S-72.333 Postgraduate Course in Radio Communications, Autumm 2004

Adaptive transmit beamforming with closed loop feedback in MIMO systems

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Outline

- Introduction.
- System model.
- WCDMA's closed loop modes.
- Signed gradient algorithms.
- Q&A.
- Homework.

Introduction

- Goal is to maximize received power at the user equipment (UE) by adapting baseband antenna weights.
- Feedback rate limits the amount of CSI that can be transmitted in Frequency Division Multiplexing (FDD) systems.
- Different approaches: update weights in turns or all of them simultaneously.
- At higher speeds, more tracking capabilities are required for a fixed feedback rate.

System model

System model described by:

$$\mathbf{y} = \mathbf{HWX} + \text{noise} \quad (1)$$

(output of receive antenna array, beamforming matrix, modulation matrix, i.i.d. white noise). $\mathbf{H}^\dagger$ refers to the conjugate transpose of $\mathbf{H}$.

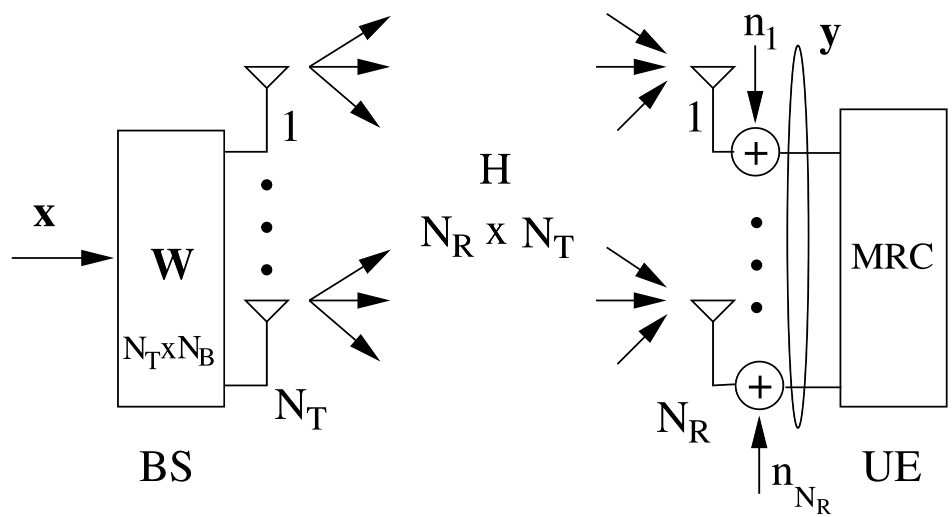

Figure 1: System model.

WCDMA’s closed loop modes

- Two schemes defined in the standard [1] for $N_T = 2$.
- Both attempt to maximize the received power, across multipath components and receive antennas.
- Quantization of feedback message is different (they still use same feedback rate).
- System is single beam.

WCDMA's closed loop modes: optimal weights

- L multipath components.
- Channel coefficients from each transmit antenna and multipath component arranged in an $L \times N_T$ matrix $\mathbf{H}_n$, associated to the receive antenna n.
- $\mathbf{R} := \sum_{n=1}^{N_R} \mathbf{H}_n^{\dagger} \mathbf{H}_n$
- Choose $\mathbf{W}$ so that

$$\mathbf{W} = \underset{\mathbf{W}\,:\,||\mathbf{W}||^2=1}{\operatorname{argmax}} \mathbf{W}^{\dagger}\mathbf{R}\mathbf{W} \qquad (2)$$

The optimal weight vector is the dominant eigenvector of $\mathbf{R}$.

WCDMA's closed loop modes: optimal weights

- For flat fading and single reception antenna, angle of first weight is not relevant, assumed real.

$$\mathbf{W} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{3}$$

$$|w_2| = \frac{|h_2|}{\sqrt{|h_1|^2 + |h_2|^2}} \tag{4}$$

$$\arg(w_2) = \arg(h_2) - \arg(h_1) \tag{5}$$

$$w_1 = \frac{|h_1|}{\sqrt{|h_1|^2 + |h_2|^2}} \tag{6}$$

WCDMA's closed loop modes: Mode 1

- $w_1 = 1/\sqrt{2}$.
- w_2 is quantized to two bits, therefore it is approximated to elements of a QPSK constellation. $w_2[k] = \frac{1}{\sqrt{2}} e^{j\phi[k]}$
- Bit filtering:

$$\phi[k] = \arg(j^{k \mod 2}\operatorname{sgn}(b_k) + j^{(k-1) \mod 2}\operatorname{sgn}(b_{k-1})) \tag{7}$$

Can be further improved by “bit verification” [2].

WCDMA's closed loop modes: Mode 1

$$\phi[k] = \arg\left(j^{k \mod 2}\,\mathrm{sgn}(b_k) + j^{(k-1) \mod 2}\,\mathrm{sgn}(b_{k-1})\right) \qquad (8)$$

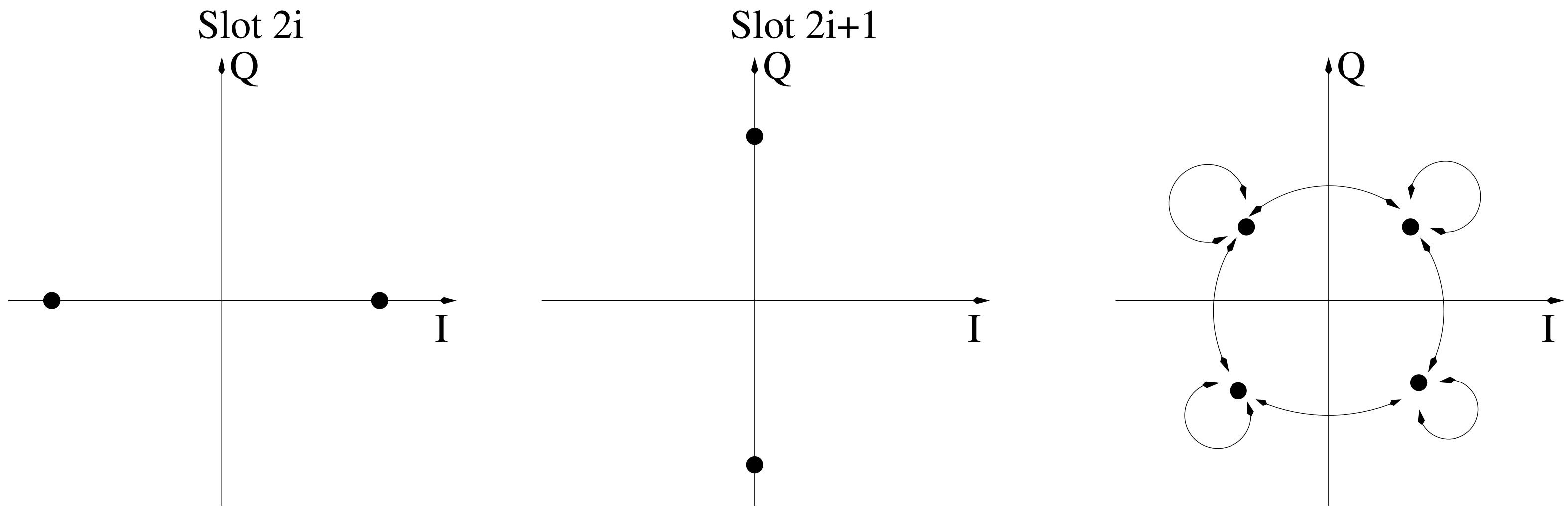

Figure 2: Weight calculation at BS for Mode 1, with filtering over two slots (source: [2])

WCDMA's closed loop modes: Mode 2

- Four bits long message.
- First bit controls the ratio of the powers being transmitted through the antennas.
- Three control the phase of w_2 (w_1 assumed to be real, but magnitude not fixed).
- w_2 lies on a constellation consisting of two circles (circular 16QAM).

WCDMA's closed loop modes: Mode 2

1st bit	Tx1 power	Tx2 Power
0	0.2	0.8
1	0.8	0.2

Table 1: Power adjustment according to first feedback bit in Mode 2

Phase (degrees)	180	-135	-90	-45	0	45	90	135
Fb. bits 2,3,4	000	001	011	010	110	111	101	100

Table 2: Phase of second weight, according to feedback bits 2,3,4 in Mode 2

WCDMA's closed loop modes: Mode 2

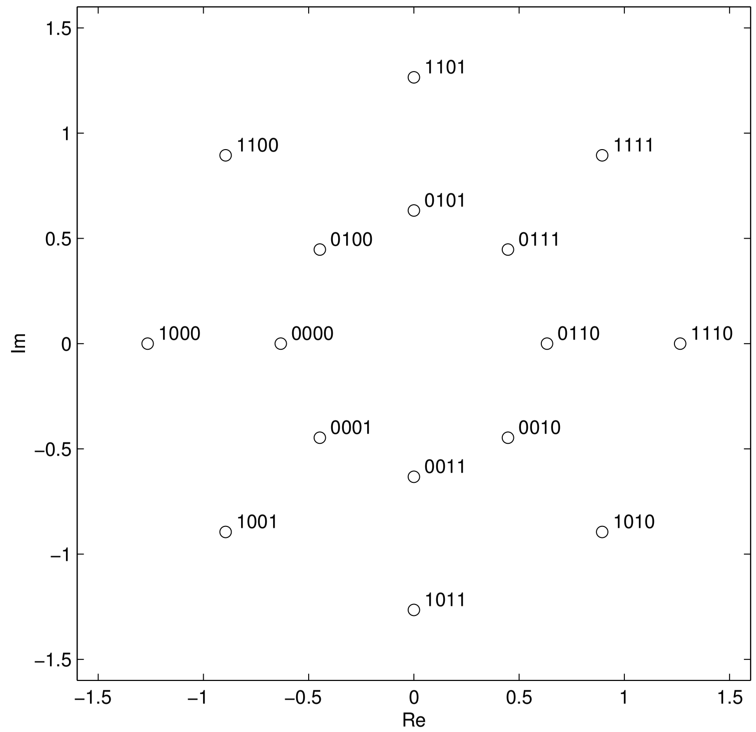

Figure 3: Weight states in Mode 2 (w_2), with associated labels (source: [2] and courtesy of authors).

WCDMA’s closed loop modes: mobile speed considerations

- Tracking capabilities diminish as the fading rates increase.
- For Rayleigh fading, analytical results available ([2], section 11.6).
- Extension to more than two transmit antennas: co-phase algorithm (analysis available also on [2]). Most natural extension of Mode 1.

WCDMA's closed loop modes: SNR gain v/s speed, no feedback errors

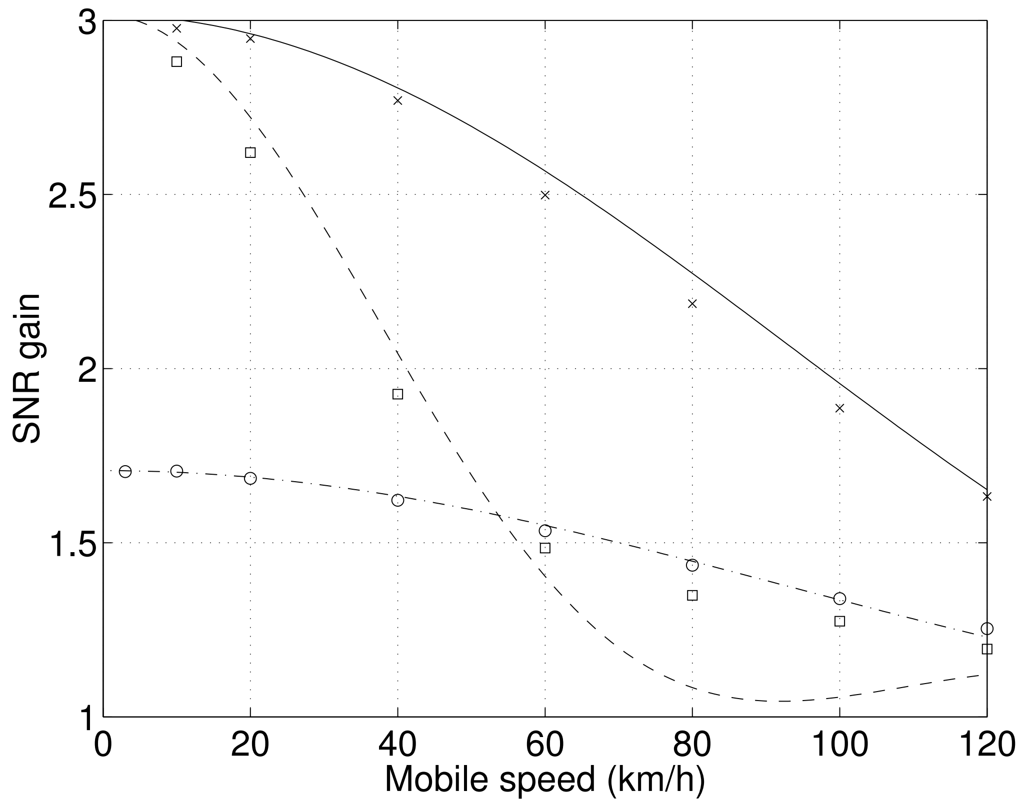

Figure 4: SNR gain. (o) and (-.-): Mode 1 (sim., analytical). Co-phase algorithm (extension to Mode 1, $N_T = 4$): at 1500 (- -) (sim.: □) and 4500 bps feedback rate (-) (and sim. (x)). Source: [2].

WCDMA's closed loop modes: SNR gain v/s speed, 4% feedback errors

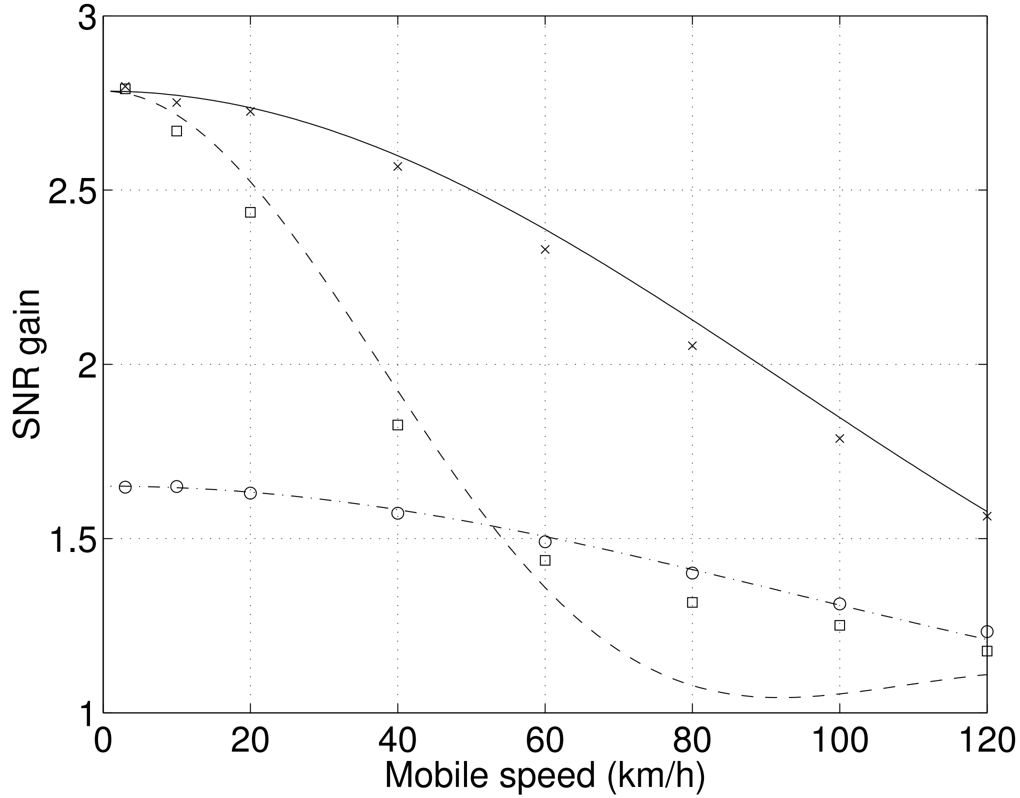

Figure 5: SNR gain. (o) and (-.-): Mode 1 (sim., analytical). Co-phase algorithm (extension to Mode 1, $N_T = 4$): at 1500 (- -) (sim.: □) and 4500 bps feedback rate (-) (and sim. (x)). Source: [2].

WCDMA's closed loop modes: Bit Error Probability (BEP)

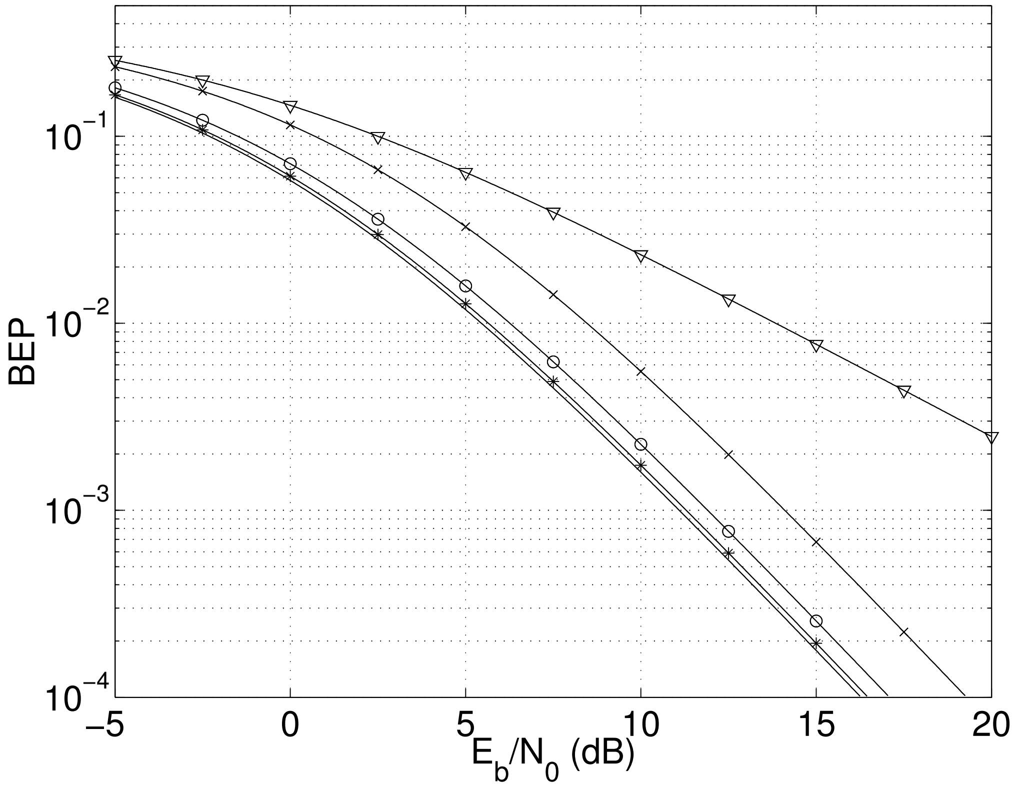

Figure 6: Mode 1 (-o-), Mode 2 (-*-) and Space-Time Transmit Diversity (STTD) ($- \times -$), single antenna transmission ($-\nabla-$). System is 2x1, feedback delay neglected. (source: [2] and courtesy of authors).

Signed gradient methods

- Maximize instantaneous power, $J(\mathbf{W}) := ||\mathbf{HW}||^2$ subject to $||\mathbf{W}||^2 = 1$.
- Solution is also dominant eigenbeam of $\mathbf{H}^\dagger\mathbf{H}$ (flat fading, sum over multipath otherwise).
- Distributed search through gradient descent:

$$\mathbf{W}_{k+1} = \mathbf{W}_k + \beta\mathbf{g}_k \tag{9}$$

- $\mathbf{g}_k$ denotes the current estimate of the gradient of $J(\mathbf{W})$.
- $\mathbf{g}_k$ built based on the feedback bits.

Signed gradient methods

- This methods update all the weights at the same time, as opposed to extensions to WCDMA Mode 1 and 2.
- Limitations in feedback channel's capacity. Not possible to feed back the gradient or the dominant eigenbeam.
- Coordination between transmitter and receiver.
- If feedback message length is fixed, performance of the algorithm depends on the quality of the gradient estimate.
- Consider:
  - b_k: feedback bit computed in the current slot
  - $\mathbf{g}_k$: gradient estimate applied in the current slot
  - $\mathbf{p}_k$: perturbation to be tested in the current slot
  - $\mathbf{W}_k$: beamforming matrix applied in the current slot
  - $\hat{\mathbf{R}}_k$: sample estimate of the channel covariance matrix (e.g. $\mathbf{H}^\dagger\mathbf{H}$).

Signed gradient methods

Signed gradient algorithms ([3], [4],[5]), in pseudo-code, at slot k:

1. BS: Update $\mathbf{g}_k$.
2. BS: $\mathbf{W}_k = \mathbf{W}_{k-1} + \beta \mathbf{g}_k$; $\mathbf{W}_k = \mathbf{W}_k / ||\mathbf{W}_k||$
3. BS: Generate new $\mathbf{p}_k$.
4. BS: $\mathbf{W}_e = \mathbf{W} + \beta \mathbf{p}_k$; $\mathbf{W}_0 = \mathbf{W} - \beta \mathbf{p}_k$;
5. UE: $p_e = \mathbf{W}_e^\dagger \hat{\mathbf{R}}_k \mathbf{W}_e / ||\mathbf{W}_e||^2$; $p_o = \mathbf{W}_o^\dagger \hat{\mathbf{R}}_k \mathbf{W}_o / ||\mathbf{W}_o||^2$
6. UE: if ($p_e > p_o$): $b_k = 1$ else: $b_k = -1$
7. UE: transmit b_k, so that it becomes b_{k-1} at next slot
8. BTS: receives b_k
9. Go to first step, b_k becomes b_{k-1}, $\mathbf{g}_k$ becomes $\mathbf{g}_{k-1}$

Signed gradient methods: Stochastic gradient approximation

- Scheme described in [4].
- Perturbation vectors always random vectors selected i.i.d. by the BS: $\mathbf{p}_k = \texttt{new random vector}$.
- Gradient estimate is only based on the decision fed back by the mobile, i.e. $\mathbf{g}_k = b_{k-1}\mathbf{p}_{k-1}$.

Signed gradient methods: Deterministic gradient approximation

- Scheme described in [3].
- $\mathbf{p}_k$ comes from a fixed set of orthogonal vectors, $\mathbf{p}_k \in \{\mathbf{v}_i\}_{i=1}^{2N_T}$.
- Vectors are any orthogonal set, and the same set multiplied by the imaginary unit j.
- Gradient estimate is only based on the decision fed back by the mobile, i.e. $\mathbf{g}_k = b_{k-1}\mathbf{p}_{k-1}$.

For example, for $N_T = 2$ and using the DFT vectors:

$$\{\mathbf{v}_i\}_{i=1}^{2N_T} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} j \\ j \end{bmatrix}, \begin{bmatrix} j \\ -j \end{bmatrix} \right\} \tag{10}$$

Signed gradient methods: Filtered gradient

- Scheme described in [5].
- Perturbation vectors are random vectors orthogonal to past gradient. $\mathbf{p}_k$ = aux - $(\mathbf{g}_{k-1}^{\dagger}\mathtt{aux}/||\mathbf{g}_{k-1}||^2)\mathbf{g}_{k-1}$ where aux is a random vector drawn in an independent fashion.
- Gradient estimate is filtered to exploit correlation between update directions (accelerate convergence): $\mathbf{g}_k = b_{k-1}\mathbf{p}_{k-1} + \lambda_G \mathbf{g}_{k-1}$.

Signed gradient methods: convergence speed

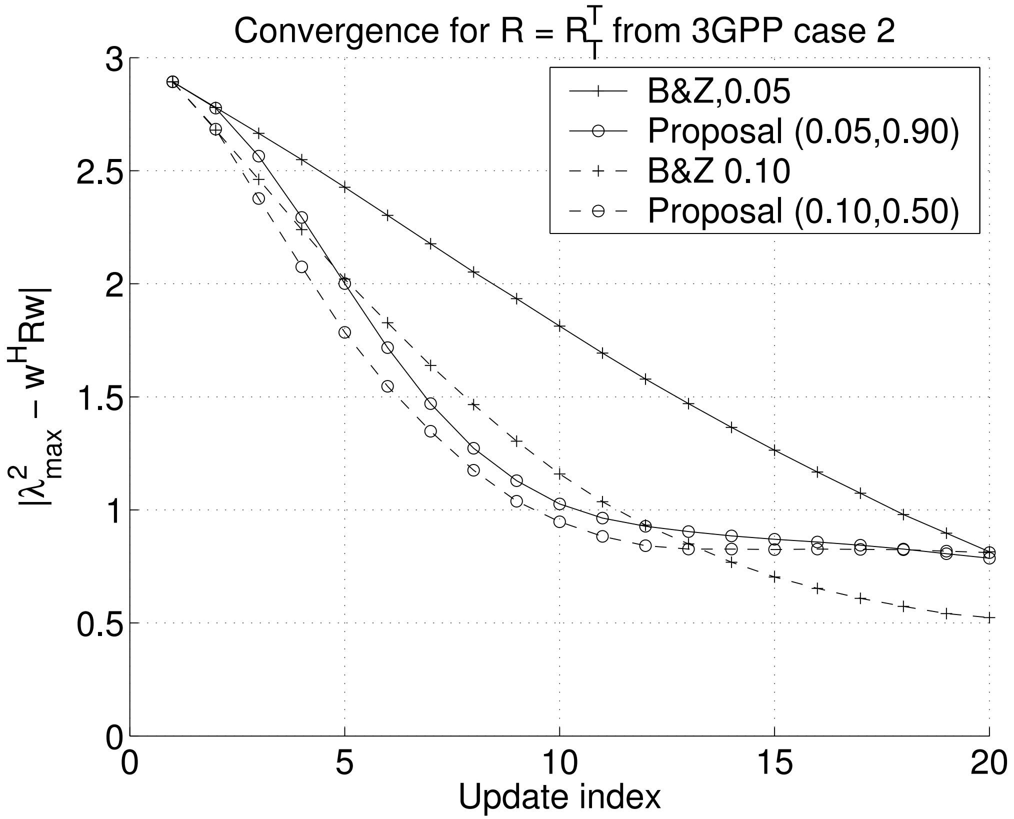

Figure 7: Convergence in static channel. “B&Z” ≡ [4]. “Proposal” ≡ [5].

Signed gradient methods: BEP at fixed SNR

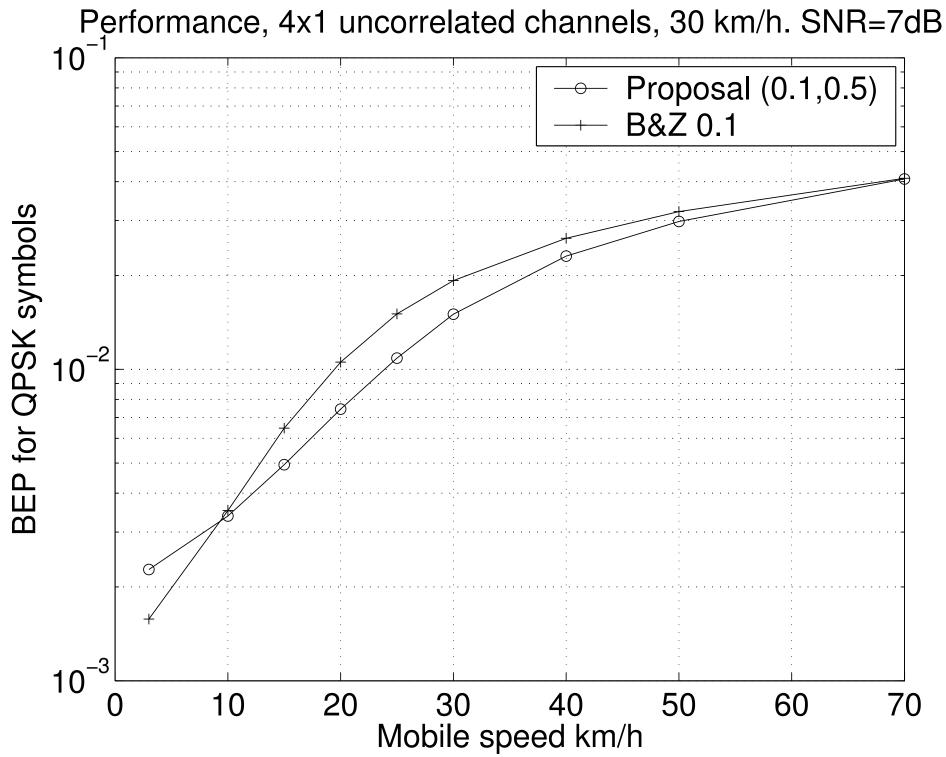

Figure 8: Bit Error Rate as function of speed in 4x1 uncorrelated channels. “B&Z” $\equiv$ [4]. “Proposal” $\equiv$ [5].

Signed gradient methods: BEP at 30 Km/h

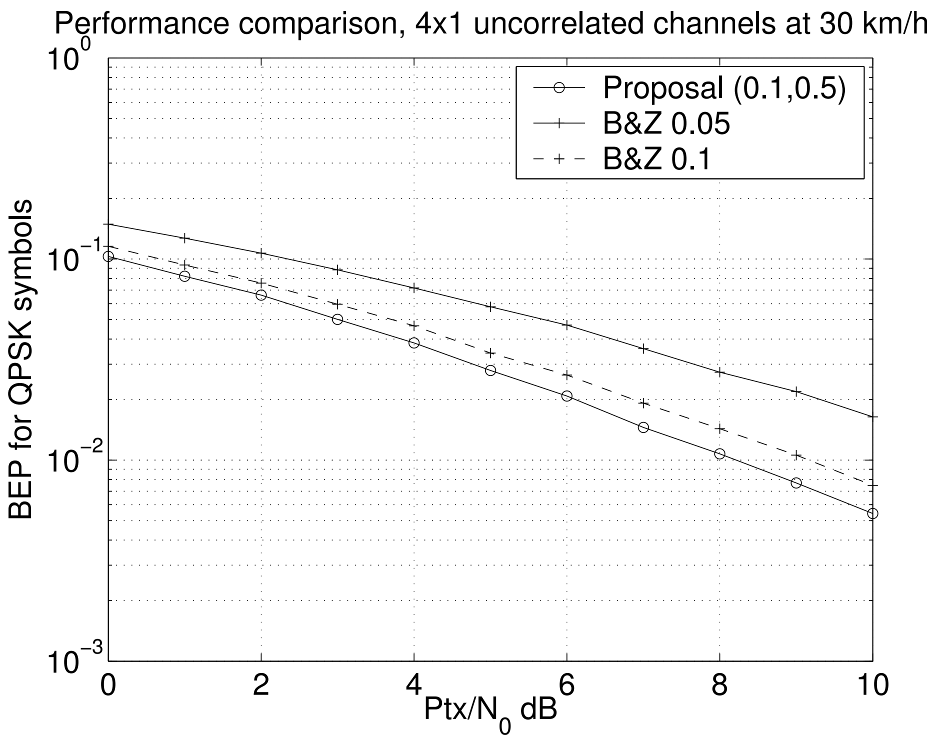

Figure 9: Bit Error Rate curves for 4x1 uncorrelated channels at 30 km/h."B&Z" $\equiv$ [4]. "Proposal" $\equiv$ [5].

Conclusions

- Overview of adaptive transmit beamforming based on the closed loop feedback.
  - Concepts already present in the current WCDMA standard.
  - Proposed techniques aiming to increase the performance in fast fading channels.
  - Emphasis made in schemes employing short feedback messages (one bit).
- Left out some considerations about correlated channels (see the text version).

Homework

- Briefly describe WCDMA’s closed loop modes, as presented here.
- What are the main differences between WCDMA’s closed loop modes and the sign gradient algorithms presented here? (please refer to weight updating strategy, feedback message lengths).

References

[1] 3GPP RAN WG1, "TS25.201: Physical layer - general description," 3GPP, Tech. Rep., June 2000.

[2] A. Hottinen, O. Tirkkonen, and R. Wichman, *Multi-antenna transceiver techniques for 3G and beyond*. John Wiley and Sons, January 2003.

[3] B. Raghothaman, "Deterministic perturbation gradient approximation for transmission subspace tracking in FDD-CDMA," *IEEE International Conference on Communications, 2003.*, vol. 4, pp. 2450–2454, May 2003.

[4] B. C. Banister and J. R. Zeidler, "A simple gradient sign algorithm for transmit antenna weight adaptation with feedback," *IEEE Transactions on Signal Processing*, vol. 51, no. 5, pp. 1156–1171, May 2003.

[5] E. Zacarías B., R. Wichman, and S. Werner, "Filtered gradient algorithm for closed loop mimo systems," *IEEE 61st Semiannual Vehicular Technology Conference (spring)*, May 2005.