S-72.340 INFORMATION THEORY

Exercise 11, (the last one) November 29, 2000

1. An additive white gaussian noise (AWGN) channel has the output $Y = X + G$, where X is the channel input and G is the probability density function

$$p(n) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-n^2/2\sigma_n^2} .$$

If X is a white gaussian input with $E(X) = 0$ and $E(X^2) = \sigma_X^2$, determine the conditional (differential) entropy $H(X|G)$.

2. Determine the average mutual information $I(X;Y)$ for the AWGN channel defined in Problem 1.

3. Find the capacity of the cascade connection of n binary symmetric channels with the same crossover (error) probability ϵ. What is the capacity when the number of channels goes to infinity?

4. Show that if n Z channels are cascaded, the resulting channel will be equivalent to a Z channel with $\epsilon_1 = \epsilon^n$.

5. A first-order binary Markov source shown in Figure 1 is characterized by the (limiting) state probabilities $P(x_1)$ and $P(x_2)$, and the transition probabilities $P(x_2|x_1)$ and $P(x_1|x_2)$. The entropy of the Markov source is

$$H(X) = \sum_{k=1}^{2} P(x_k) H(X|x_k) ,$$

where $H(X|x_k)$ is the entropy conditioned on the source being in state x_k. The conditional entropies $H(X|x_1)$ and $H(X|x_2)$ are given by the binary entropy functions $H[P(x_2|x_1)]$ and $H[P(x_1|x_2)]$, respectively. Determine $H(X)$ when $P(x_2|x_1) = 0.2$ and $P(x_1|x_2) = 0.3$. Compare the result to the entropy of a binary discrete memoryless source (DMS) with the same output letter probabilities $P(x_1)$ and $P(x_2)$.

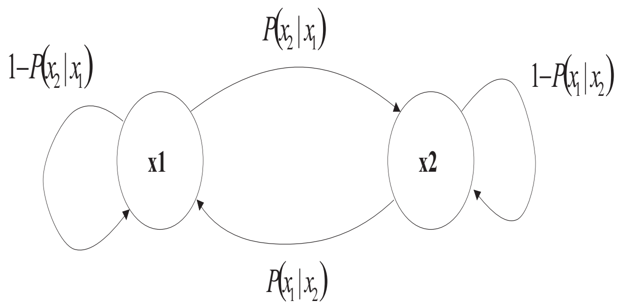

Figure 1: A binary Markov source

Try to solve as many problems as you can. Return your solutions by the time that the exercise begins. You can bring them to the exercise or put them into the 'return box' below the information board of our laboratory on the 2nd floor.