S-72.340 INFORMATION THEORY

Exercise 1, September 20, 2000

1. Three events E_1, E_2 and E_3 are defined in the same sample space Ω. Their probabilities are: $P(E_1) = P(E_2) = P(E_3) = 0.25$. Let E_0 denote the event that at least one of these three events occurs. What is $P(E_0)$ if

   a) E_1, E_2 and E_3 are mutually exclusive?

   b) E_1, E_2 and E_3 are independent?

   c) E_1, E_2 and E_3 are different names for the same event?

2. a) In an experiment consisting of 10 throws of a pair of fair dice, find the probability of the event that at least one double 6 occurs.

   b) An urn contains 8 white balls and 4 red balls. The experiment consists of drawing 2 balls from the urn without replacement. Find the probability that both balls drawn are white.

3. Show that if $P(A) > P(B) > 0$, then $P(A|B) > P(B|A)$.

4. There are 100 patients in a hospital with a certain disease. Of these, 10 are selected to undergo a drug treatment that increases the percentage cured rate from 50 percent to 75 percent. What is the probability that the patient received a drug treatment if the patient is known to be cured.

5. The continuous random variable X has the probability density function (pdf)

$$f_X(x) = \begin{cases} k(2x - x^2) & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

   Find the value of k, the cumulative distribution function (cdf) of X and $P(X > 1)$.

6. Given that X is a Poisson random variable and $p_X(0) = 0.0498$ (the probability that $X = 0$), find $E(X)$ and $P(X \geq 3)$.

Try to solve as many problems as you can. Return your solutions by the time that the exercise begins. You can bring them to the exercise or put them into the 'return box' below the information board of our laboratory on the 2nd floor.