S-72.340 INFORMATION THEORY

Exercise 4, October 11, 2000

1. A source produces a sequence of statistically independent binary digits with the probabilities $P(\mathbf{1}) = 0.005$ and $P(\mathbf{0}) = 0.995$. These digits are taken 100 at a time and a binary code word is provided for every sequence of 100 digits containing 3 or fewer **1**'s.

   a) If the code words are all of the same length, find the minimum length required to provide the specified set of code words.

   b) Find the probability of getting a source sequence for which no code word has been provided.

2. A source produces statistically independent binary digits with the probabilities $P(\mathbf{0}) = 3/4$ and $P(\mathbf{1}) = 1/4$. Consider sequences $\mathbf{u}_K$ of K successive digits and the associated inequality

$$P\left[\left|\frac{I(\mathbf{u}_K)}{K} - H(U)\right| \geq \epsilon\right] \leq \delta(K, \epsilon).$$

   Find K_0 such that this inequality holds for $K \geq K_0$ when $\delta = 0.05$ and $\epsilon = 0.1$. *Hint:* Use the Chebyshev inequality.

3. Use the binary Huffman algorithm to construct a prefix code for source symbols $a_1, \ldots, a_5$ with probabilities $(1/3, 1/5, 1/5, 2/15, 2/15)$. What is the average length of a codeword for this encoding?

4. Same as Problem 3, but use the Shannon-Fano algorithm.

5. Assume that we are transmitting binary digits over a binary *erasure channel*, where transmitted digits may be erased, but they can never be erroneously received. So, the input symbol X is **0** or **1**, and the corresponding output symbol Y is either the same as the input or the erasure symbol E. An erasure occurs with probability ϵ for both input symbols. Let $p_X(\mathbf{0}) = p$ and $p_X(\mathbf{1}) = 1 - p$. Calculate the average mutual information $I(X;Y)$ in terms of p and find the value of p that maximizes $I(X;Y)$. For this maximizing distribution, calculate $I(x;y)$ at all points of the XY ensemble in addition to finding $I(X;Y)$.

Try to solve as many problems as you can. Return your solutions by the time that the exercise begins. You can bring them to the exercise or put them into the 'return box' below the information board of our laboratory on the 2nd floor.