S-72.340 INFORMATION THEORY

Exercise 7, Tuesday October 31, 2000, 10 a.m.-12 a.m. in E208

1. Let $Q_1(x)$ and $Q_2(x)$ be any two probability assignments (distributions) over the same sample space and let $H_1(X)$ and $H_2(X)$ be the corresponding entropies. According to the lectures, for $0 \leq \lambda \leq 1$,

$$Q(x) = \lambda Q_1(x) + (1-\lambda)Q_2(x)$$

is also a probability assignment over the same sample space. Let $H(X)$ be the entropy corresponding to $Q(x)$. Prove that

$$H(X) \geq \lambda H_1(X) + (1-\lambda)H_2(X)\,.$$

and state the conditions for equality.

2. Consider the function of two variables

$$f(\boldsymbol{\alpha}) = \alpha_1(2-\alpha_1) - (\alpha_2+1)^2$$

over the region $\alpha_1 \geq 0\,, \alpha_2 \geq 0$. Show that $f(\boldsymbol{\alpha})$ is concave (convex $\cap$) over the given region.

3. Find the maximum of $f(\boldsymbol{\alpha})$ (of Problem 2) over the given region and indicate clearly why it is a maximum.

4. Find the capacity and an optimizing input probability assignment for the binary non-symmetric channel with the following transition probabilities $P(y|x)$:

$$P(0|0) = 1-\epsilon\,, \quad P(1|0) = \epsilon\,,$$
$$P(1|1) = 1-\delta\,, \quad P(0|1) = \delta\,.$$

Try to solve as many problems as you can. Return your solutions by the time that the exercise begins. You can bring them to the exercise or put them into the 'return box' below the information board of our laboratory on the 2nd floor.