Modeling of the linear time-variant channel

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1. Characterization of the linear time-variant channel

The transmission channel (radio path) of a radio communication system is in most cases a multipath channel. When changes take place in the propagation environment e.g. the radio stations are mobile, reflectors and scatterers are moving, or the medium itself (troposphere, ionosphere) is changing, then the channel response will also change as a function of time, the radio path is fading.

If the changes are slow so that the channel is almost constant under the duration of a single data symbol or even under the duration of a frame containing hundreds of symbols, the channel can be characterized as quasi-invariant, and the channel response to one symbol or one frame can be calculated using the formulas of time invariant systems. In the transmission of a long message the channel is, however, gradually changing and the changes can be represented e.g. with the joint density function of the multipath channel tap path amplitudes and delays and their correlation functions.

When the channel is significantly changing during the transmission of a data frame new channel representations must be used. The features of these must also be known, so it can be decided when the channel can be modeled as quasi-invariant.

The impulse response $h(t)$ or the transfer function $H(f)$, which is the Fourier transform of the impulse response, fully describes the linear time-invariant system. In the characterization of linear time-variant systems several new system functions are introduced. Their physical interpretation is not always easy, and their conception is rather laborious.

Hereafter we will use the abbreviations **LTV-channel** for the **Linear Time-Variant channel** and **LTI-channel** for the **Linear Time-Invariant channel**.

Bello has developed the theory of LTV-system already 1963 [1] for characterization of the troposcatter channel. Good descriptions are also included in references [2] and [3]. In Fig. 1 the approach used here is presented.

1

[1] P.A.Bello: Characterization of randomly time-variant linear channels. IEEE Trans., Vol CS-11, No. 4, Dec. 1963, pp. 360 - 393.

[2] R.Steele (editor): Mobile radio communications. London 1992, Pentech Press, 779p.

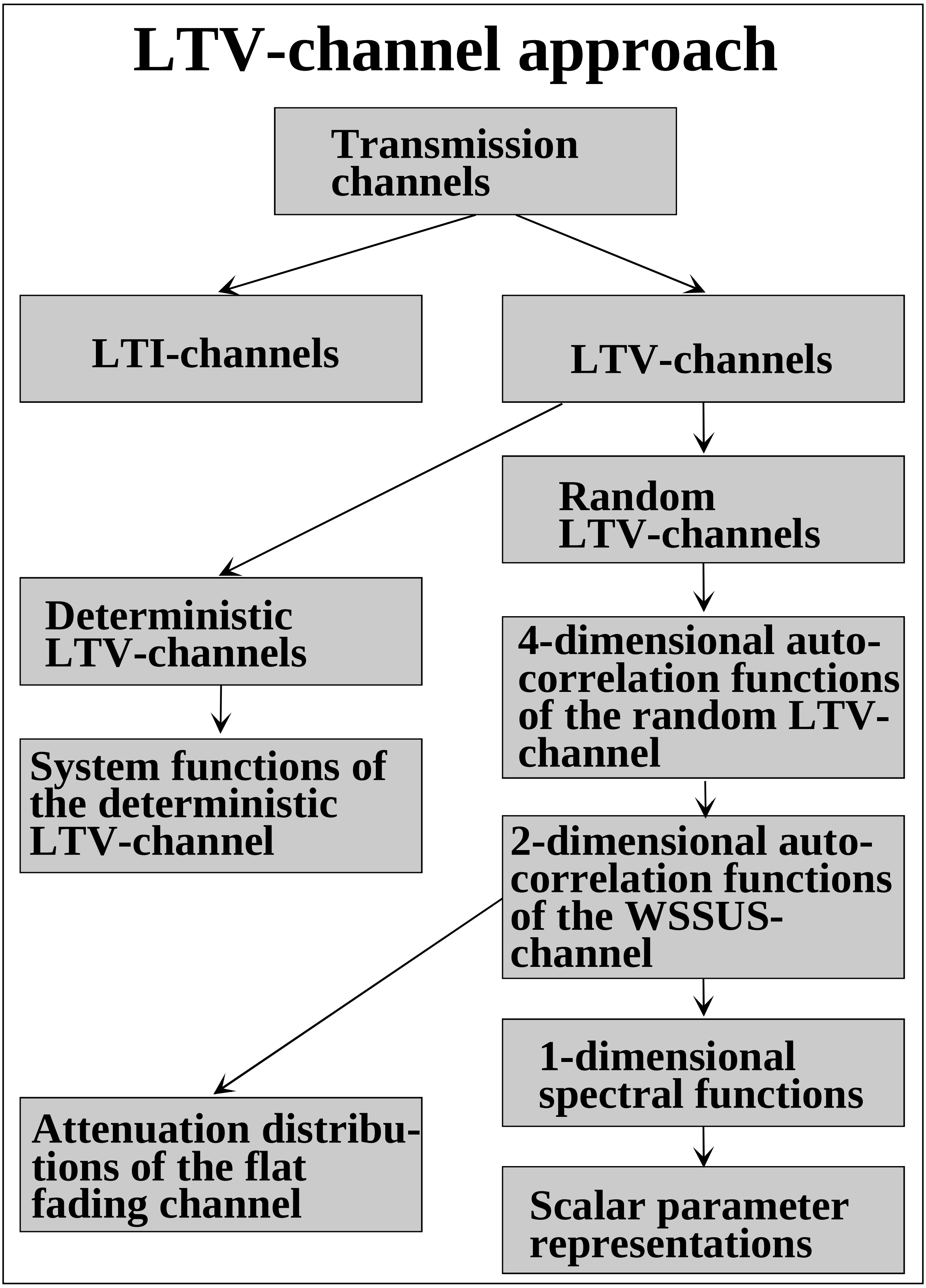


Fig. 1

[3] D.Parsons: The mobile radio propagation channel. London 1992, Pentech Press, 316p.

1.1 Calculation of LTV-channel signal response.

The calculation of the signal response is first presented for the discrete LTV-multipath channel. The result is then generalized for an arbitrary LTV-channel. The signal analysis is performed for complex low-pass signals.

Fig. 2 shows the behavior of the LTV-multipath channel in an idealized situation. The idealisation means e.g. that the signal passes undistorted over all paths. The signal duration is so short that the signals over different path do not overlap at the receiver. The input signal is a narrow pulse, which is transmitted at the time instants t_1, t_2, t_3, and t_4. The channel has of course some minimum delay, before the response starts. On the first time instant t_1 the channel is a three-path channel and the pulse is attenuated individually on each path but the pulse shape remains unchanged. The time-variant nature of the channel can be seen from the varying number, attenuation, and delay of the paths at different time instants. From this can be concluded that the multipath channel response depends both on the time of arrival of the input pulse (time) and of the time past since that (delay). The impulse response is a function of both time and delay, while in the time-invariant channel it is only a function of delay.

The input and output signals are real in the figure but generally the output signal is complex when low-pass representation of the system is used. Also the input signal can be complex. If the delay difference between the paths is less than the signal duration, the path responses will overlap and the response will be very complex and the multipath structure cannot be directly observed.

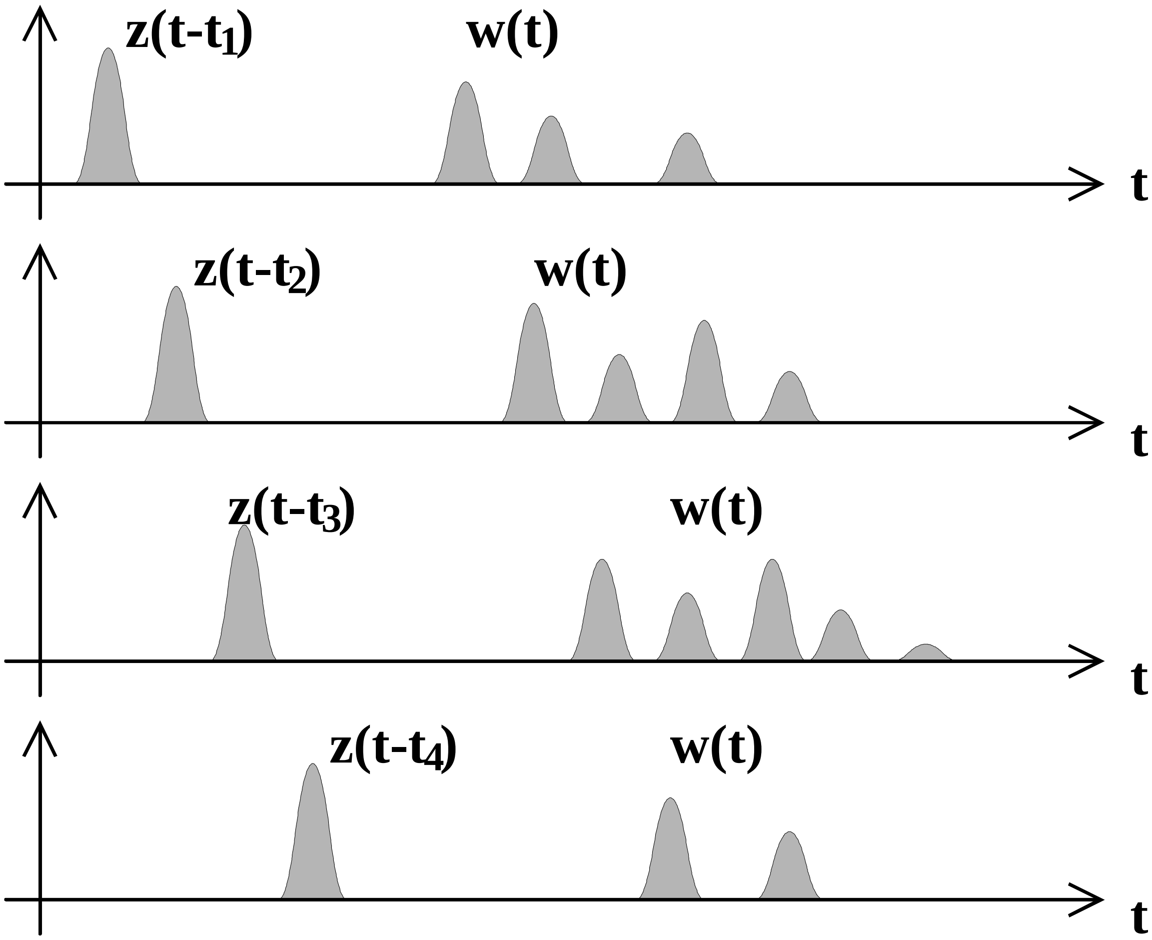
Complex low-pass system representation of the signals is used. The physical band-pass input and output signals are thus:

$$s(t) = \mathrm{Re}\left\{ z(t) e^{j2\pi f_c t} \right\} = \frac{1}{2}\left[z(t) e^{j2\pi f_c t} + z^*(t) e^{-j2\pi f_c t} \right] \tag{1}$$

$$r(t) = \mathrm{Re}\left\{ w(t) e^{j2\pi f_c t} \right\} = \frac{1}{2}\left[w(t) e^{j2\pi f_c t} + w^*(t) e^{-j2\pi f_c t} \right] \tag{2}$$

where f_c is the carrier frequency in use.

The time-variant channel pulse response at four different time instants



$$w(t) = \sum_{n=0}^{M(t)-1} \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} z\left(t - \tau_n(t)\right)$$

Fig. 2

From Fig. 2 can be seen that the physical output signal of the LTV-multipath channel can be expressed as

$$r(t) = \sum_{n=0}^{M(t)-1} \alpha_n(t) s\left(t - \tau_n(t)\right) \tag{3}$$

where

- $\alpha_n(t)$ is the gain of the n:th propagation path as function of time,
- $\tau_n(t)$ is the propagation delay of the n:th propagation path as function of time,
- $M(t)$ is the number of propagation paths as function of time.

Insertion of Eq. (1) in Eq. (3) gives:

$$\begin{aligned} r(t) &= \sum_{n=0}^{M(t)-1} \alpha_n(t) \operatorname{Re}\left\{ z\left(t - \tau_n(t)\right) e^{j2\pi f_c \left(t - \tau_n(t)\right)} \right\} \\ &= \operatorname{Re}\left\{ \sum_{n=0}^{M(t)-1} \alpha_n(t) z\left(t - \tau_n(t)\right) e^{j2\pi f_c \left(t - \tau_n(t)\right)} \right\} \\ &= \operatorname{Re}\left\{ \sum_{n=0}^{M(t)-1} \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} z\left(t - \tau_n(t)\right) e^{j2\pi f_c t} \right\} \end{aligned} \tag{4}$$

In the last form of Eq. (4) the complex low-pass output signal can be recognized:

$$w(t) = \sum_{n=0}^{M(t)-1} \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} z\left(t - \tau_n(t)\right) \tag{5}$$

Eq. (5) can also be written as:

$$w(t) = \left[\sum_{n=0}^{M(t)-1} h_n(t) \delta\left(\lambda - \tau_n(t)\right) \right] \otimes z(\lambda) \tag{6}$$

where $\otimes$ denotes convolution and

$$h_n(t) = \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} \tag{7}$$

From this follows the impulse response of the LTV-multipath channel:

$$h(\lambda,t) = \sum_{n=0}^{M(t)} h_n(t)\delta\left(\lambda - \tau_n(t)\right) \tag{8}$$

where $h(\lambda,t)$ is the response at time t to an impulse arrived λ seconds earlier.

Eqs. (5) - (8) are valid for a discrete multipath channel. In some radio channels e.g. the troposcatter channel a time-continuous impulse response is a more adequate model. Then the signal response is calculated with a generalized convolution integral:

$$w(t) = \int_{-\infty}^{\infty} h(\lambda,t)z(t-\lambda)d\lambda \tag{9}$$

The response of the multipath channel to a sinewave

$$z(t) = e^{j2\pi f_z t} \tag{10}$$

is obtained by inserting the input signal in Eq. (10) in Eq. (5), which gives

$$\begin{aligned} w(t) &= \sum_{n=0}^{M(t)-1} \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} e^{j2\pi f_z \left(t-\tau_n(t)\right)} \\ &= \sum_{n=0}^{M(t)-1} \alpha_n(t) e^{-j2\pi \left(f_c + f_z\right)\tau_n(t)} e^{j2\pi f_z t} \end{aligned} \tag{11}$$

A simple example will show that the path delay variation and not the amplitude variation will mainly determine the dynamic characteristics of the LTV-channel.

Example 1. In Fig. 3 it is assumed that the reflection coefficient is -1. The geometrical length of the propagation paths is calculated using the right-angled triangles and the image principle. The path amplitude gains and delays follow directly from the length of the propagation paths.

The logarithmic amplitude of the received signal as a function of time is calculated assuming the carrier frequency 2 GHz and the speed of the receiver 15 m/s. The geometrical values are given in Fig. 3.

The graph in Fig. 3 shows the logaritmic signal amplitud as function of time assuming first that only the delays depend on time, and then that both delays and gains are changing. In this simple situation the time dependence is very regular, which is nearly equal under both assumptions. From this can be concluded that the behavior is determined almost solely by the delay changes and the changes in path gains only have minor effects. This can be generalised to almost all radio channels. The reason for this behaviour is of course that delay changes are multiplied by the carrier frequency in the exponential function in Eq. (11).

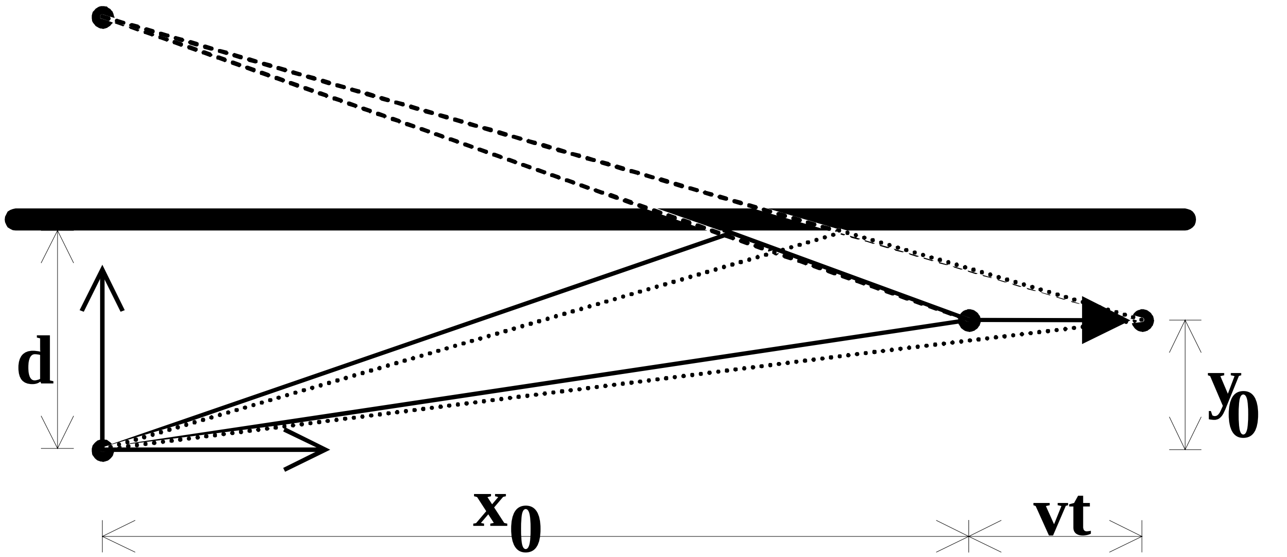
Example 2. The mobile channel can in a small area be modeled with the impulse response

$$h(\lambda, t) = \sum_{k=0}^{M-1} h_k e^{j2\pi \nu_k t} \delta\left(\lambda - \tau_k\right) \tag{12}$$

This is a discrete M-path channel where the complex path gains h_k and the delays τ_k do not change with time. As a consequence of the constant speed each path has its own constant Doppler-shift

$$\nu_k = \frac{v}{c} f_c \cos\left(\alpha_k\right) \tag{13}$$

where v on speed of the mobile station, c is the speed of the radio wave, f_c is the carrier frequency, and α_k is the angle between the k:th propagation path and the mobile station velocity vector. In reality h_k, τ_k, α_k, and M are functions of time but in the small region where the mobile station moves under one transmission frame they are approximately constant. The randomness of this channel appears so that the model parameters after some few frames have obtained new values.



$$r_0(t) = \sqrt{\left(x_0 + vt\right)^2 + y_o^2}$$

$$r_1(t) = \sqrt{\left(x_0 + vt\right)^2 + \left(2d - y_0\right)^2}$$

$$\alpha_o(t) = \frac{k}{r_0(t)} \qquad \tau_0(t) = \frac{c}{r_o(t)}$$

$$\alpha_1(t) = \frac{k}{r_1(t)} \qquad \tau_1(t) = \frac{c}{r_1(t)}$$

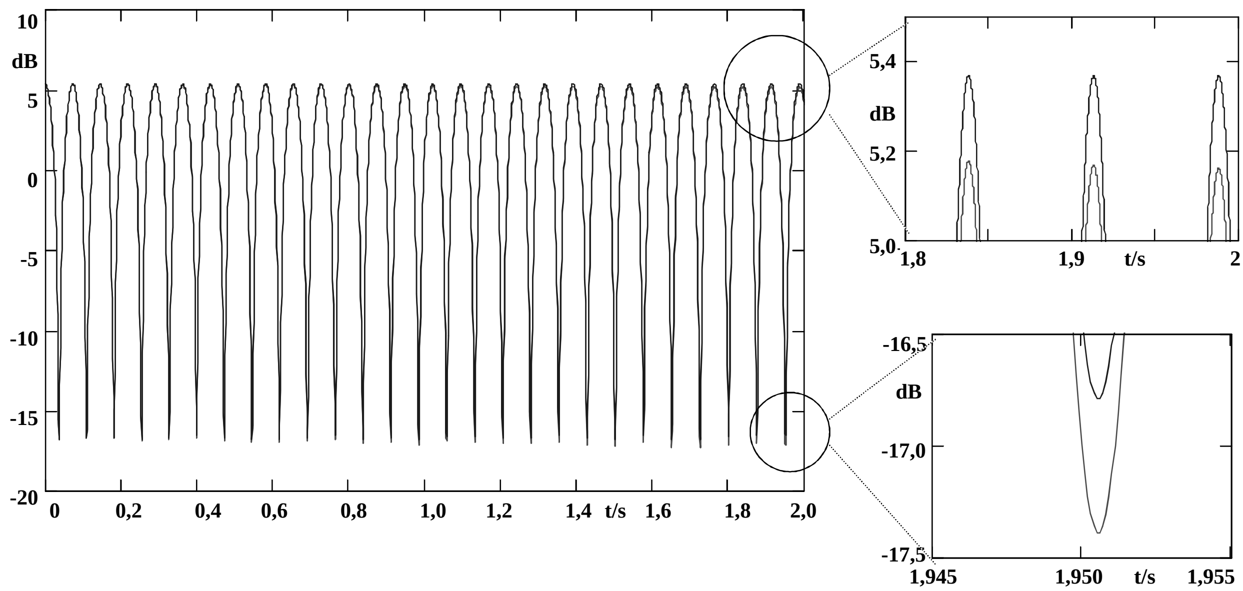


Fig. 3

3.2.2 The system functions of a LTV-channel

The basic system function of a deterministic LTV-channel and of a sample function of a random LTV-channel is time variant impulse response $h(\lambda,t)$ described above, also known as the channel input delay spread function.

Other frequently used system functions are:

- the time-variant channel transfer function:

$$H(f,t) = \mathcal{F}_{\lambda}\left\{h(\lambda,t)\right\} = \int_{-\infty}^{\infty} h(\lambda,t)\exp\left(-j2\pi f\lambda\right)d\lambda \qquad (14)$$

which is obtained by Fourier-transforming the impulse response with respect to the delay variable λ. It describes the complex envelope of the output signal when the input signal is $e^{j2\pi ft}$.

- the channel output Doppler spread function:

$$D(f,\nu) = \mathcal{F}_{t}\left\{H(f,t)\right\} = \int_{-\infty}^{\infty} H(f,t)\exp\left(-j2\pi\nu t\right)dt \qquad (15)$$

which is obtained by Fourier-transforming the transfer function with respect to the time variable t, and which describes the channel frequency response on the frequency $f+\nu$, when the input signal is $e^{j2\pi ft}$.

- the delay-Doppler spread function

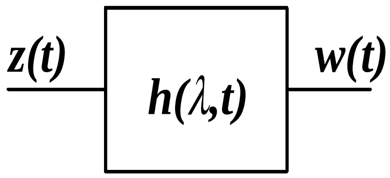
$$S(\lambda,\nu) = \mathcal{F}_{t}\left\{h(\lambda,t)\right\} = \int_{-\infty}^{\infty} h(\lambda,t)\exp\left(-j2\pi\nu t\right)dt \qquad (16)$$

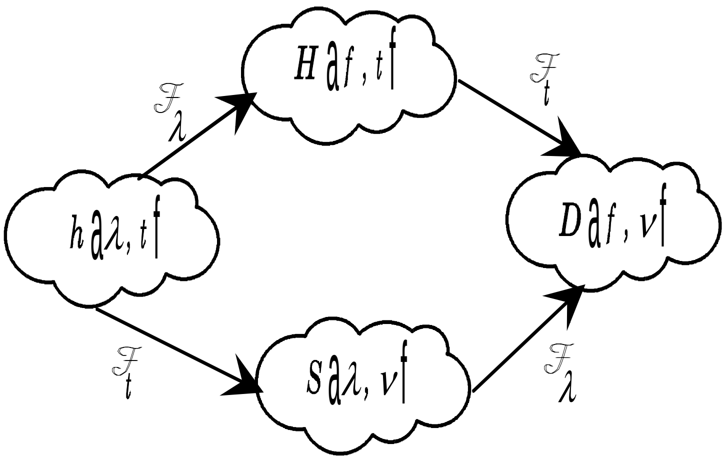
is obtained by Fourier-transforming $h(\lambda,t)$ with respect to the time variable t or by taking the inverse Fourier-transform of the Doppler-spread function with respect to frequency f. It gives the complex gain of the channel on the delay interval $[\tau + d\tau]$ and the Doppler-shift interval $[\nu + d\nu]$.

The Fourier-transform relations of these four system functions are shown in Fig. 4. In addition four other dual system functions can be defined.

REPRESENTATION OF THE LTV-CHANNEL

SYSTEM FUNCTIONS OF A DETERMINISTIC CHANNEL





$$\boldsymbol{w(t) = \int_{-\infty}^{\infty} h(\lambda, t) z(t-\lambda) d\lambda}$$

$$\boldsymbol{= \int_{-\infty}^{\infty} H(f, t) Z(f) e^{j2\pi ft} df}$$

$$\boldsymbol{= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(\lambda, \nu) z(t-\lambda) e^{j2\pi\nu\lambda} d\nu d\lambda}$$

$$\boldsymbol{= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D(f-\nu, \nu) Z(f-\nu) e^{j2\pi ft} d\nu df}$$

Fig. 4

Example 2 continues
The instantaneous system functions of this channel model are:

- the time-variant impulse response (model definition):

$$h(\lambda,t)=\sum_{k=0}^{M-1} h_k e^{j2\pi\nu_k t}\delta(\lambda-\tau_k) \tag{17}$$

- the time-variant transfer function:

$$H(f,t)=\mathcal{F}_\lambda\{h(\lambda,t)\}=\sum_{k=0}^{M-1} h_k e^{j2\pi\nu_k t} e^{-j2\pi f\tau_k} \tag{18}$$

- the output Doppler-spread function:

$$D(f,\nu)=\mathcal{F}_t\{H(f,t)\}=\sum_{k=0}^{M-1} h_k \delta(\nu-\nu_k) e^{-j2\pi f\tau_k} \tag{19}$$

- the delay-Doppler-spread function:

$$S(\lambda,\nu)=\mathcal{F}_t\{h(\lambda,t)\}=\sum_{k=0}^{M-1} h_k \delta(\nu-\nu_k)\delta(\lambda-\tau_k) \tag{20}$$

In Fig. 5 the impulse response, amplitude frequency response, and the delay-Doppler-spread functions are shown for a 2-path channel with the parameter values:

$$h_1=1,\quad h_2=0{,}9$$
$$\tau_1=1\mu s,\quad \tau_2=5\mu s$$
$$\nu_1=100Hz,\quad \nu_2=-50Hz$$

From the figure or from Eqs. (17) - (20) it appears that the channel impulse response dependence of time can be seen only from the phase behaviour of the complex path gain, which is a linear function of time (In the figure drawn modulo 2π). In the transfer function it can be seen as a gliding of the transmission minimum through the signal bandwidth. the delay-Doppler-spread functions contains impulses which tell the delay and Doppler-shift of each path.

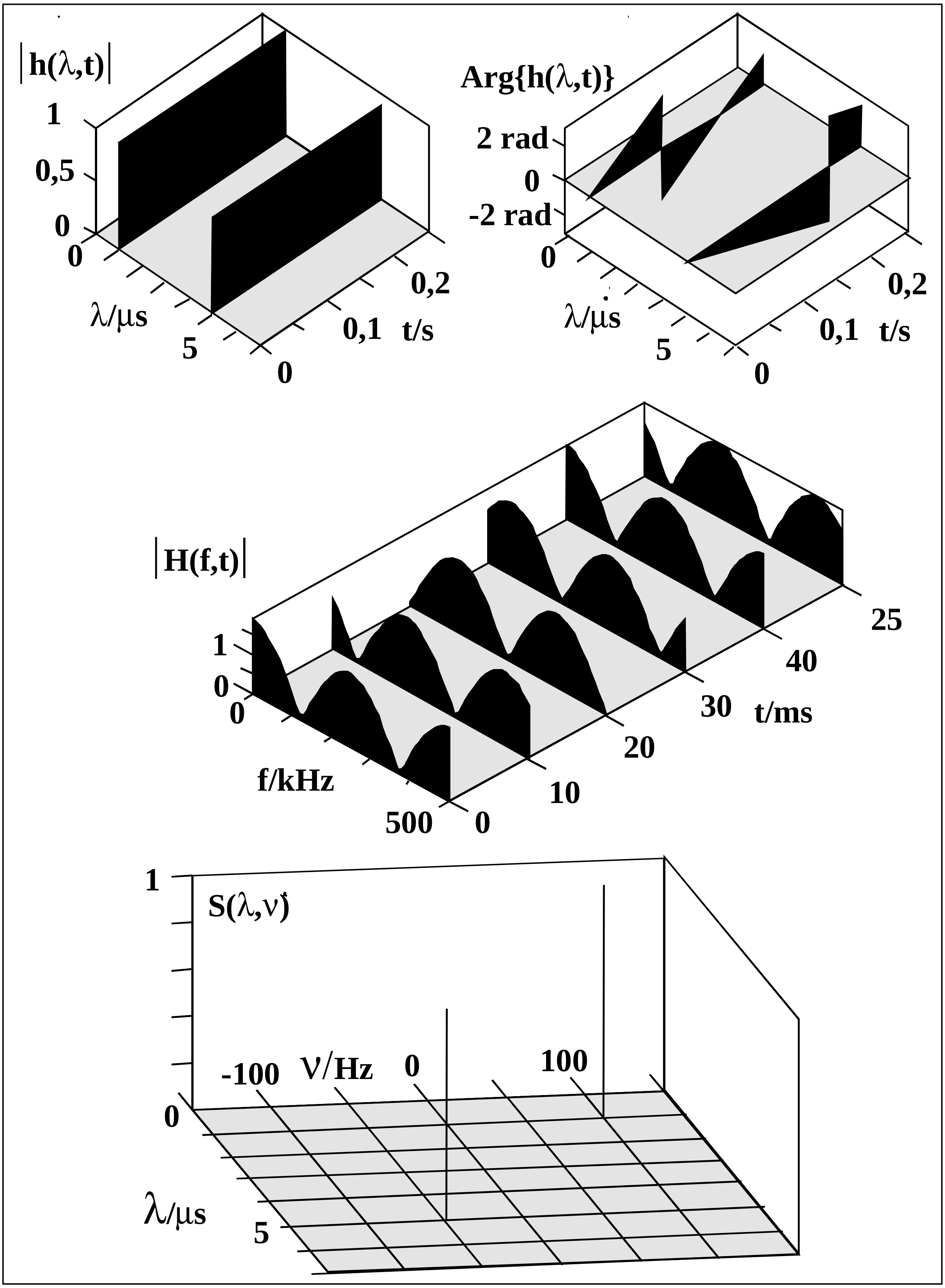

|h(λ,t)|
1
0,5
0
0
λ/µs
5
0
0,1
0,2
t/s
Arg{h(λ,t)}
2 rad
0
-2 rad
0
λ/µs
5
0
0,1
0,2
t/s
|H(f,t)|
1
0
0
f/kHz
500
0
10
20
30
40
25
t/ms
1
S(λ,ν)
-100
ν/Hz
0
100
0
λ/µs
5

Fig. 5