MRRC.

Figure 2 shows the performance of Alamouti code compared with a MRCC system in a Rayleigh fading channel using BPSK modulation. From the figure, we can conclude that the performance of Alamouti code with two transmitters and a single receiver is 3 dB worse than two-branch MRRC. The 3-dB penalty is incurred because is assumed that each transmit antenna radiates half the energy in order to ensure the same total radiated power as with one transmit antenna. If each transmit antenna was to radiate the same energy as the single transmit antenna for MRRC, the performance would be identical

5. GENERALIZED STBC

The Alamouti scheme works only with two transmit antennas. This scheme was later generalized in to an arbitrary number of transmit antennas. Similarly to the Alamouti code, the general STBC is defined by a code matrix with orthogonal columns. Just like in the Alamouti scheme, a simple linear receiver is also obtained due to the orthogonality of the columns of the code matrix. In general, an STBC is defined by a $(p \times n_T)$ matrix G. The entries of the matrix G are linear (possibly complex) combinations of the variables $x_1; x_2; \ldots; x_k$ (representing symbols). The columns of the matrix represent antennas and the rows time slots [2].

Therefore, p time slots are needed to transmit k symbols, resulting in a code rate $R = k/p$.

It is of special interest code matrices achieving the maximum transmission rate permitted by the STC theory, i.e, R = 1 symbol/channel use. For a fixed n_T , among the code matrices that achieve the maximum rate, we will be interested in those with minimum values of p or equivalently, minimum number of time slots needed to transmit a block. These code matrices are referred as delay optimal and they are interesting because they minimize the memory requirements at the transmitter and at the receiver [5].

The construction of STBC using the generalized complex orthogonal for a rate equal to $1/2$ give the following matrix:

$$G_3 = \begin{bmatrix} s_1 & s_2 & s_3 \\ -s_2 & s_1 & s_4 \\ -s_3 & s_4 & s_1 \\ -s_4 & -s_3 & s_2 \\ s_1^* & s_2^* & s_3^* \\ -s_2^* & s_1^* & s_4^* \\ -s_3^* & s_4^* & s_1^* \\ -s_4^* & -s_3^* & s_2^* \end{bmatrix} \quad (12)$$

G_3 code is designed for 3 transmit antennas with a code rate $R = 1/2$ and G_4 is designed for 4 antennas with the same code rate.

$$G_4 = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2 & s_1 & s_4 & s_3 \\ -s_3 & s_4 & s_1 & -s_2 \\ -s_4 & -s_3 & s_2 & s_1 \\ s_1^* & s_2^* & s_3^* & s_4^* \\ -s_2^* & s_1^* & s_4^* & s_3^* \\ -s_3^* & s_4^* & s_1^* & -s_2^* \\ -s_4^* & -s_3^* & s_2^* & s_1^* \end{bmatrix} \quad (13)$$

Compared with Alamouti code, the STBC G_3 and G_4 have two disadvantages: The bandwidth efficiency is reduced by a factor of two. The number of time slots across which the channel is required to have a constant fading envelope is increased by a factor of four. Figure 3 shows the BER performance of G_3,G_4 and G_2 (Alamouti). 16 QAM modulation is used for G_3 and G_4 and 4 DPSK modulation for Alamouti in order to have the same data rate (2 bits/sec) [6]. BER results for 1 bit/sec is shown in figure 4.

6. ERROR PROBABILITY ON SLOW FADING CHANNELS

On slow fading channels, the fading coefficients within each frame are constant. So we can ignore the subscript of the fading coefficients

$$h_{j,i}^1 = h_{j,i}^2 = \ldots = h_{j,i}^L = h_{j,i}$$
$$i = 1, 2, \ldots, n_T, \; i = j, 2, \ldots, n_R \quad (14)$$

Let us define a codeword difference matrix $B(X, \hat{X})$ as

$$B = \begin{bmatrix} x_1^1 - \hat{x}_1^1 & x_2^1 - \hat{x}_2^1 & \ldots & x_L^1 - \hat{x}_L^1 \\ x_1^2 - \hat{x}_1^2 & x_2^2 - \hat{x}_2^2 & \ldots & x_L^2 - \hat{x}_L^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{nT} - \hat{x}_1^{nT} & x_2^{nT} - \hat{x}_2^{nT} & \ldots & x_L^{nT} - \hat{x}_L^{nT} \end{bmatrix} (15)$$
(16)

We can construct an $n_T \times xn_T$ codeword distance matrix $A(X, \hat{X})$ defined as

$$A(X, \hat{X}) = B((X, \hat{X}).B^H(X, \hat{X})$$

Exist a unitary matrix V and a real diagonal matrix Δ such that

$$VA(X, \hat{X})V^H = \Delta$$

where the rows of V are the eigenvectors of $A(X, \hat{X})$, forming a complete orthogonal basis of an N-dimensional vector space. The diagonal elements of Δ are the eigenvalues $\lambda_1 > 0, i = 0, 1, \ldots, n_T$ of $A(X, \hat{X})$.

The modified Euclidean distance between the two space-times codeword matrices X and $\hat{X}$ can be written as

$$\begin{aligned} d_h^2(X, \hat{X}) &= \sum_{j=1}^{n_R} h_j A(X, \hat{X}) h_j^H \\ &= \sum_{j=1}^{n_R} \sum_{i=1}^{n_T} \lambda_i |\beta_{j,i}|^2 \end{aligned} \quad (17)$$

where $\beta_{j,i} = h_j.v_i$ and . denotes the inner product. In the case of Rayleigh fading and high SNR, the upper bound of the error probability is [2]

$$P(X, \hat{X}) \leq \left(\prod_{i=1}^{r} \lambda_i \right)^{-n_R} (\frac{E_s}{4N_0})^{-rn_R} \quad (18)$$

where r denotes the rank of the matrix $A(X, \hat{X})$ and λ_i are the nonzero eigenvalues of matrix $A(X, \hat{X})$. The exponent of the SNR term, rn_R is called diversity order and the product of the eigenvalues is called coding gain.

When a space time code is designed, the following criteria can be considered:

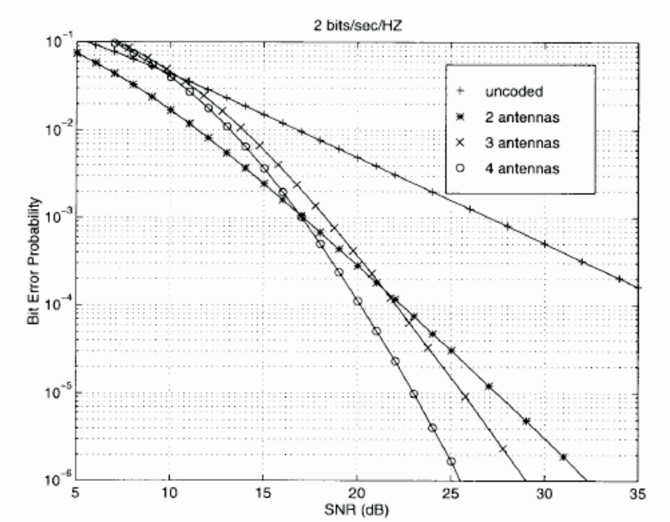

Fig. 3. BER G_3,G_4 and G_2 codes - 2 bits/sec

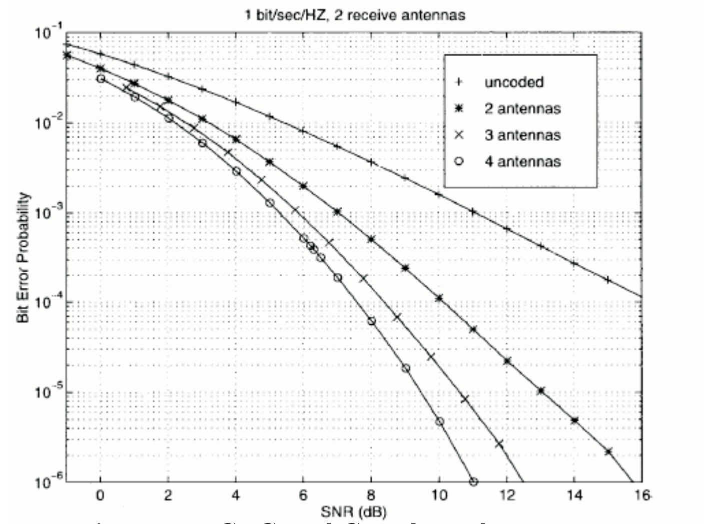

Fig. 4. BER G_3,G_4 and G_2 codes - 1 bits/sec

- Maximize the minimum rank r of the matrix $A(X, \hat{X})$ over all pairs of distinct codewords.
- Maximizes the minimum product $\prod_{i=1}^{r} \lambda_i$ of matrix $A(X, \hat{X})$ along the pairs of distinct codewords with the minimum rank.

7. DIFFERENTIAL STBC

When the channel changes slowly compared to the symbol rate, the transmitter can send pilot sequences that enable the receiver to estimate the channel accurately. However, in some situations, such as high mobility channels or channel fading conditions changing rapidly, it may be difficult to estimate the channel with good precision. For such situations, it is useful to develop space time codes that do not requires channel estimates either at the receiver or at the transmitter [7].

7.1. Encoding Algorithm

The transmitter begins the transmission with sending arbitrary symbols s_1 and s_2 at time 1 and symbols $-s_2^*$ and s_1^* at time 2 unknown to the receiver. These two transmissions do not convey any information. A mapping function $\mathbf{M}$ is defined The transmitter subsequently encodes the rest of the data in an inductive manner. Suppose that s_{2t-1} and s_{2t} are sent, respectively, from transmit antennas one and two at time $2t-1$, and that $-s_{2t}^*$ and s_{2t-1}^* , are sent respectively, from antennas one and two at time $2t$. At time $2t+1$, a block of $2b$ bits B_{2t+1} arrives at the encoder. The transmitter uses the mapping function $\mathbf{M}$ and computes $\mathbf{M}(\mathbf{B_{2t+1}}) = \mathbf{A}(\mathbf{B_{2t+1}}) + \mathbf{B}(\mathbf{B_{2t+1}})$ Then it computes

$$(s_{2t+1}s_{2t+2}) = A(B_{2t-1})(s_{2t-1}s_{2t}) + B(B_{2t+1})(-s_{2t}^* s_{2t-1})$$

The transmitter then sends s_{2t+1} and s_{2t+2}, respectively, from transmit antennas one and two at time $2t+1$ and $-s_{2t+2}^*$ and s_{2t+1}^* from antennas one and two at time $2t+2$. This process is inductively repeated until the end of the frame. Block diagram of the differential encoder and decoder is shown in Figure 5.

Example Consider a BPSK constellation of two signal points $1/\sqrt{2}$ and $-1/\sqrt{2}$. The coefficient vector set is given by

$$V = [(1,0),(0,1),(-1,0),(0,-1)] \tag{19}$$

At each encoding operation, a block of $2m = 2$ bits arrives at the encoder and is mapped into V. The mapping function can be computed using

$$\begin{aligned} M(00) &= (1,0) \\ M(10) &= (0,1) \\ M(01) &= (0,-1) \\ M(11) &= (-1,0) \end{aligned}$$

Assuming that at time $2t-1$, $x_{2t-1} = -1/\sqrt{2}$ and $x_{2t} = -1/\sqrt{2}$ are sent from antennas one and two, respectively, and at time $2t$, $-x_{2t}^* = 1/\sqrt{2}$ and $x_{2t-1}^* = -1/\sqrt{2}$ are sent from antennas one and two. If the two information bits at the encoder input at time $2t+1$ are 11, according to the mapping function $M(11) = (-1,0)$, the coefficients used to compute the transmitted signals for the next two transmissions are $R_1 = -1$ and $R_2 = 0$. Thus, we have

$$\begin{aligned} x_{2t+1}, x_{2t+2} &= -1(-1/\sqrt{2}, -1/\sqrt{2}) + 0(+1/\sqrt{2}, -1/\sqrt{2}) \\ &= -1(+1/\sqrt{2}, +1/\sqrt{2}) \end{aligned} \tag{20}$$

At time $2t+1$, $x_{2t+1} = +1/\sqrt{2}$ and $x_{2t+2} = -1/\sqrt{2}$ are sent from antennas one and two and $-x_{2t+2}^* = -1/\sqrt{2}$ and $x_{2t+1}^* = +1/\sqrt{2}$ at time $2t+2$.

In [2], details related with decoding process can be found. The DSTBC detection scheme is 3 dB worse than that of the transmit diversity scheme of employs coherent detection at high SNR [7]. However, it is an excellent option in high mobility environments like cellular systems.

8. OFDM AND ST CODES

Space-time (ST) coding has been proved effective in combating fading, and enhancing data rates. Exploiting the presence of spatial diversity offered by multiple transmit and/or receive antennas, ST coding relies on simultaneous coding across space and time to achieve diversity gain without necessarily sacrificing bandwidth. Two typical examples of ST codes are ST trellis codes and ST block codes .

Multipath diversity becomes available when frequency selectivity is present, which is the typical situation for broadband wireless channels. Multiantenna transmissions over frequency-selective fading channels can potentially provide a maximum diversity gain that is multiplicative in the number of transmit antennas, receive antennas, and the channel length. Inspired by this result, a number of coding schemes have been proposed recently to exploit multipath diversity. Because they offer low-complexity equalization decoding and facilitate the support of multirate services, multicarrier transmissions are typically adopted by those schemes.

Diversity techniques designed for single carrier (SC) modulation are easily extended to OFDM modulation with the time index for SC modulation replaced by the tone index in OFDM. For example, considering the Alamouti scheme which requires that the channel remains constant over consecutive symbols periods. In the OFDM context, this translates to the channel remaining constant over consecutive tones, i.e, $H(k) = H(k+1)$ [4]. Consider two data symbols, s_1 and s_2, to be transmitted over antennas 1 and 2 respectively on tone k, and s_2^* and s_1^* are transmitted over antennas 1 and 2 respectively on tone $k+1$ within the same OFDM symbol.

The receiver detects the transmitted symbols from the received signal on the two tones using the Alamouti detection techniques. As in SC modulation, the effective channel is orthogonalized irrespective of the channel realization and the vector detection problem collapses into scalar detection problems with the effective input-output relation for symbols is given by

$$y_i = \sqrt{\frac{E_s}{2}}\, \|H[k]\|_f^2\, s_i + n_1,\ i = 1, 2 \qquad (21)$$

where n_i is a noise component with variance $\|H[k]\|_f^2 N_0$. Assuming that the $2M_R$ elements of $H[k]$ undergo independent fading, the Alamouti scheme extracts $2M_R$ order diversity, just as SC modulation.

The use of consecutive tones is not strictly necessary, any pair of tones can be used as long as the associated channel are equal.

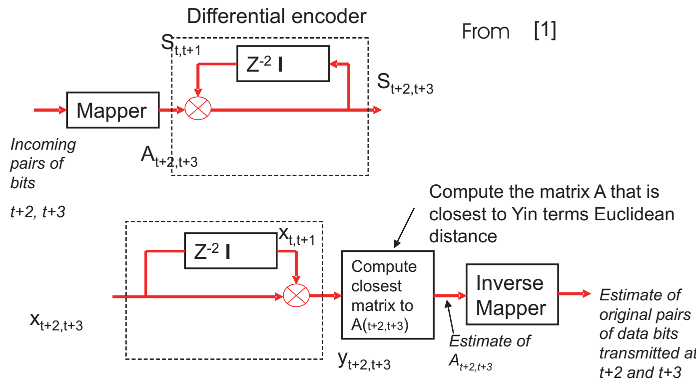

Fig. 5. Differential Encoder - Decoder

The technique can be generalized to extract spatial diversity over a large number of antennas by using STBC techniques. In this case, we need a block size $T \geq M_T$ and the channel must be identical over the T tones.

9. CONCLUSIONS

As conclusion of this paper we can mention:

- Alamouti code is the best option when 2 Transmission antennas is considered.
- Low complexity receiver is a good characteristic for STBC.
- DSTBC can be considered in high mobility channels.

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