S-72.340 INFORMATION THEORY

Exercise 5, October 18, 2000

1. Prove the *chain rule for entropy:*

   Let $X_1, \dots, X_n$ be drawn according to some joint distribution $P(x_1, \dots, x_n)$. Then

$$H(X_1, \dots, X_n) = \sum_{i=1}^{n} H(X_i | X_{i-1}, \dots, X_1)\,.$$

2. Let X, Y and Z be joint random variables. Show that

$$I(X;Z|Y) \geq I(Z;Y|X) - I(Z;Y) + I(X;Z)\,.$$

3. Consider a sequence of n binary random variables $X_1, \dots, X_n$. Each sequence with an even number of 1's has a probability $2^{-(n-1)}$ and each sequence with an odd number of 1's has probability 0. Find the mutual informations

$$I(X_1;X_2),\ I(X_2;X_3|X_1), \dots, I(X_{n-1};X_n|X_1, \dots, X_{n-2})\,.$$

4. Consider an M-ary symmetric DMC (discrete memoryless channel). Both the input X and the output Y can have one of the M values $0, \dots, M-1$. The transition probabilities are:

$$P(y|x) = \begin{cases} 1-p & \text{when } x = y \\ \frac{p}{M-1} & \text{when } x \neq y\,. \end{cases}$$

   Calculate the average mutual information $I(X;Y)$ assuming that the probability of each input symbol is $1/M$.

5. Calculate the average mutual information $I(X;Y)$ for the channel shown in Figure 1. All the non-zero transition probabilities $P(y|x)$ and also the probabilities of the input symbols are equal to $1/3$.

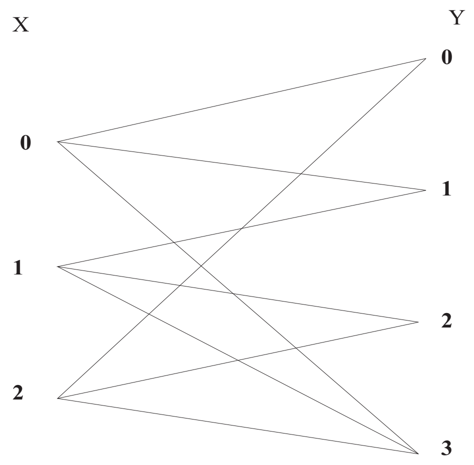

Figure 1: A channel model

Try to solve as many problems as you can. Return your solutions by the time that the exercise begins. You can bring them to the exercise or put them into the 'return box' below the information board of our laboratory on the 2nd floor.