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# RF Distortion Analysis for OFDM WLAN

**S-72. 333 Postgraduate Course in Radio Communications**

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## Line out

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- 1.2 Methods for estimating the inverse characteristic for the HPA

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- 2.2 Selective mapping
- 2.3 Block coding

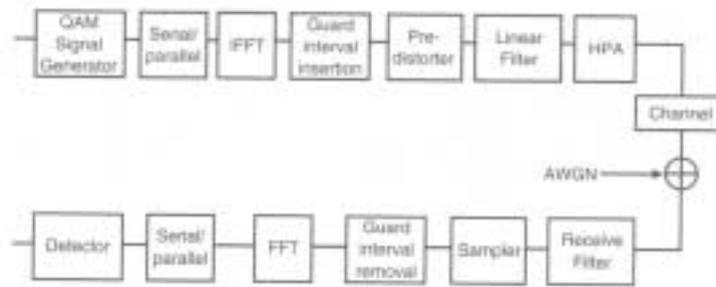
### **3 Phase noise**

### **4 IQ imbalance**

# 1 Adaptive predistortion techniques

## 1.1 Predistortions algorithm for a cascade of HPA and linear filter

*Simplified block diagram of the baseband equivalent OFDM transceiver*



**Serial-to-parallel block:** converts a QAM input data stream to a block of N symbols

**IFFT:** OFDM modulation

**Guard interval:** longer than the largest delay spread : remove ISI and ICI

**Linear filter :** transmitter pulse shaping filter

**HPA :** fully characterized by AM/AM and AM/PM conversions

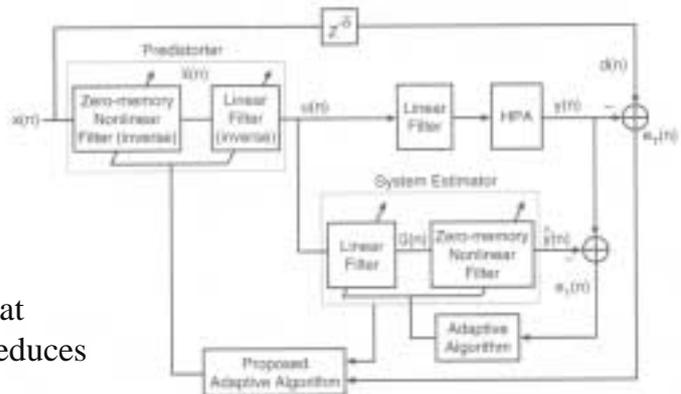
## 1 Adaptive predistortion techniques

### Adaptive algorithm :

**1<sup>st</sup> step:** System classification or estimation (system estimator block)

HPA: memoryless nonlinear subsystem preceded by adaptive linear filter

**2<sup>nd</sup> step:** develop adaptive predistorter that compensates constellation warping and reduces ISI effects.



Adaptive predistorter for HPA preceded by linear filter

Use of the polynomial approximation for modeling HPA :  $\hat{y}(n) = \sum_{l=1}^{N_a} a(l) \left( \sum_{k=1}^{N_h} h(k) u(n-k) \right)^l$

$N_h$  = memory length of the linear filter  $h(k)$  ;  $N_a$  = order of the nonlinear filter  $a(l)$

$U(n)$  = input signal of the linear filter

Coefficients of the system estimator  $h(k)$  and  $a(l)$  : adjusted to minimize mean squared error (MSE) ,  $\Gamma_1 = E\{e(n)^2\}$  where  $e(n) = y(n) - \hat{y}(n)$

**Design of the adaptive predistorter:**

In the previous figure: predistorter is constructed by a memoryless nonlinear inverse filter followed by a linear filter

Expression of the predistorter using polynomial form of finite order for the

$$\text{memoryless nonlinear inverse filter : } u(n) = \sum_{i=1}^{N_p} p(i) \sum_{j=1}^{N_s} s(j) x(n-i)^j$$

$x(n)$  = input OFDM signal ;  $N(p)$  = memory length of the linear inverse filter  $P(I)$  ;

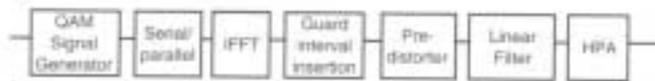
$N_s$  = order of the nonlinear inverse filter  $S(j)$

Error of the total system :  $e_T(n) = d(n) - y(n)$  ;

$d(n)$  = delay version of the input signal  $x(n)$  by  $\delta$  samples to account for causality of the predistorter

Coefficients of the predistorter are obtained by minimizing the MSE,  $\Gamma_T = E\{e_T(n)^2\}$

Validity of this predistortion technique : demonstrated via computer simulation using a block-oriented model



Serial to parallel converter : transfers a block of 1024 16-QAM symbols to the OFDM modulator, which uses an 800 of 1024 subchannels of IFFT to modulate them.

The first and last 112 subcarriers are set to avoid spectrum overlapping.

➡ a 224 subcarrier guard band between adjacent OFDM systems

**Learning curve of the system estimator :**

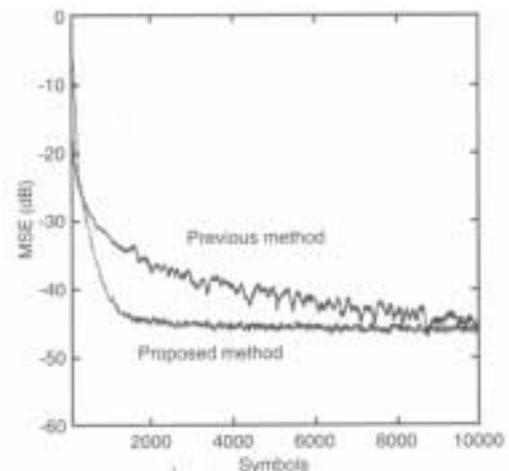
Obtained by averaging 200 independent trials

Order of the nonlinear filter :  $N_a = 5$  ;  
memory length of the linear filter :  $N_h = 3$

With zero initial conditions, about  $-45\text{dB}$  in the MSE was obtained

➡ accurate estimation of filter coefficients

**Proposed approach has faster convergence speed and smaller fluctuation**



Although the results are encouraging : several questions left unanswered :

-How much does the performance degrade when fading characteristics are considered in the problem ?

-Is there sufficient time or data available for the various algorithms to converge in a real-time implementation ?

-What happens if one of the adaptive algorithms misconverge to a local not global minimum ?

➔ **In practice, the actual benefits remain uncertain**

## 1.2 Methods for estimating the inverse characteristic for the HPA

The combined characteristics of the transmit filter and the HPA can be approximate using Volterra series

In this case, the adaptive predistortion can be regarded as an inverse nonlinear estimation with memory problem

 This approach is unsuitable when the input signal constellation is infinite.

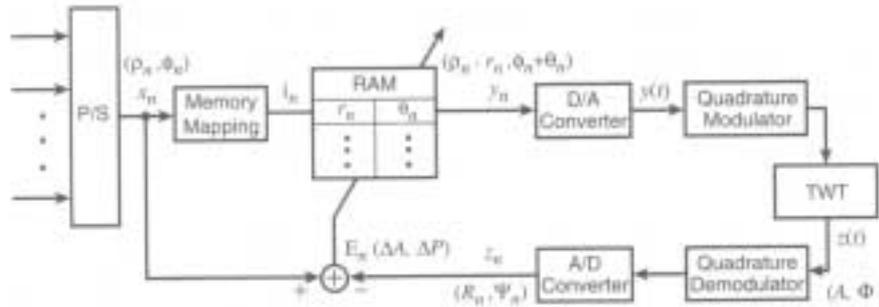
By limiting the number of possibilities for the input levels of the HPA, the data predistortion problem becomes more tractable

In order to design data predistorter for the HPA, the magnitude and phase of the input and output signals of the HPA is quantized uniformly over Q bits :

$$0 \leq i_n = \sum_{j=0}^{Q-1} b_{n,j} 2^j \leq 2^Q - 1$$

$b_{n,j}$  =  $j^{\text{th}}$  bit corresponding to the magnitude of the  $n^{\text{th}}$  input signal  $x_n$

**Block diagram of adaptive data predistorter using RAM or memory lookup :**



Polar form of the complex input signal  $x_n$  :  $x_n = \rho_n e^{j\phi_n}$

Ideally, the content of the RAM ( $r_n, \theta_n$ ) addressed by the corresponding index  $i_n$  will represent the amplitude and phase required to linearize the HPA

➔ Predistorted value applied to the HPA :  $y_n = \gamma_n e^{j\psi_n}$  where  $\gamma = \rho_n r_n$  and  $\psi = \theta + \phi_n$

Response of HPA to the predistorted signal :  $z_n = R_n e^{j\Psi_n}$

Where  $R_n = A(\gamma_n) + v_n$  and  $\Psi_n = \Phi(\gamma_n) + \phi_n + w_n$

$A(.)$  = AM/AM characteristics of HPA and  $\Phi(.)$  = AM/PM Characteristics of HPA

$v_n$  = measurement noises for amplitude and  $w_n$  = measurement noises for phase

Amplitude error : difference between desired signal and HPA output =  $\Delta A_n = \chi \rho_n - R_n$

$\chi$  = desired gain

Phase error : difference between desired signal and HPA output =  $\Delta P_n = \Phi_n - \Psi_n$

Least mean squared algorithm to update the RAM :  $r_{n+1} = r_n + \mu_\alpha \Delta A_n$

$$\theta_{n+1} = \theta_n + \mu_\beta \Delta P_n$$

Only the content of RAM corresponding to the input level is update each time

For block size =  $N \geq 2^Q$  the content of each address will be update  $N/2^Q$  times during  $NT_s$  seconds (1 block) on the average

➔ **Extremely slow convergence characteristic limits its real-time implementation**

The validity of this adaptive data predistortion technique is demonstrated by computer simulation

Typical performance measure for quantifying the effect of nonlinear distortion in HPA : total degradation (in dB) :  $TD = SNR_{HPA} - SNR_{AWGN} + OBO$  (dB)

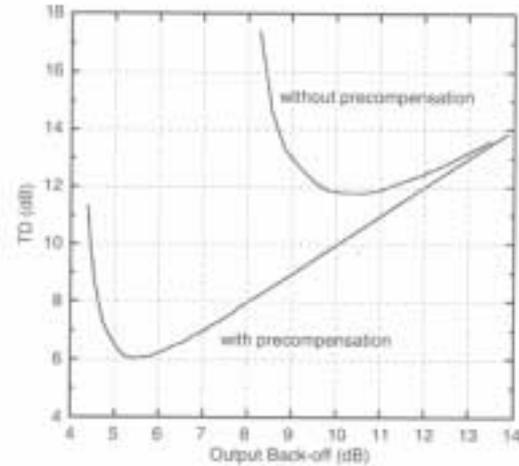
$SNR_{HPA}$  = SNR for specific BER when the HPA is used

$SNR_{AWGN}$  = SNR for the same BER over a AWGN channel without HPA

Total degradation versus HPA OBO ( $BER = 10^{-4}$ ) :

Optimum backoff : 5.5 dB

Gain of 5 dB with the data predistortion system



2 Coding techniques for amplifier nonlinear distortion mitigation

**2 Coding techniques for amplifier nonlinear distortion mitigation**

**2.1 Partial transmit sequences**

Notation : the  $n^{\text{th}}$  transmit symbol modulating the  $j^{\text{th}}$  subcarrier :  $X_j(n)$

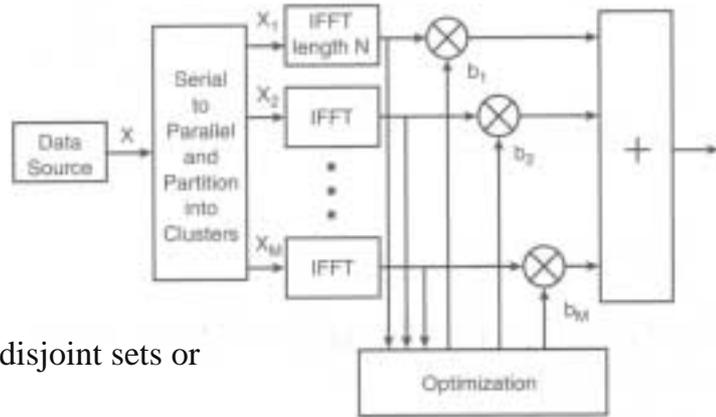
Input at time instance  $n$  is formed as :  $\mathbf{X}(n) = [X_0(n), X_1(n), \dots, X_{N-1}(n)]^T$

Corresponding OFDM symbol :  $\mathbf{x}(n) = \text{IFFT} \{ \mathbf{X}(n) \}$

OFDM symbol crest factor is defined as :  $\zeta_n = \frac{\max |X(n)|}{\sqrt{E\{|X(n)|\}}}$  for  $n = 0, \dots, N-1$

Performance criterion used to assess the algorithm : complementary cumulative distribution function :  $P_\zeta(\zeta_0) = \Pr(\zeta_0 > \zeta_n)$

PTS functional block diagram :



**PTS algorithm :**

Partition the input data  $\mathbf{X}$  into  $M$  disjoint sets or clusters :  $X_m, m = 1, 2, \dots, M$

Minimize the crest factor  $\zeta_n : X' = \sum_{m=1}^M b_m X_m$

where  $\{b_m, m = 1, 2, \dots, M\}$  are weighting factor of pure phase rotations

If  $b_1 = 1$ , the remaining phase terms can be found from :  $\hat{b}_m = \arg \min \left( \max \left\{ b_1 x + \sum_{m=2}^M b_m X_m \right\} \right)$

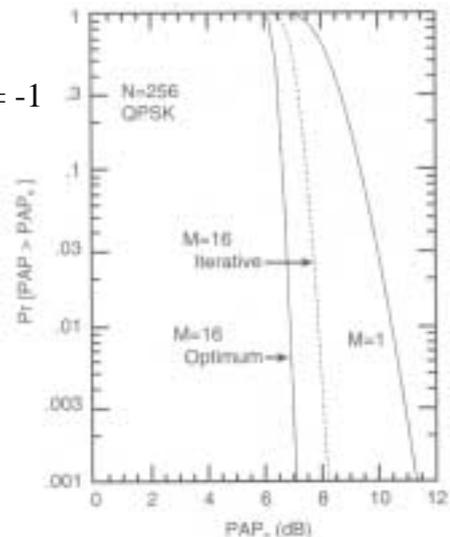
Where  $\arg \min(\cdot)$  achieves the global minimum for the peak power optimized transmit sequence

### 2.1.1 Modification of the PTS algorithm

Suboptimal combining algorithm which uses only binary weighting factors :

1. Assume  $b_m = 1$  for  $m = 1, 2, \dots, M$  and compute  $\text{PAPR}^{(+)}$
2.  $m = 2$
3. Invert :  $b_m = -1$  and recompute the  $\text{PAPR}^{(-)}$
4. If  $\text{PAPR}^{(+)} > \text{PAPR}^{(-)}$ , retain  $b_m = 1$ ; otherwise,  $b_m = -1$
5.  $m = m+1$
6. Repeat steps 3-5 until  $m = M$

Comparison of iterative and optimum combining strategies :



### 2.2 Selective mapping

Concept : given  $M$  statistically independent OFDM symbols conveying the same information, select the symbol with the lowest PAPR for transmission

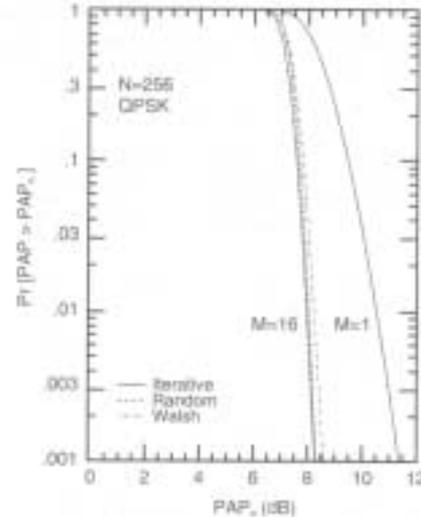
One possibility : use Walsh sequences

Other possibility : use a random interleaver on the data sequence

Comparison between these 2 methods:

Using 16 random sequences  $\Rightarrow$  same performances of the iterative approach

Walsh sequence of length 16  $\Rightarrow$  only additional 0.3 dB degradation



### 2.3 Block coding

Block code should provide error protection as well as PAPR reduction

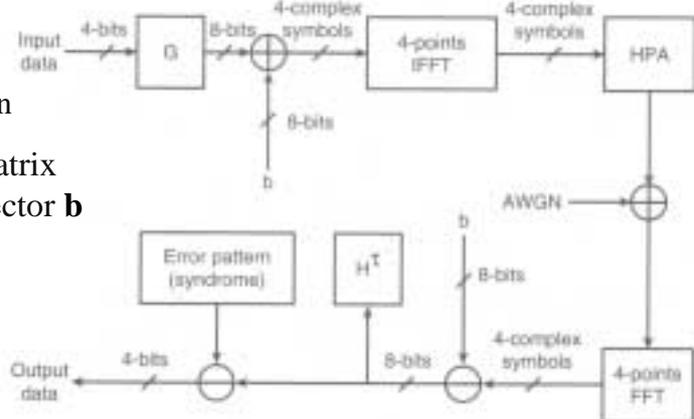
Functional block diagram of an OFDM system using the proposed block coding :

Achieve the minimum PAPR for system using 4 subcarriers and QPSK modulation

The algorithm utilizes 4x8 generator matrix  $\mathbf{G}^*$  followed by a 1x8 phase rotation vector  $\mathbf{b}$  to yield the encoded output  $\mathbf{x} = \mathbf{u}\mathbf{G} + \mathbf{B}$

$\mathbf{u}$  : input data vector to be encoded

$\mathbf{G}$  and  $\mathbf{B}$  are found by examining all possible  $4^4 = 256$  combinations of QPSK input sequences



Next, the 16 symbol combinations, which yield the minimum PAPR of 7.07W after the IFFT is selected.

In this case  $G = \begin{pmatrix} 10010110 \\ 01010101 \\ 00110011 \\ 00001111 \end{pmatrix}$  and  $b = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1]$

Choice of the decoder  $H$  :

$s = GH^T = 0$  where  $s$ : syndrome vector ;  $0 = 4 \times 4$  zero vector

**Comparison :**

Conventional OFDM : PAPR = 6.02dB; Proposed scheme: PAPR = 2.58dB

**Conclusion :** various coding techniques provide methods for PAPR reduction at the expense of bandwidth

Usefulness of these techniques are limited to OFDM systems with small number of subcarriers and small constellation sizes. Otherwise : code rate become very low

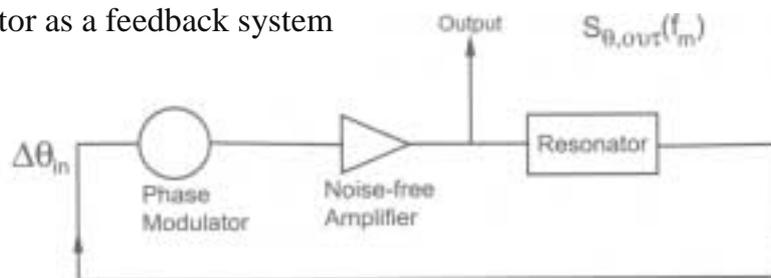
Actual benefits of coding for PAPR reduction for practical OFDM systems are very small

3 Phase noise

**3 Phase noise**

Like all real devices : oscillator is a source of noise in the system

Suitable model : oscillator as a feedback system



Transfer characteristics of the phase modulator are determined by the flicker noise

(noise that varies inversely with frequency) :  $S_{\theta}(\Delta f) = \frac{N_0}{P} \left( 1 + \frac{f_z}{\Delta f} \right)$

$N_0$  = noise density at the output of real unity gain amplifier

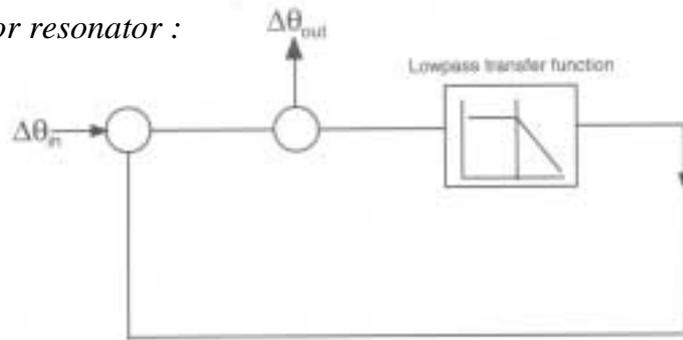
$\Delta f$  = frequency offset

$f_z$  = corner frequency of zero in the magnitude response of the phase modulator

Resonator : low pass transfer function :  $L(\omega_m) = \frac{1}{1 + j(\omega_p/\omega_m)}$

$\omega_p$  = pole resonator (rad/s) = half bandwidth of the resonator

Equivalent lowpass representation for resonator :



Closed loop response of the phase feedback loop :  $\Delta\theta_{out}(\Delta f) = \left(1 + \frac{\omega_p}{j\omega_m}\right) \Delta\theta_{in}(\Delta f)$

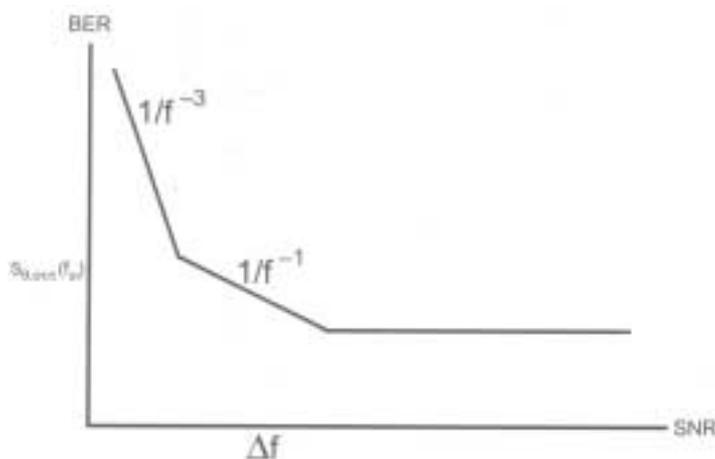
Power spectral density of the phase noise at the output of the resonator :

$$S_{\theta_{out}}(\Delta f) = \left[1 + \left(\frac{f_p}{\Delta f}\right)^2\right] S_{\theta_{in}}(\Delta f)$$

Overall transfer characteristics :  $S_{\theta_{out}}(\Delta f) = \frac{N_0}{P} \left(1 + \frac{f_z}{\Delta f}\right) \left[1 + \left(\frac{f_p}{\Delta f}\right)^2\right]$

Remark : this equation does not account for the thermal noise floor, which is present in all physical devices

Power spectral density for phase noise :



Performance evaluation of BPSK and 16 QAM with phase noise bandwidths of 40KHz and 100KHz :

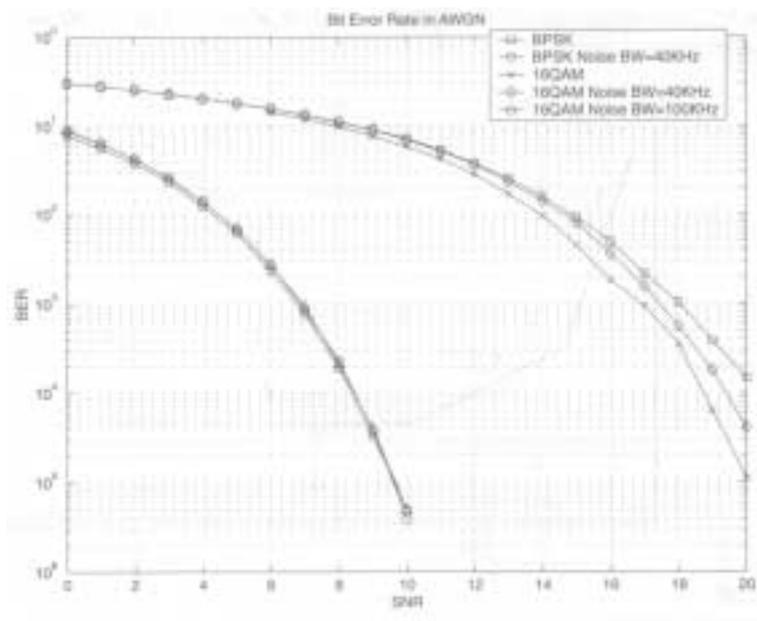
**For BPSK :**

no perceivable degradation in performance for the phase noise generated

**For 16 QAM :**

About 0.5dB degradation at noise bandwidth of 40 KHz

About 1.5dB degradation at noise bandwidth of 100KHz



4 IQ imbalance

**4 IQ imbalance**

Block diagram of a real IQ modulator

Amplitude imbalances :  $\epsilon$   
Phase imbalances :  $\phi$

**Effects of these imbalances :**

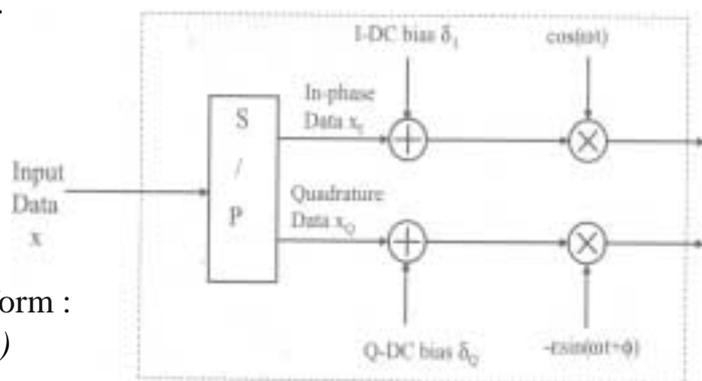
If  $x$  is input data sequence:

Ideal complex modulate carrier waveform :  
 $y(t) = x_I(nT)\cos(\omega t) + jx_Q(nT)\sin(\omega t)$

IQ imbalanced modulated waveform :  
 $\tilde{y}(t) = x_I(nT)\cos(\omega t) + \delta_I + \delta_Q - \epsilon x_Q(nT)\sin(\omega t + \phi)$

$\tilde{y}(t)$  simplifies after adjustment for the DC offset terms to  
 $\hat{y}(t) = [x_I(nT) - \epsilon x_Q(nT)\sin(\phi)]\cos(\omega t) - \epsilon[1 + \cos(\phi)]x_Q(nT)\sin(\omega t)$   
 $\hat{y}(t) = \tilde{y}(t) - (\delta_I + \delta_Q)$

At the receiver : synchronization must be performed with  $\hat{y}(t)$  rather  $y(t)$



**Problem :**

Imbalanced signal is in general subjected to frequency offset, channel fading and received in the presence of additive white noise process

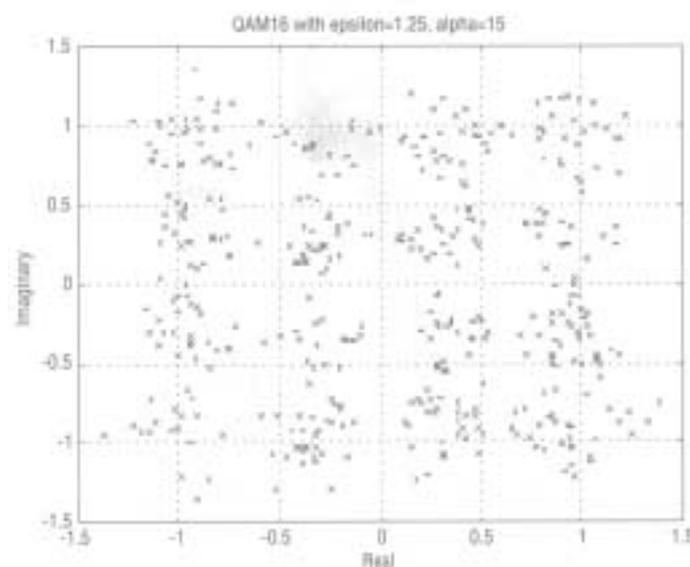
Furthermore, the channel estimation, packet detection and frequency offset algorithms exploit the phase information of the training sequence

Difference between the phase of the imbalanced signal and the balanced signal :

$$\phi_{IM} = -\tan^{-1}\left(\frac{[x_I(nT) - \epsilon x_Q(nT)\sin(\phi)]}{\epsilon[1 + \cos(\phi)]x_Q(nT)}\right)$$

$$\phi_{BAL} = -\tan^{-1}\left(\frac{x_I(nT)}{x_Q(nT)}\right)$$

*Impacts of IQ imbalance on the signal constellation :*



Potentially IQ imbalance can limit the ability of the receiver to achieve synchronization ➡ would be devastating !

## Summary

### 1 Adaptive predistortion techniques

Predistortions algorithm for a cascade of HPA and linear filter

Methods for estimating the inverse characteristic for the HPA

### 2 Coding techniques for amplifier nonlinear distortion mitigation

Partial transmit sequence techniques

Modification of the PTS algorithm

Selective mapping

Block coding

### 3 Phase noise

### 4 IQ imbalance

## Homework

In slide 6 is shown the learning curves of the system estimator. Please, give an example of important issue for fast convergence of an adaptive data predistorter, and give the solution.

Hint : you can see :

*Karam, G.; Sari, H.;*

**A data predistortion technique with memory for QAM radio systems**

Communications, IEEE Transactions on , Volume: 39 , Issue: 2 , Feb. 1991

Pages:336 - 344