Stability-throughput-delay tradeoff in CSMA

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I. Introduction(1)

- The assumptions in previous studies for non-persistent and *p*-persistent CSMA (Carrier Sense Multiple Access)
- Infinite number of users having Poisson input
- Steady state condition prevail
- Empirically determined stationary state-steady state (e.g., after 10,000 packet transmission time)
- These merely represent approximation to the physical situation
- Moreover, the extensive simulation runs performed on ALOHA channels with an infinite population have shown that the assumption of channel equilibrium may not always be valid
- In fact, after some finite period of quasi-stationary condition, the channel will drift into saturation with probability one

Introduction(2)

- A more representative measure of channel performance is the stability-throughput-delay trade-off
- Stable channel: the equilibrium throughput-delay results are achievable over an infinite time horizon
- Unstable channel: such channel performance is achievable only for some finite time period before the channel goes into the saturation
- Applying the stability theory defined in [2] in order to predict the behavior of CSMA channels
- Discuss the conditions under which we can guarantee stability and finally give the performance of these guaranteed stable channels

II. The model and analysis(1)

- Assumptions: the slot size τ, the packet fixed length T, terminals M (users)
- Each user can be in one of two states: *backlogged* or *thinking*
- In the thinking state, a user generates a new packet in a slot with probability σ
- In backlogged state, a user is scheduled to re-sense the channel in the current slot with a probability ν
- They are assumed to be memoryless and time invariant
- The discrete states space of the system consists of the integers *{1,2,3,…,M}*

The intervals time of a cycle between two consecutive idle periods (in slots)

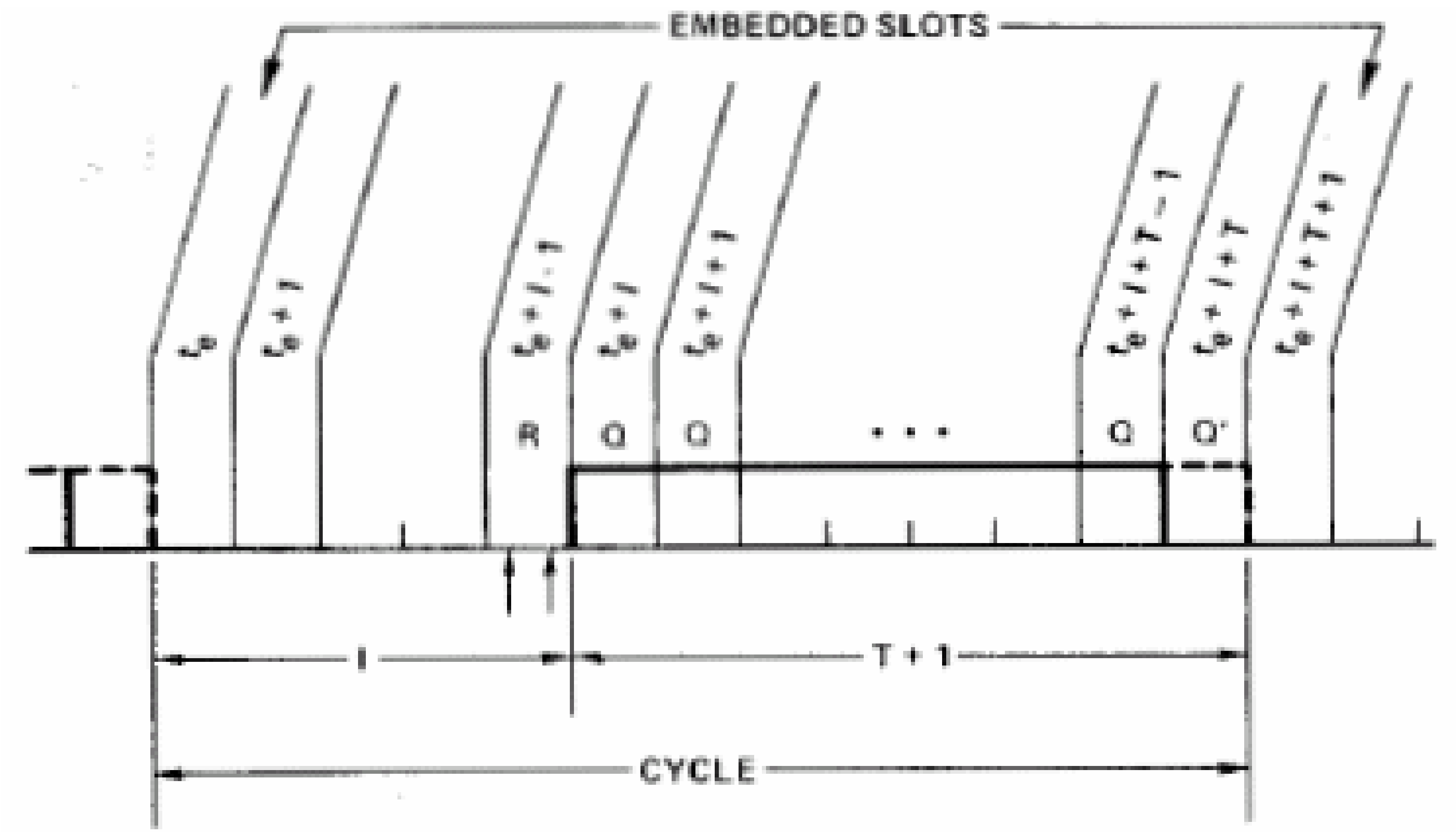


Fig. 1. The Imbedded Markov Chain in Slotted Nonpersistent CSMA.

The model and analysis(2)

- The probability that some terminal is ready to transmit is given by

$$\Pr\{\text{some terminal is ready}/N^{t_e+l-1} = i\}$$
$$= 1 - (1-\nu)^i (1-\sigma)^{M-i}.$$

- Condition on the fact that a transmission starts at slot t_e+l be one-step transition matrix between slot t_e+l-1 and t_e+l

$$r_{ik} = \begin{cases} 0, & k < i \\ \dfrac{(1-\sigma)^{M-i}[1-(1-\nu)^i]}{1-(1-\nu)^i(1-\sigma)^{M-i}}, & k = i \\ \dfrac{\binom{M-i}{k-i}(1-\sigma)^{M-k}\sigma^{k-i}}{1-(1-\nu)^i(1-\sigma)^{M-i}}, & k > i. \end{cases}$$

The model and analysis(3)

- The probability of a successful transmission over a cycle when $N^{t_e} = n$

$$P_s(n) = \frac{(1-\nu)^n (M-n)\sigma(1-\sigma)^{M-n-1} + n\nu(1-\nu)^{n-1}(1-\sigma)^{M-n}}{1-(1-\nu)^n(1-\sigma)^{M-n}}$$

- The stationary backlog distribution

$$\Pi = \{\pi_0, \cdots, \pi_j, \cdots, \pi_M\}$$

where

$$\pi_j = \lim_{t_e \to \infty} \Pr\{N^{t_e} = j\}.$$

The model and analysis(4)

- Distribution of the length of the idle period. If we note that no terminals become ready during slot *t* is, $\delta_i = (1-\nu)^i (1-\sigma)^{M-i}$ then
- Stationary average channel backlog
- Stationary average channel throughput
- Expected packet delay

$$D = \frac{\bar{N}}{S_{out}}$$

$$\eta_k(i) = (1-\delta_i)\delta_i^{k-1}.$$

The average idle period is $1/(1-\delta_i)$.

$$\bar{N} = \frac{\sum_{i=0}^{M} \pi_i \left[\frac{1}{1-\delta_i} i + A(i)\right]}{\sum_{i=0}^{M} \pi_i \left[\frac{1}{1-\delta_i} + T + 1\right]}$$

$$S_{out} = \frac{\sum_{i=0}^{M} \pi_i TP_s(i)}{\sum_{i=0}^{M} \pi_i \left[\frac{1}{1-\delta_i} + T + 1\right]}$$

III. Stability considerations

- The expected number of new packets generated (input rate) over the cycle normalized with respect to *T*

$$S_{in}(n, \sigma) \cong S_{in}'(n, \sigma) = (M - n)\sigma \left[\frac{1}{1 - \delta_n} + T + 1 \right]$$

$$\overline{S_{in}(n, \sigma)} \cong \overline{S_{in}'(n, \sigma)} = (M - n)\sigma T$$

- As *M* going to infinite, and having *Mσ=s,* (s is the poisson distribution of input rate) the successful transmission probability

$$P_s(n) \cong P_s'(n) = \frac{(1 - \nu)^n s e^{-s} + n\nu(1 - \nu)^{n-1} e^{-s}}{1 - (1 - \nu)^n e^{-s}}$$

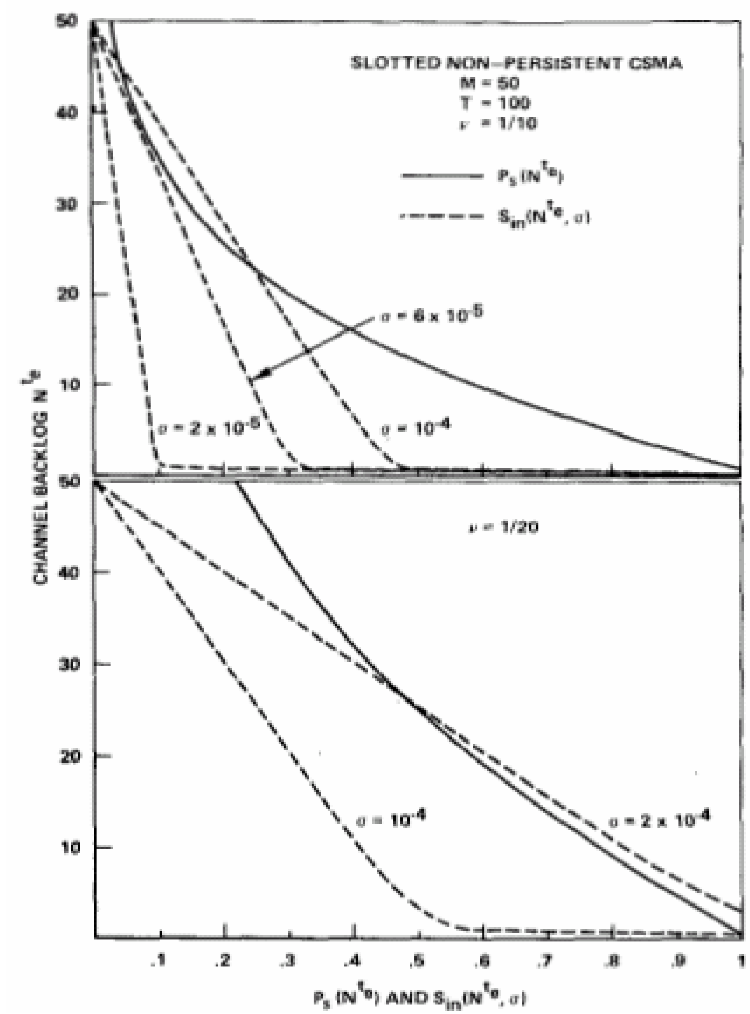
Expected number of successful packets *Ps(n)* and new packets *Sin(n, σ)*

- The equilibrium points can be observed when

  Ps(n) = Sin(n, σ),* thus the *equilibrium contour* can be defined for a given ν

- The system has to increase the channel backlog (this occurred when *Ps(n) < Sin(n, σ),* lies outside of equilibrium contour) or to decrease the channel backlog

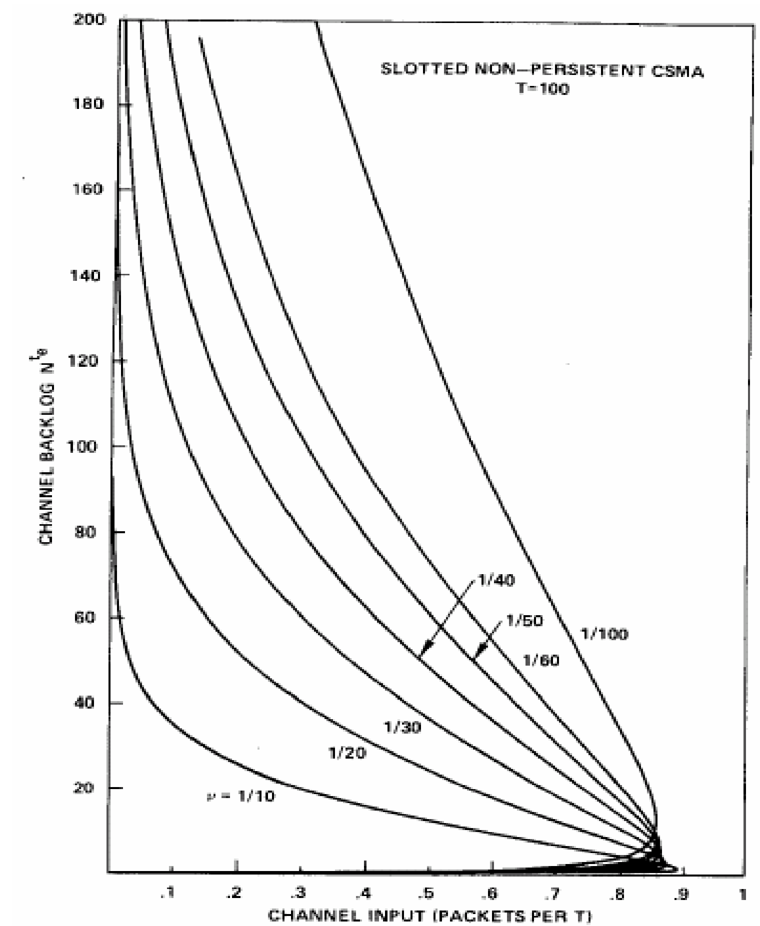
  (*Ps(n) > Sin(n, σ),* when the points outside the equlibrium contour)



Equilibrium contour

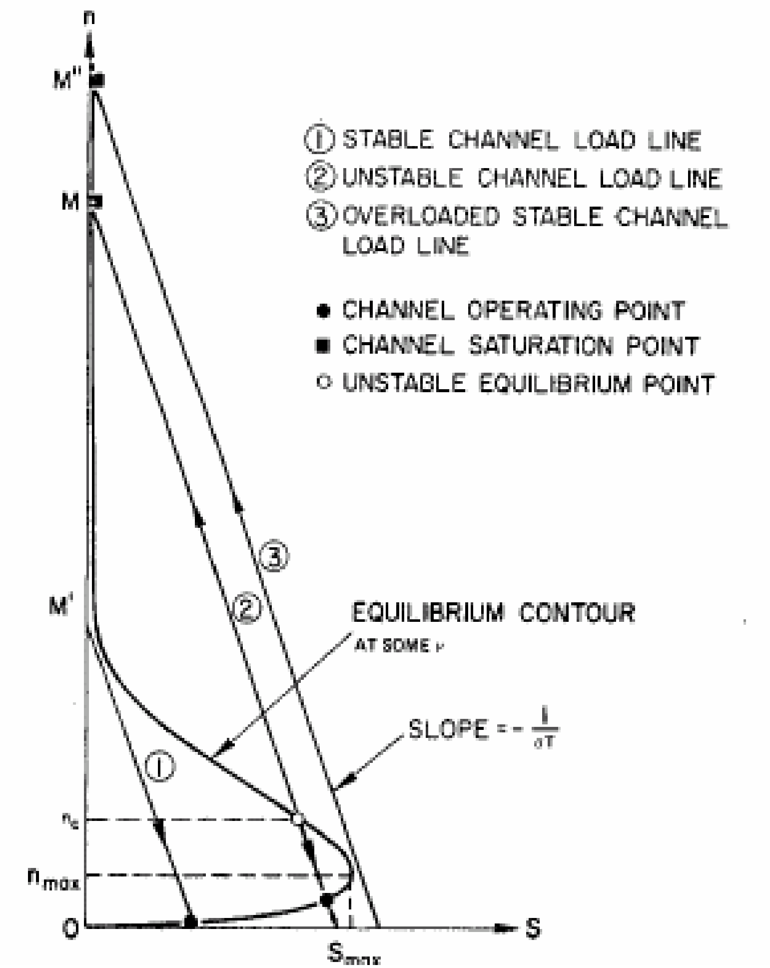
- Using σ^* for each n, we may plot a family of equilibrium contours for various of ν
- And the instantaneous system output

$$\overline{S_{\text{out}}(n)} \triangleq \frac{P_s(n)T}{1/(1-\delta_n)+T+1}$$



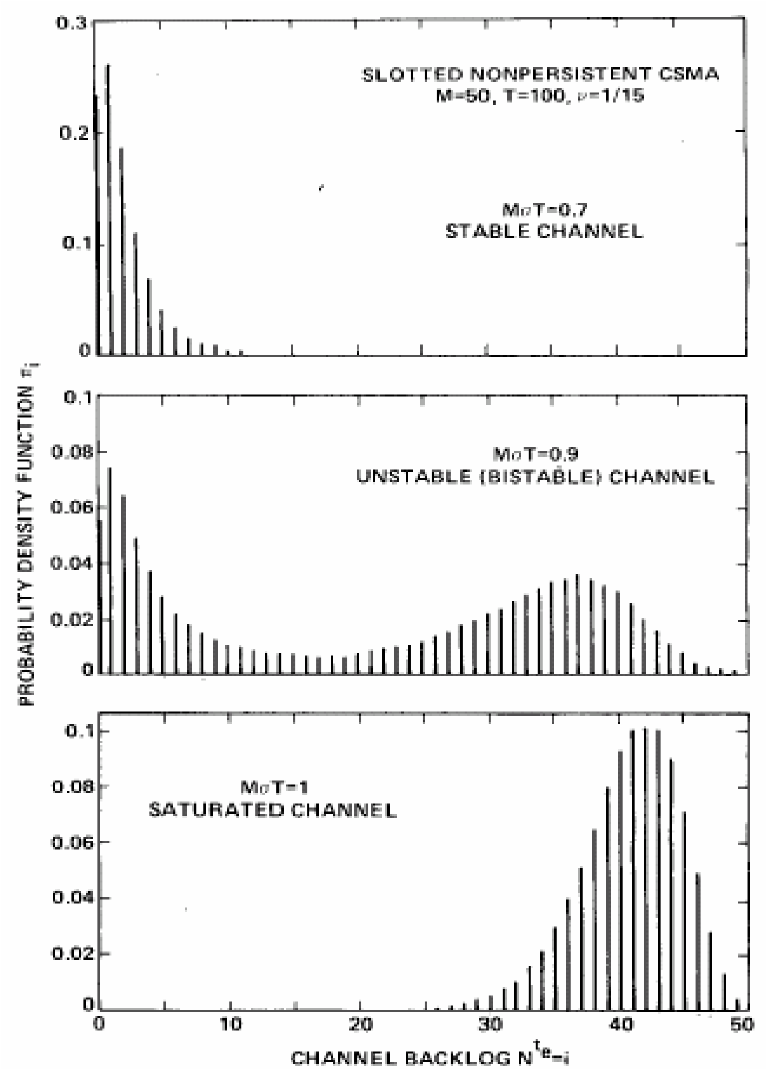
Hypothetical stable and channels

- If we define the channel load line as
  $S = (M-n)\,\sigma T$, then we have



Stationary backlog distribution for stable, unstable, and saturated channels

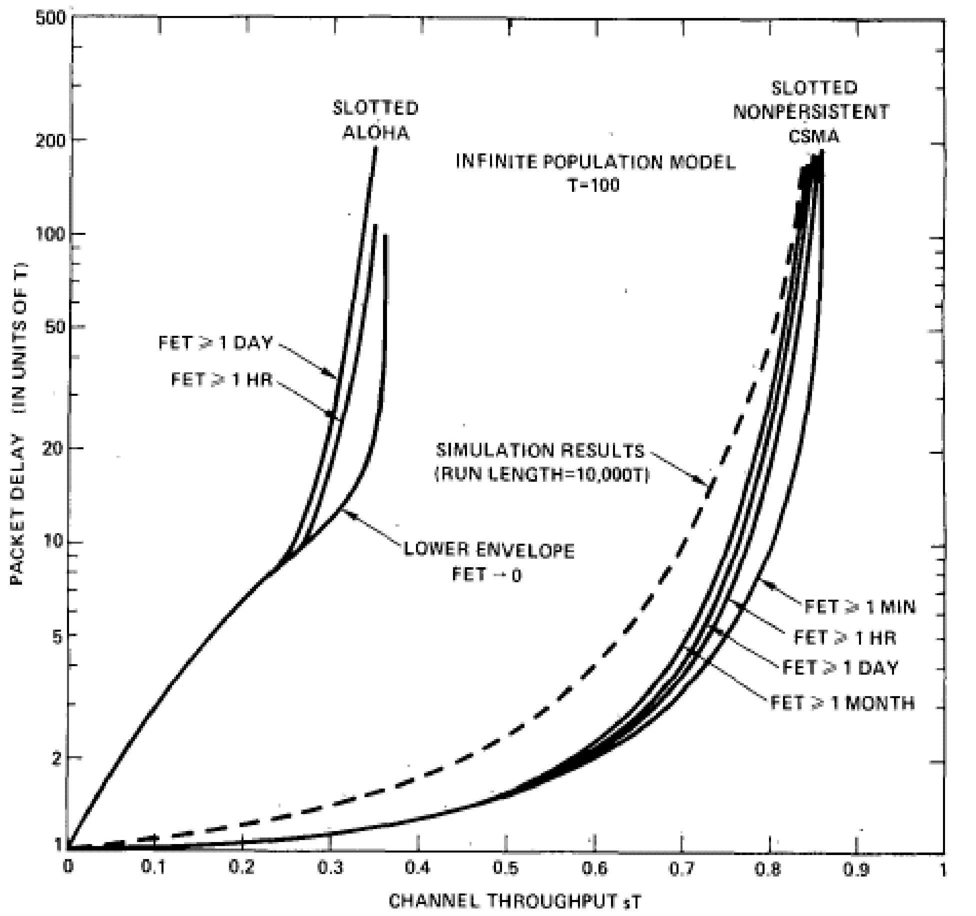
- The statements in previous slide are verified by observing the figure (on left), the density function of the stationary backlog distribution



FET: A stability measure

- FET (first exit time): the average time to exit the stable states (safe region) into unsafe region

- On the left we give a comparison between slotted ALOHA and non-persistent CSMA
- Stability-delay-throughput tradeoff for the infinite population model



IV. Dynamic control for improved performance in CSMA channels

- In the context of slotted ALOHA, two classes of control actions were considered [3]. The input control and the retransmission control
- The input control procedure allows the channel to either accept or reject new packets from their sources
- The retransmission control procedure allows the channel transmitters to impose either large or small retransmission delay on previously collided packets
- Here, we limit ourselves to the retransmission control
- The aim is to find a optimal ν^* which will maximizing the instantaneous throughput

The simplification of averaged S_{out}

- As we known for infinite population case

$$\overline{S_{out}(n)} = \frac{(1-\nu)^n s e^{-s} T + n\nu(1-\nu)^{n-1} e^{-s} T}{1+(T+1)[1-(1-\nu)^n e^{-s}]}$$

- It is also very accurate expression for finite *M* as $s = M\sigma$
- And by the approximation that

$(1-\nu)^k \; by \; 1-k\nu$

$$\bar{S}_{out}(n) \cong \frac{n\nu[1-(n-1)\nu]T}{1+(T+1)n\nu}$$

The simplification of averaged S_{out} and the optimal ν

- The derivative of this last expression yields the second degree equation
- The optimal solution is found as
- Furthermore it can be simplified as

$$(T+1)n(n-1)\nu^2 + 2(n-1)\nu - 1 = 0$$

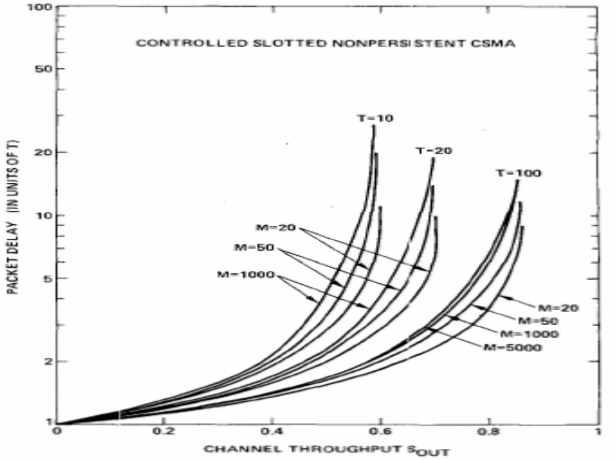
$$\nu^* = \frac{-1 + \sqrt{1 + \frac{n}{n-1}(T+1)}}{(T+1)n}$$

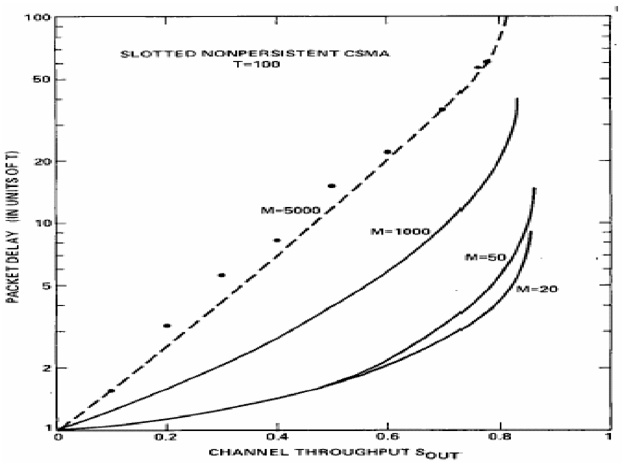
$$\nu^*(n) = \begin{cases} 1, & \text{if } n = 0 \\ \frac{1}{n\sqrt{T}}, & \text{if } n \geq 1 \end{cases}$$

Controlled slotted non-persistent CSMA: Throughput-Delay Performance

- The figures showed us the steady-state performance of controlled CSMA channels for various of M and T
- The significant improvement (upper figure) in delay, particularly with large populations compared to uncontrolled channels (lower figure)

- Thank you!





References

- [1] *Tobagi, F.; Kleinrock, L.;* ***Packet Switching in Radio Channels: Part IV-Stability Considerations and Dynamic Control in Carrier Sense Multiple Access,*** IEEE Transactions on Communications, [legacy, pre - 1988], Volume: 25, Issue: 10, Pages:1103 – 1119, October 1977.
- [2] L. Kleinrock and S. Lam, ***“Packet switching in a multiaccess broadcast channel: Performance evaluation,”*** IEEE Trans. Cornmun., vol. COM-23; pp. 410-423, Apr. 1975.
- [3] S. Lam and L. Kleinrock, ***“Packet switching in a multiaccess broadcast channel: Dynamic control procedures,”*** IEEE Trans. Commun., vol. COM-23,pp. 891-904, Sep. 1975.

Homework

(1) The probability of backlogged packets *i* at time of an idle period defined as $\delta_i = (1-\nu)^i (1-\sigma)^{M-i}$ That meant no terminals become ready during slot *t*; thus giving the probability of idle length *I* with

$\eta_k(i) = (1-\delta_i)\delta_i^{k-1}$

prove that the average length of the idle period is $1/(1-\delta_i)$

(2) In paper[1], the stationary average channel throughput is given as below. Explaining the meanings of the symbolic terms in the numerator and denominator such as $\pi_i, T, P_s(i)$ *and* ν, σ *in* δ_i

$$S_{out} = \frac{\sum_{i=0}^{M} \pi_i T P_s(i)}{\sum_{i=0}^{M} \pi_i [\frac{1}{1-\delta_i} + T + 1]}$$