S-72.333 Postgraduate Course in Radio Communications

2.3.2004

Capture Effects on ALOHA and CSMA

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  - SNR
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  on the throughput with capture effect

Capture Effect (1/2)

- Conventional analysis of random access protocols assumes all packets are lost in collision
- In real radio channels large variation of signal levels
  => strongest signal may be correctly decoded
- Difference in signal levels caused by
  - different Tx powers
  - different distances (near-far effect)
  - fading
- Strongest packet *captures* the receiver in presence of overlapping packets
- Described by capture probability

Capture Effect (2/2)

- Capture probability depends on
  - distribution of user terminals (near-far effect)
  - fading
  - power control
  - average SNR
  - modulation and coding
  - packet length

=> Numerical evaluation often used

Throughput Analysis with Capture Effect (1/3)

Assumptions :

- Constant packet duration T_p (seconds) and length L (bits)
- Infinite population of users and Poisson packet arrival model
- Constant average arrival rate G (steady-state condition)
- Negligible propagation delay
- Perfect acknowledgments from the receiver
- Base station in the center and terminals distributed around it with a given distribution
  - ring distribution (same distance or average-power control)
  - bell-shaped distribution (convenient approximation of uniform distribution) $\rho(r) = 2re^{-(\pi/4)r^4}$

Throughput Analysis with Capture Effect (2/3)

- Case: slotted ALOHA
- With probability $P_C(k)$ a randomly chosen test packet captures k interfering packets in the same slot
- Throughput is given by

$$S = \sum_{k=0}^{\infty} P(k+1) P_C(k)$$

  where $P(k+1)$ is the probability of $k+1$ overlapping packets
- For Poisson arrival process with traffic load G

$$P(k) = \frac{G^k e^{-G}}{k!}$$

- Lower bound $S=Ge^{-G}$ (no capture: $P_C(0)=1$ and $P_C(k)=0$ for $k>1$)
- Upper bound $S=1-e^{-G}$ (perfect capture: $P_C(k)=1$ for all k)

Throughput Analysis with Capture Effect (3/3)

- Case: nonpersistent CSMA
- Assuming perfect acknowledgments (Zdunek et al. '89)

$$S = \frac{\sum_{k=0}^{\infty} \frac{1}{k!} P_C(k) e^{-aG} (aG)^k}{1 + 2a + \frac{e^{-aG}}{G}}$$

where *a* is the ratio of maximum transmission delay to packet duration

- Typically *a* is small (*a<0.01*)

Power Capture Model

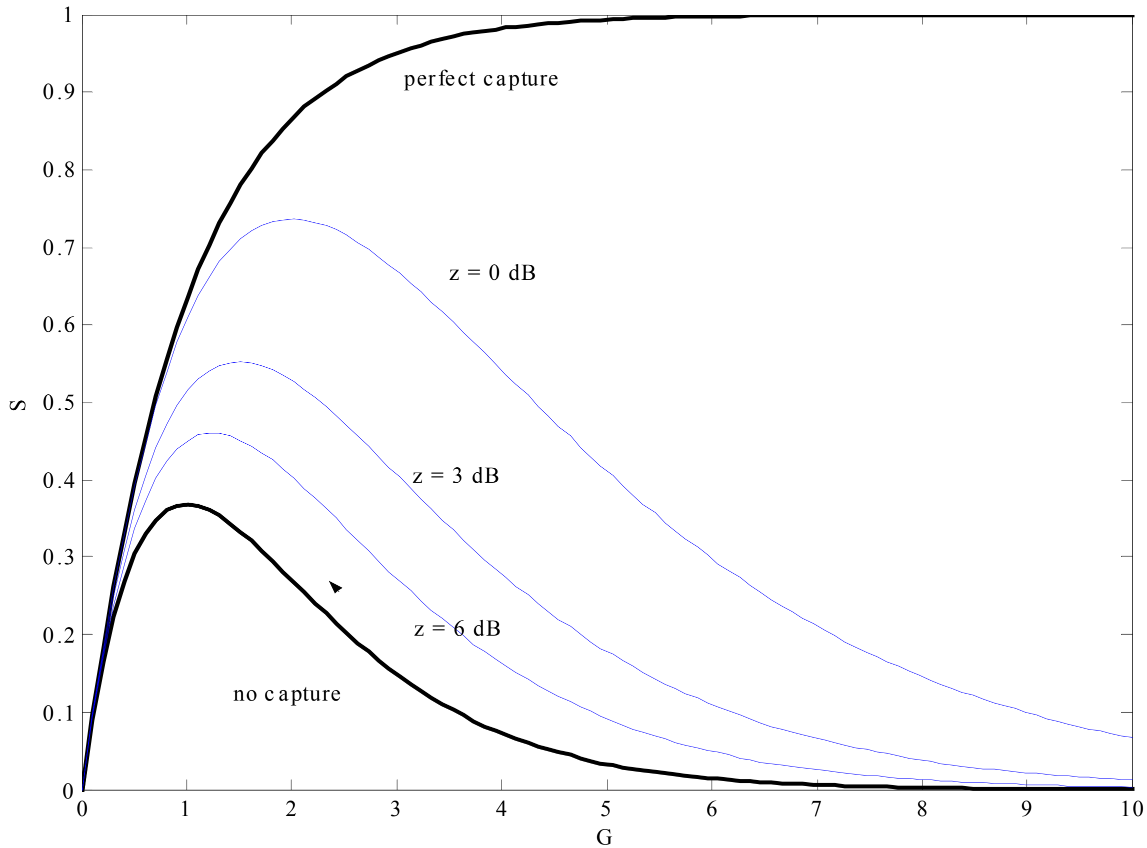
- Power capture model / Capture ratio model
- Test packet is received successfully if its instantaneous power is larger than the instantaneous joint interference power by a minimum certain threshold factor z

$$\gamma = \frac{p_s}{p_I} > z$$

=> Capture probability $P_C(k)$ is obtained from distributions of p_s and p_I

- Threshold factor z is called capture ratio
- Instantaneous power is assumed to remain constant for packet duration
- Capture ratio model is simple and relatively reliable (Linnartz et al. '92)

Throughput of Slotted ALOHA in Rayleigh Fading with Capture Effect (Power Capture Model)



Exact Evaluation of Capture Probability

- Assume a test packet phase-locked to the receiver and k interfering packets
- Let a_0 be the amplitude of test packet bits and $\mathbf{G}_k=[g_1,\ldots,g_k]$ define the interference

$$P_C(k) = \int_0^{\infty} da_0 \int_{-\infty}^{\infty} dg_1 \cdots \int_{-\infty}^{\infty} dg_k f_{A_0}(a_0) f_{G_1}(g_1) \cdots f_{G_k}(g_k) P_C(k \mid a_0, \mathbf{G}_k)$$

- If each packet of L bits is protected by BCH block code capable of correcting up to t bit errors

$$P_C(k \mid a_0, \mathbf{G}_k) = \sum_{i=0}^{t} \binom{L}{i} \left(1-\overline{P}_b\right)^{L-i} \overline{P}_b^{\,i}$$

where
$$\overline{P}_b = \frac{1}{2^k} \sum_{\alpha_1=\pm 1} \cdots \sum_{\alpha_k=\pm 1} \tfrac{1}{2}\operatorname{erfc}\left[\frac{a_0 + \sum_{i=1}^{k} \alpha_i g_i}{\sqrt{N_0}}\right]$$

Effect of SNR

- Exact evaluation of throughput of slotted ALOHA in Rayleigh fading with capture effect
- Different levels of SNR and ring and bell-shaped terminal distributions
- Bell-shaped distribution shows better performance and is less sensitive to SNR

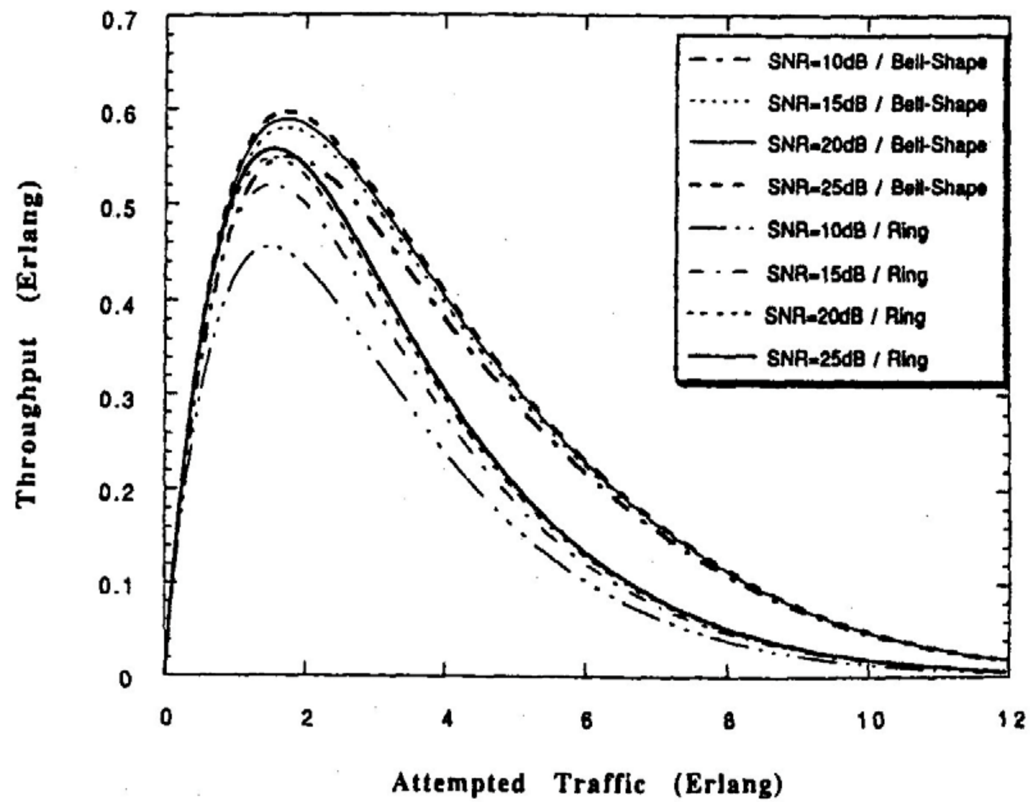


Figure 11.23 Exact calculations of throughput versus attempted traffic for slotted ALOHA in Rayleigh fading at different levels of SNR, assuming two distributions of terminals. For both bell- and ring-shaped distributions, the modulation is BPSK and the packet length is $L = 16$ bits.

Effect of packet length

- Exact evaluation of throughput of slotted ALOHA and nonpersistent CSMA with and without capture effect
- Different packet lengths and bell-shaped terminal distribution
- *a=0.01*
- Minimal sensitivity to packet length (due to slow fading)
- Throughput increase due to capture effect larger in slotted ALOHA than CSMA

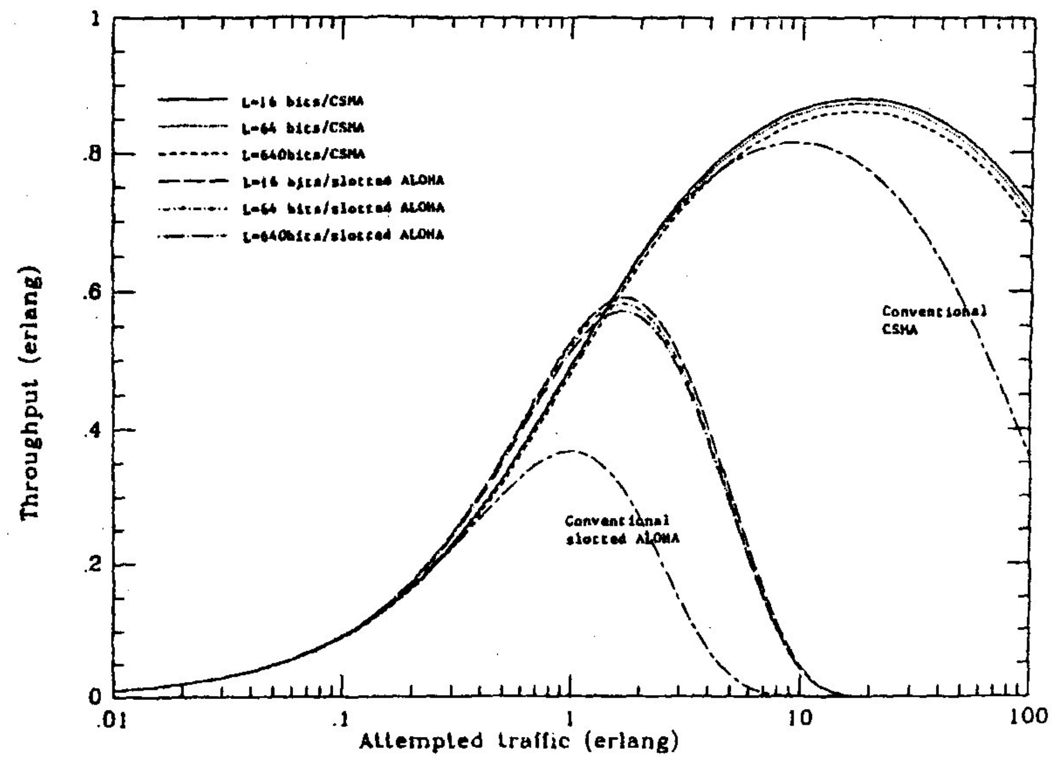


Figure 11.28 Effects of packet length on throughput for CSMA and slotted ALOHA with capture. The modulation is BPSK, and SNR = 20 dB.

Effect of modulation

- Exact evaluation of throughput of slotted ALOHA in Rayleigh fading with capture effect
- Different modulation methods and ring and bell-shaped terminal distributions
- Bell-shaped distribution shows better performance and is less sensitive to modulation
- Effect of modulation is small

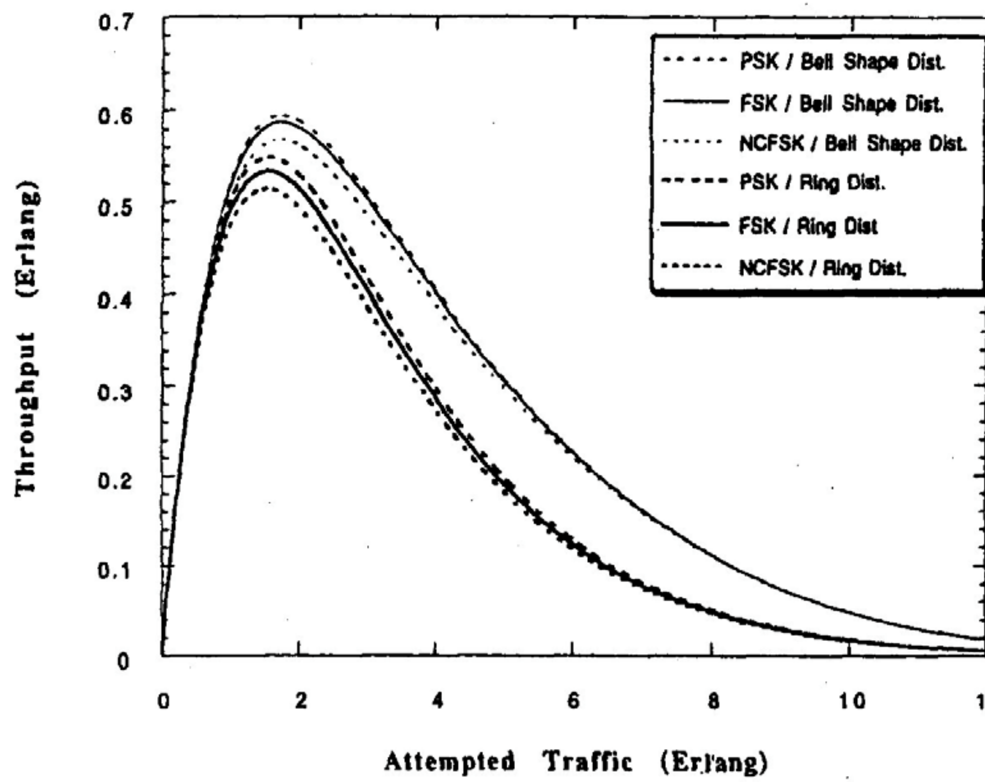


Figure 11.25 Effect of different choices of modulation method on throughput versus offered traffic. Slotted ALOHA in Rayleigh fading is assumed, the packet length is $L = 16$, and the SNR = 20 dB. The modulation schemes are PSK, FSK, and NCFSK, and both the bell-shaped and ring-shaped distributions are shown.

Summary

- Capture effect increases throughput significantly when signal levels vary due to
  - fading
  - near-far effect
- Larger effect on ALOHA than CSMA
- Packet length, SNR, modulation and coding have little effect on throughput

References

- Pahlavan, Levesque, "Wireless Information Networks," Chapter 11, 1995
- Arnbak, Blitterswijk, "Capacity of Slotted ALOHA in Rayleigh-Fading Channels," IEEE J. SAC, Feb. 1987
- Linnartz, Hekmat, Venema, "Near-Far Effects in Land Mobile Random Access Networks with Narrow-Band Rayleigh Fading Channels," IEEE Trans. VT, Feb. 1992

Homework

In a slotted ALOHA network the received signal amplitudes from each terminal are independent Rayleigh distributed random variables with mean power P. The probability density functions of the test packet power and interference power from n other terminals are then given by

$$f_{P_s}(p_s) = \frac{1}{P} e^{-p_s/P}, \quad f_{P_I,n}(p_I) = \frac{1}{P} \frac{(p_I/P)^{n-1}}{(n-1)!} e^{-p_I/P}$$

a) Show that the cumulative distribution function of the signal-to-interference ratio

$$\gamma = \frac{p_s}{p_I}$$

is given by

$$F_{\Gamma,n}(\gamma) = 1 - (1+\gamma)^{-n}$$

b) In a collision of $n+1$ packets, the test packet is destroyed if $\gamma < z$, where z is the capture threshold. Show that with Poisson distributed packet arrival the throughput is given by

$$S = Ge^{-Gz/(1+z)}$$